American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-10, pp-36-46
www.ajer.org
Research Paper Open Access

Numerical Analysis of Radiation Effect on Heat Flow through Fin of Rectangular Profile

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ABSTRACT:*This paper presents reports on numerical analysis of radiation effects on heat flow through fin of rectangular profile. Mathematical model was devised by applying the concept of energy balance on an element of the fin normal to direction of heat flow. Galerkin method was applied to express the temperature distribution along the fin surface in terms of finite element formulation. The influence of physical parameters which include: length, thickness, fin metal type and emissivity of the fin on temperature distribution over the rectangular fin profile surface with and without considering radiation heat loss are comparatively studied. The numerical solution results revealed that heat dissipation rate for the fin with thermal radiation is higher than those without thermal radiation. By employing the value of temperature distribution along the fin longitudinal surface for the case considering thermal radiation effect (Model 1), appreciable enhancement of the fin thermal performance was observed for aluminium and copper materials compared to the stainless steel material.For the case of excluding radiation heat loss (Model 2) in fin design, the incurred errors observed for both numerical and analytical solution methods is up to 20% of the total heat loss contributed by radiation. Therefore, the thermal radiation effect has to be considered in the thermal analysis of fin metal type such as aluminium and copper as neglecting this effect will reduce the fins heat dissipation capacity rate and hence the fin performance.*

Keywords:*Galerkin method, Heat Dissipation, Physical parameters,Rectangularfin,Thermal Radiation.*

Date of Submission: 13-09-2017 Date of acceptance: 06-10-2017

I. INTRODUCTION

The demand for improvement in geometric design of heat exchanger with a view to deliver energy efficient toward producing compact and miniaturized equipment in particular in the area of microelectronics and microcooling has prompted several researchers and scholars in recent years to research in the area of heat transfer enhancement techniques both by experimental and numerical analysis. Findings from various studies [1-5] have observed that high performance surface will enhance the heat transfer that takes place within the heat exchanger without incurring penalties on friction and pressure drop that are enough to negate the benefits of heat transfer augmentation. **Manneti et al.** [6] also pointed out that new technologies are giving rise to higher operating temperatures on even smaller devices, and therefore the necessity of better heat transfer performance surface for extending the service life of electronic and structural components.One of the alternatives to achieve this objective is the use of fin attachment.

Fins are the extended surfaces that are widely utilized in many engineering systems to increase the heat transfer area of the system, dissipate heat generated within the system and prolong their functionality [5, 7]. **Aziz and Bouaziz** [8] also reported that a longitudinal fin of constant cross-sectional area (rectangular, circular, elliptic, etc.) is widely used in practice to enhance heat dissipation from a heated primary surface. In many physical situations, the fin is attached to one side of a wall of finite thickness while the other side of the wall is in contact with a hot fluid from which heat transmitted through the wall is ultimately rejected by convection and radiation from the surface of the fin to the environment (sink). Due to its easy of manufacturing process, low cost and simplicity of its designing, the rectangular fin type is widely used and preferred among other various types of the fins [9-10]. In principle, heat transfer takes place through conduction within the fin surface boundaries while convection and radiation occur between its boundaries and surroundings. It is also interesting to point out that the arrangement of this device is most effective when it operates in a natural convection environment where the convection heat transfer coefficient is low. In this circumstance, the radiative component of heat loss from the fins is comparable to the natural convection heat loss. Therefore, the three modes of heat transfer, i.e., conduction, convection, and radiation should be considered in thermal analysis of fin heat sink. However, the reports from many researchers in the open literature as pointed out in the study of **[**11] have revealed that most of the existing studies, either theoretical or experimental, failed to recognize the effects of thermal radiation by considering only natural convection in the thermal performance analysis of the fin heat sink. As reported by **Hatami, et al.** [12] in the effort to finding the best approach to solving problems involving removal of excess damaging heat from system component many research work are being embarked upon with various extended surface geometries such as in plain fins, wavy and corrugated channels, offset-strip fins, louvered fins and vortex generators.Efficiency of horizontal single pin fin subjected to free convection and radiation heat transfer was reported by **Czesław et al** [13].Higher results were observed by the authors for both measured and numerical solution results in comparison with analytical solution results. Studies on optimization of variable cross-section convective pin fins with variable heat transfer coefficient and temperature dependence of the thermal conductivity were also investigated by **Reardon and Razani** [14] without considering the effects of radiation.Considering a convective–radiative fin tip and allowing the thermal conductivity of the fin to vary with temperature, Chiu and Chen [9] utilized Adomian's decomposition procedure to evaluate the heat transfer characteristics of a convecting–radiating longitudinal fin of rectangular profile.

Aziz and Beers-Green [15] presented reports on performance and optimum design of convective–radiative rectangular fin with convective base heating, wall conduction resistance, and contact resistance between the wall and the fin base. Similarly, considering temperature and humidity ratio differences as the driving forces for heat and mass transfer, **Sharqawy and Zubair** [16] carried out analytical solutions for temperature distribution over the fin surface when the fin is fully wet and concluded that the overall fin efficiency is dependent on the atmospheric pressure thus yield an increase in overall fin efficiency with increasing the atmospheric pressure. **Zhang et al.** [17] modelled a fin tube evaporator heat exchanger for ORC cycle and achieved between 60% and 70% waste recovery efficiency for most of the engine's operating regional in their study involving heat transfer analysis of a finned tube evaporator for engine exhaust heat recovery.

Recently, thermal analysis of convective fin with temperature-dependent thermal conductivity and heat generation was carried out by **Ghasemi et al**. [10]. The authors solved the nonlinear temperature distribution equation in the longitudinal fin with temperature dependent internal heat generation and thermal conductivity using Differential Transformation Method (DTM). Adopting similar method considered by [10], **Torabi et al.[**18] in their study, observed decrease in the fin base temperature with increasing both radiation and convection effects. Results from previous study of [19] also revealed that radiation contributes up to 20% of the total heat dissipation from the fin under natural convection, a situation that can affect the fin thermal performance additionally [20] reported that for many practical engineering problems the importance of the fin's weight initiated an optimization problem such as to maximize the fin heat dissipation an appropriate fin dimension will have to be determined for a given fin volume.

Therefore, in the present study, numerical analyses ofradiation effect on heat flow through fin of rectangular profile taking into account the influence of physical parameters which include: thermal conductivity (K), emissivity (ε), length (L) and thickness (t) of the fin on fin thermal performance are studied. Galerkin method was applied to provide the analytical solution in terms of finite element formulation. Galerkin method is a numerical analysis technique which mainly deals with precise mathematical calculation for determining the approximate solution. It takes simpler elements from the main problem, calculates separately and thus minimizes the probabilities of errors. The observed heat transfer performance interns of temperature distributions over the rectangular fin profile surface with and without considering radiation heat loss are comparatively reported.

II. METHODOLOGY

2. Problem Definition

Consider a straight fin of rectangular profile depicted schematically in Fig. 1 with cross sectional area A, length, L, constant thermal conductivity, k and surface emissivity, ε. The fin is attached to a primary surface with a constant temperature T_b and losses heat to the surrounding medium with temperature, $T_{\propto}$. The analysis of the problem is based on the following simplified assumptions:

- Steady state heat conduction.
- One-dimensional heat conduction.
- The temperature at the base of the fin is uniform.
- There is no heat transfer from the tip of the fin.
- The fin radiates according to the Stefan-Boltzmann law.
- There are no heat sources or sinks in the fin.
- Constant convective heat transfer coefficient.

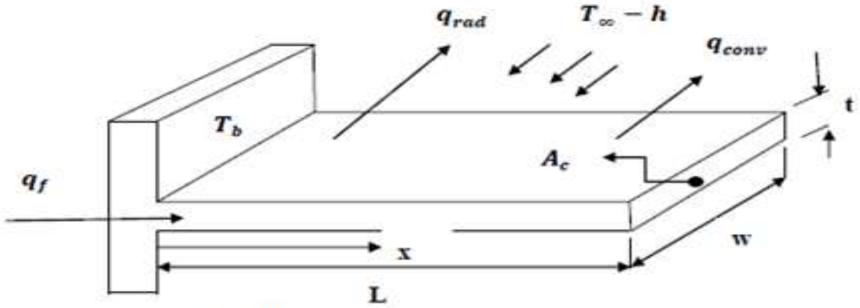

Fig1 Rectangular fin of the case study

On the basis of the above assumptions, and assuming that the rate at which energy transfers to the environment by compound effect of convection and radiation from any point on the fin surface must be balanced by the rate at which energy reaches that point due to the conduction in transverse (y, z) , thus for the model taking radiation effect and convection into consideration and the model without considering radiation, the conservation of energy balance, yields the following respective governing differential equations that must be satisfied by the fin temperature.

2.1 Mathematical Models

The general heat conduction equation in rectangular coordinates according to Fourier Law of heat conduction is given by expression (1):

$$K\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial y^2}\right]+\dot{q} = \rho C\frac{\partial T}{\partial t} \tag{1}$$

Taken into account the above stated assumptions, Eq. (1) is reduce to

$$K\frac{\partial^2 T}{\partial x^2} = 0 \tag{2}$$

Thus the conductive heat energy transfer through the fin is obtained as:

$$q_{cond\,.} = K\frac{\partial^2 T}{\partial x^2} \tag{3}$$

The convective heat energy transfer from the fin to the surrounding is obtained based on Newton law of cooling equation given by:

$$q_{conv\,.} = h(T - T_\propto) \tag{4}$$

The heat energy transfer by radiation from surrounding to the fin due to suppose low convective heat energy transfer from the fin to the surrounding is modelled as:

$$q_{rad\,.} = \in \nabla(T^4 - T_\propto^4) \tag{5}$$

By linearization Eq. (5) becomes

$$q_{rad} \cong 4\varepsilon\nabla \mathrm{T}_{\mathrm{m}}^3(T - T_\propto) \tag{6}$$

where, $T_m = \left(\frac{T_i + T_\propto}{2}\right)$

The boundary conditions considered in this study are as given in Eqns. (7) and (8):

$$x = 0, \quad T = T_b \tag{7}$$

$$x = LT = T_L \tag{8}$$

With the boundary conditions given in Eqns. (7) and (8), the general governing differential equation results from an energy balance on an element of the fin normal to direction of heat flow shown in Fig. 1 is expressed as:

$$KA_{CS}\frac{d^2T}{dx^2} = h_{sf}A_{sf}(T - T_\alpha) + \varepsilon\nabla A_{sf}\left(T^4 - {T_\alpha}^4\right) \tag{9}$$

Taking into account an infinite log fin, Eq. (9) becomes:

$$KA_{cs}\frac{d^2T}{dx^2} = h_{sf}A_{sf}(T - T_\propto) + 4\varepsilon\nabla A_{sf}T_m^3(T - T_\propto) + h_tA_t(T - T_\propto) + 4\varepsilon\nabla A_tT_m^3(T - T_\propto) \tag{10}$$

For ease of understanding the numerical solution of the theoretical equation given in Eq. (10) is denoted as Model 1 for fin taking radiation effect and convection into consideration.

2.1.1 Finite Element Solution using Galerkin method for Model 1

In the present study, with a view to analyse numerically the radiation effect given in Eq. (10), Galerkin finite element method was applied to expressthe temperature distribution along the fin surface in the form:

$$\int N_i R_{es}dx = 0 \tag{11}$$

where N_i and R_{es} is a shape function and residuals respectively

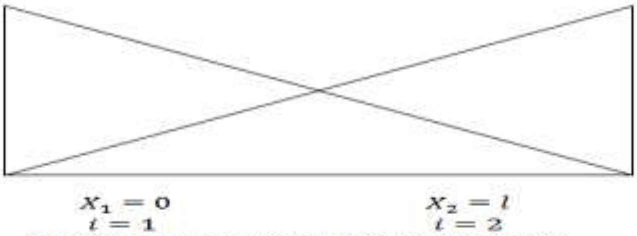

Fig2 A 2-node element used as case study

The analysis of the shape function N_i with respect to Fig. 2 is obtained by using Lagrange method of interpolation as follows:

$$N_1 = \frac{x-x_2}{x_1-x_2} = \frac{x-l}{0-l} = \frac{l-x}{l} = 1 - \frac{x}{l} \tag{12}$$

$$N_2 = \frac{x-x_1}{x_2-x_1} = \frac{x-0}{l-0} = \frac{x}{l} \tag{13}$$

$$N_1^2 = 1 - \frac{2x}{l} + \frac{x^2}{l^2} \tag{14}$$

$$N_2^2 = \frac{x^2}{l^2} \tag{15}$$

$$N_1N_2 = \frac{x}{l} - \frac{x^2}{l^2} \tag{16}$$

$$\frac{dN_1}{dx} = -\frac{1}{l} \tag{17}$$

$$\frac{dN_2}{dx} = \frac{1}{l} \tag{18}$$

$$R_{es} = KA_{cs}\frac{d^2T}{dx^2} - h_{sf}P(T-T_\propto)dx - 4\varepsilon\nabla T_m^3 P(T-T_\propto)d_x \tag{19}$$

Thus, application of the Galerkin finite element method to Eq. (10) yields:

$$KA_{cs}\int_{x_1}^{x_2} N_i \frac{d}{dx^2}\left(\frac{dT}{dx}\right)dx - h_{sf}P\int_{x_1}^{x_2} N_i(T-T_\propto)dx - 4\varepsilon\nabla T_m^3 P\int_{x_1}^{x_2} N_i(T-T_\propto)d_x = 0 \tag{20}$$

In the present study, the temperature distribution along the fin obtained in a matrix form by integrating term by term, the stiffness function, the forcing function and the gradient function for the 2-node element considered in Fig. 2 are given in Eqns. (21), (22) and (23) respectively as follows:

$$K_{Model\,1}^e = KA_{cs}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{h_{sf}Pl}{6}\begin{bmatrix}21\\ 12\end{bmatrix} + \frac{4\varepsilon\nabla PlT_m^3}{6}\begin{bmatrix}21\\ 12\end{bmatrix} \tag{21}$$

$$f_{Model\ 1}^e = \frac{h_{sf}PlT_\propto}{2}\begin{bmatrix}1\\ 1\end{bmatrix} + \frac{4\varepsilon\nabla PT_m^3T_\propto}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \tag{22}$$

$$f_{gModel\ 1}^e = KA_{cs}N_1\frac{dT}{dx}\Big|_{x_1}^{x_2} \tag{23}$$

It is interesting to point out that the stiffness matrix or conductance matrix governs all the elements except the last element as the boundary condition (B.C.) at the tip of fin comes to play in addition. Thus, by imposing the boundary condition at the tip of the fin considering the last element, we have:

$$K_{BC}^e = \begin{bmatrix}0 & 0\\ 0 & (\mathrm{h_t} + 4\varepsilon\nabla PT_m^3)\mathrm{A_t}\end{bmatrix} \tag{24}$$

$$f_{BC=}^e\begin{bmatrix}0\\ (\mathrm{h_t} + 4\varepsilon\nabla PT_m^3)\mathrm{T_lA_t}\end{bmatrix} \tag{25}$$

Combining Eqns. (21) to (25) yield general matrix form of the element as follows**:**

$$K_{Model\ 1}^e = KA_{cs}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{h_{sf}Pl}{6}\begin{bmatrix}21\\ 12\end{bmatrix} + \frac{4\varepsilon\nabla PlT_m^3}{6}\begin{bmatrix}21\\ 12\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & (\mathrm{h_t} + 4\varepsilon\nabla PT_m^3)\mathrm{A_t}\end{bmatrix} \tag{26}$$

and

$$f_{Model\ 1}^e = \frac{h_{sf}PlT_\propto}{2}\begin{bmatrix}1\\ 1\end{bmatrix} + \frac{4\varepsilon\nabla PT_m^3T_\propto}{2}\begin{bmatrix}1\\ 1\end{bmatrix} + \begin{bmatrix}0\\ (\mathrm{h_t} + 4\varepsilon\nabla PT_m^3)\mathrm{T_lA_t}\end{bmatrix} \tag{27}$$

Temperature distribution at each nodal point along the fin is obtained with the approximate solution given as:

$$T = \sum_{v=1}^{n} N_iT_i \tag{28}$$

For a 2- node element, n=2

$$T = \sum_{i=1}^{2} N_iT_i = N_1T_1 + N_2T_2 \tag{29}$$

$$\frac{dT}{dx} = \frac{dN_1}{dx}T_1 + \frac{dN_2}{dx}T_2 \tag{30}$$

Combining Eqns. (26) to (27) and substitute for T and $\frac{dT}{dx}$ expressed by Eqns. (29) and (30), we have:

$$KA_{CS}\int_{x_1}^{x_2}\frac{dN_i}{dx}\frac{d}{dx}[N_1T_1 + N_2T_2]dx + h_{sf}P\int_{x_1}^{x_2} N_i[N_1T_1 + N_2T_2]dx + 4 \in \nabla PT_M^3\int_{x_1}^{x_2} N_i[N_1T_1 + N_2T_2]dx = h_{sf}PT_\propto\int_{x_1}^{x_2} N_i dx + 4 \in \nabla PT_M^3\int_{x_1}^{x_2} N_i dx + KA_{cs}N_i\frac{dT}{dx}\Big|_{x_1}^{x_2} \tag{31}$$

In order to verify the effect of the radiation on effectiveness of fins application for heat dissipation in thermal system considered in this study, general form of the element matrix equation was also obtained for the fin without considering radiation heat loss. Thus, the general differential equation results from energy balance on an element of the fin normal to direction of heat flow without considering radiation heat loss is expressed as follows:

$$KA_{CS}\frac{d^2T}{dx^2} = h_{sf}A_{sf}(T - T\alpha) \tag{32}$$

Also for ease of understanding the numerical solution of theoretical equation given in Eq. (32) is denoted as Model 2 for the fin without considering radiation heat loss.

2.1.2 Finite Element Solution using Galerkin method for Model 2

Application of the Galerkin finite element method following thesame procedure given in Eqns. (28) to (32) yields:

$$KA_{cs}\int_{x_1}^{x_2} N_i\frac{d}{dx^2}\left(\frac{dT}{dx}\right)dx - h_{sf}P\int_{x_1}^{x_2} N_i(T - T_\propto)dx = \tag{33}$$

Thus, the general matrix forms of the element for the fin without considering radiation heat loss are obtained as**:**

$$K^e_{Model\ 2} = KA_{cs}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{h_{sf}Pl}{6}\begin{bmatrix}21\\ 12\end{bmatrix} \tag{34}$$

and

$$f^e_{Model\ 2} = \frac{h_{sf}PlT_\propto}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \tag{35}$$

Equating Eq. (34) to (35) and applying the approximate solution given in Eq. (28), the temperature distribution at each nodal point along the fin without considering radiation heat loss is estimated using Eq. (36):

$$KA_{CS}\int_{x_1}^{x_2}\frac{dN_i}{dx}\frac{d}{dx}[N_1T_1 + N_2T_2]dx + h_{sf}P\int_{x_1}^{x_2} N_i[N_1T_1 + N_2T_2]dx = h_{sf}PT_\propto\int_{x_1}^{x_2} N_i dx \tag{36}$$

The Mathematical models described above were implemented in MATLAB software to predict the response of heat flow through fin of a rectangular profile considering both Model 1 and 2. i.ethe fin with and without considering radiation heat loss. The software input data are, operating temperature (T_b), ambient temperature ($T_\propto$), convection heat transfer coefficient(h), fin emissivity (ε), Stefan-Boltzmann constant (∇), thermal conductivity of the material (k), length of the material(L), the perimeter of the fin(P) and the sectional area of fin(Ac). It is interesting to point out that the effect radiation on heat flow through the fin of the rectangular profile can be evaluated using Galerkin method by considering Eqns. (31) and (36) respectively, thus the temperature distribution along the longitudinal direction can be achieved with these models. In this study, the fin is divided uniformly into twenty (20) finer elements with a total of twenty-one (21) nodes in order to achieve an accurate numerical solution. The simulation of both Model 1 and 2 are characterized by temperature distribution taking into consideration the influence of physical parameters which include: length, thickness, fin metal type and emissivity of the fin on the performance of the systems.

III. RESULTS AND DISCUSSION

In this section, overview of the fin thermal performance based on the use of numerical model developed to evaluate the significance of thermal radiation effects on heat flow through fin of a rectangular profile are comparatively reported. Numerical data were generated based on the influence of physical parameters which include: length (L), thickness (t), fin metal type and emissivity (ε) of the fin on temperature distribution over the rectangular fin profile surface with and without considering radiation heat loss. For ease of the discussion, Model1 and Model 2 denote terms for the fin profile surface with and without considering radiation heat loss respectively.

3.1 Comparative Analysis

Shown in Figures 3a and 3b is the variation of the fin length with the fin base temperature at 100 and 500 ^{o} C respectively. In these figures a fin metal type made of Aluminium was considered with assumed fin thickness of 0.002.m. As can be seen in these figures, the model taking radiation effect and convection into consideration (Model 1) showed a better heat transfer performance than the one without thermal radiation (Model 2); moreover, it can be notice that by applying the model with radiation effect for higher fin base temperature value (Figure 3b) increases the heat dissipating rate of the device. This behaviour implies that higher power dissipation by the device on which the fin is attached e.g (electronic components) can be achieved by considering thermal radiation effects and consequently improving the efficiency of the thermal system. In addition, the fin length used for the fin base temperature distribution also play an important role in the thermal performance of the fin, since with increasing the fin length, the fin base temperature also decreases along the longitudinal direction of the fin independently of the two models considered in this study. These behaviours are consistent with those observed in the study of **Safayet-Hossain*et al*** [21].

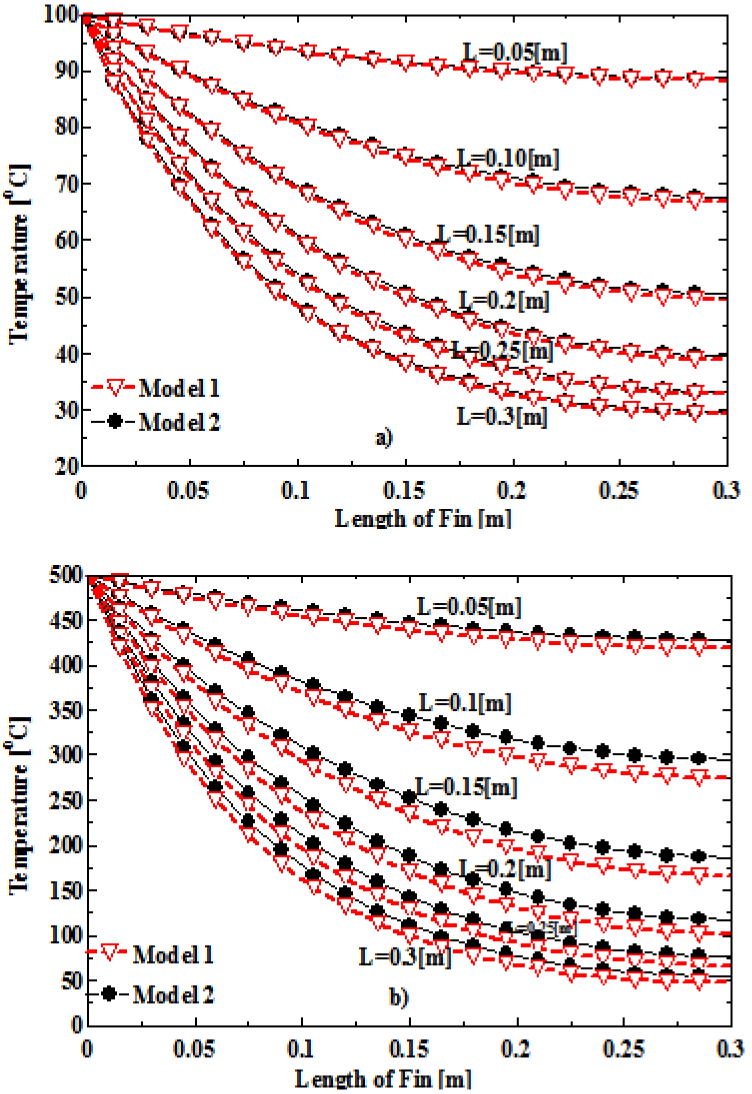

Fig3.Temperature variation in the fin for various values of fin Length obtained using Galerkin method.

Figure 4 presents the numerical data for variations of temperature distribution along the longitudinal direction of the fin at a base temperature of 500 ^{o} C. The numerical data were generate for different fin thickness values of 0.01, 0.004 and 0.002 (m) respectively to evaluate the effects of fin thickness on the fin heat dissipation rate behaviour. It is interesting to point out that in practical context; the base temperature is normally regarded as the operating temperature of electronic components (Khor *et al.* [11]). From this figure, it is observed that the fin heat dissipation rate virtually increases with reducing the fin thickness for the given base temperature. It can also be noticed that the heat dissipation rate for the case associated with considering radiation heat loss (Model 1) is higher compared to that without considering radiation heat loss (Model 2). This behaviour which is the consequence of the additional accounted radiation loss from the fin surface apart from the convection loss is observed to be more pronounced with the lowest fin thickness value considered in this study. This is conceivable and profitable as higher power dissipation needed for a compact and miniaturized thermal equipment such as for a reduced fin thickness observed in this study will give rise to higher operating temperature and subsequently improve the safety of the thermal system.

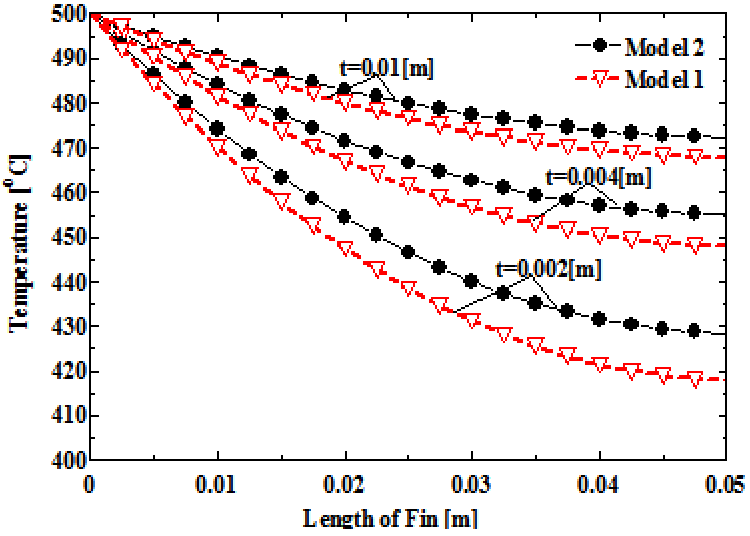

Fig 4.Temperature variation in the fin for various values of fin thickness obtained using Galerkin method.

Variations of temperature distribution along the longitudinal direction of the fin considering the effect of the fin metal type on the fin thermal performance behaviour is displayed in Figures 5a and 5b. Commonly used fin metal type such as Copper, Aluminium and Stainless steel are consider for comparison in the present study. Numerical experimentation were carried out for fin base temperature of 100 and 500 ^{o} C respectively based on the materials respective thermal conductivity values. It should be highlighted that, thermal conductivity property of a material play an important role on heat transported from the base to the tip by conduction inside the fin, concurrent heat dissipation by convection and radiation to the surroundings takes place on the fin surfaces. From these figures, it can be notice clearly that independently of the fin metal type, the thermal performance of fin for the case considering thermal radiation effect (Model 1) is higher than the case of excluding thermal radiation effect (Model 2), this behaviour is observed to be more pronounce for the fin with higher base temperature value being that at high operating temperature, the heat transported to the heat sink is high and hence more heat is dissipated from the fin to the surroundings and vice versa. By employing the value of temperature distribution along the fin longitudinal surface for the case considering thermal radiation effect (Model 1) as the basis for comparison with respect to the fin type metal, it can be observed that when thermal radiation is neglected (Model 2), the heat dissipation rate with the use of stainless steel material is underrated with temperature distribution (T ^{o}C) value of up to 15.6% at a reduced fin length, L= 0.2 m. Similar behaviour is observed for Aluminium and Copper materials at fin length of 0.25 m but with underrated temperature distribution (T ^{o}C) value of up to 21.5 and 21.8% respectively, the observed differences in the fin thermal performance results and as pointed out in the study of **Torabi*et al.*** [18] is due to the effects of thermal conductivity on heat flow through the respective material considered in this study.

It can be inferred that for a required high heat dissipation process towards the needs of compact and miniaturized equipment in thermal system application such as cooling of electronic devices, stainless steel fin material could be applicable provided the material cost implication is not put into consideration, i.e for special thermal application such as in Nuclear reactor system and others. Also, for a reduced production cost with improving thermal system efficiency by adopting the case with thermal radiation effect model proposed in this study, Aluminium fin material could be useful. This result also justifies one of the reasons why Aluminium fin material is commonly used as fin in practical context.

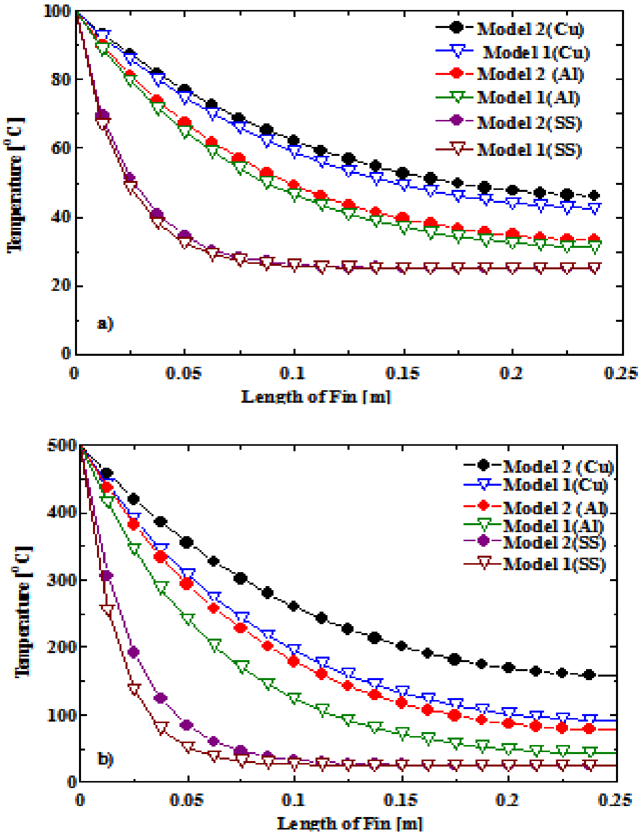

Fig5.Temperature variation in the fin for different fin metal types obtained using Galerkin method.

The fin thermal performance behaviour on effect of increase in the fin material emissivity is also displayed in Figures 6 to 8. It is clearly show that fin heat dissipation rate increases with increasing emissivity value for various fin metal type considered in this study. However, for the case with thermal radiation (Model 1), the effect of increasing emissivity value for stainless steel material is observed to have negligible effect in increasing the fin thermal performance compared to that with Aluminium and Copper materials which gave appreciable response in enhancing the fin thermal performance as also observed in Fig. 5b, this is mainly due to the fact that stainless steel is a metal of high emissivity value in nature. This behaviour agreed with **Khor*et al.***[11] who reported that the role of fin becomes less notable especially when the emissivity of the fin surface is high, and when the thermal radiation heat loss is taken into consideration.

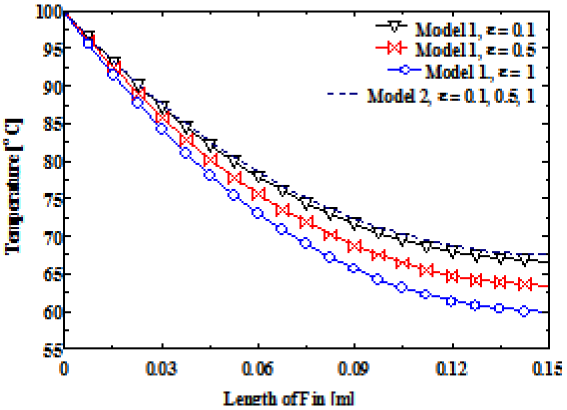

Fig6.Temperature variation in the fin metal type (Copper material) for various values of emissivity obtained using Galerkinmethod .

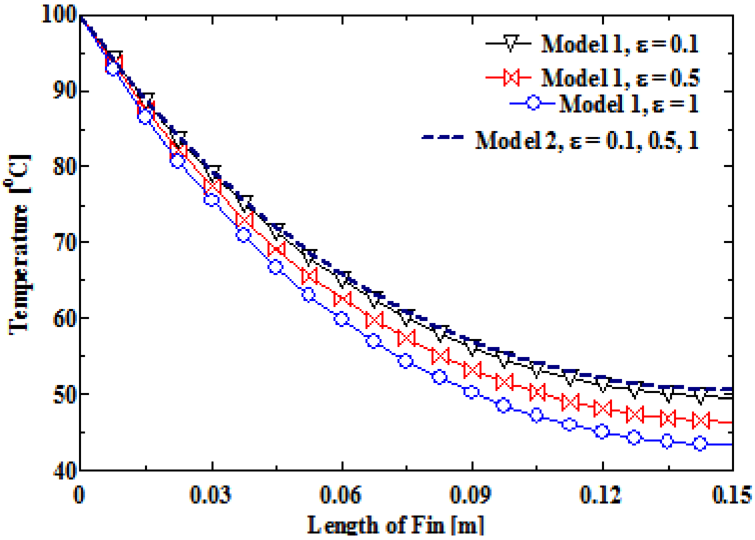

Fig7.Temperature variation in the fin metal type (Aluminium material) for various values of emissivity obtained using Galerkinmethod .

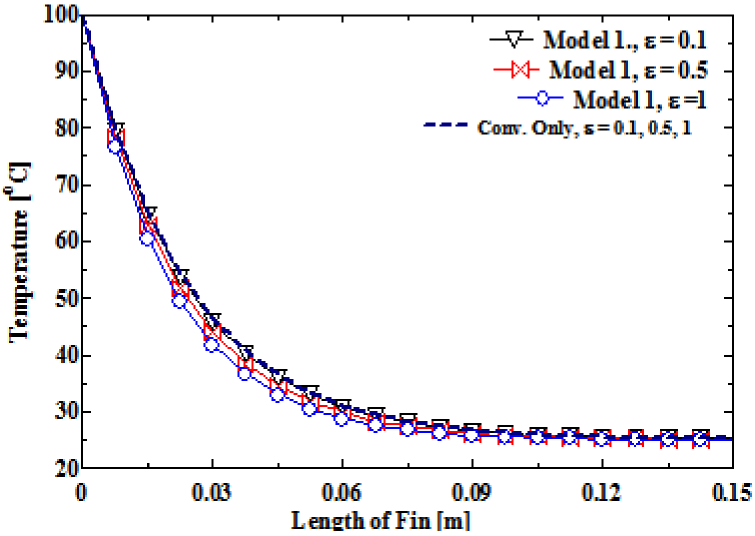

Fig8. Temperature variation in the fin metal type (Stainless steel material) for various values of emissivity obtained using Galerkin method.

3.2 Fin thermal performance evaluation

In order to show more clearly the effect associated with excluding radiation heat loss in thermal performance of a rectangular fin profile, analytical calculations are carried out to elucidate such effect in the fin heat dissipation rate capacity by considering a situation where heat flow through a fin metal by Aluminum, (Al) taking Fig.1 as a case study having a thickness, t = 0.002m, width, w= 0.01m and length, L=0.25m is to dissipate heat from the surface of an electronic device having a specific operating temperature for a particular cooling application. The fin is exposed to ambient air and its radiating and convecting heat to the environment at temperature, ($T_\propto$) of 25 °*C*, and convection heat transfer coefficient, (h_t) along the length and the end is 20W/m^2K, the fin emissivity,ε =1 and the Stefan-Boltzmann constant, ∇=5.7E-08 W/ m^2 K^4. Determine the heat dissipation performance of the solid fin for different operating temperature, T_b ranging from 100 to 500 °*C* with and without radiation heat loss.

To solve the case study, the heat dissipation performances for the cases without thermal radiation (Q_1) and the case with thermal radiation (Q_2) were evaluated using the exact solution method according to **Bergman*et al*.** [22] given by:

$$Q_1 = \sqrt{Ph_1ka}\left(\frac{Sinhm_1L+{}^{h_1}/_{mk}\,Coshm_1L}{Coshm_1L+{}^{h_1}/_{m_1k}Sinhm_1L}\right)[T_b - T_\propto] \tag{37}$$

$$Q_2 = \sqrt{Ph_2ka}\left(\frac{Sinhm_2L + {}^{h_2}/_{m_2k}Coshm_2L}{Coshm_1L + {}^{h_2}/_{m_2k}Sinhm_2L}\right)[T_b - T_\propto] \qquad (38)$$

where T_b is the base surface temperature of the fin, $T_\propto$is the ambient air surrounding temperature of the fin, P = $2(w+t)$ is the perimeter of the fin, A_c = wt is the sectional area of fin, h_{sf} is convective heat transfer coefficient of surrounding medium and K=K_{Al} is the fin metal type thermal conductivity

Thus,

$$h_1 = h_{sf} + h_t \qquad (38)$$

$$h_2 = h_{sf} + h_t + 12\varepsilon\nabla T_m^3 \qquad (39)$$

$$m_1 = \sqrt{\frac{Ph_1}{KA_c}} \qquad (40)$$

$$m_2 = \sqrt{\frac{Ph_2}{KA_c}} \qquad (41)$$

By using the value of Q_2 as the basis for comparison, the relative deviation (in percentage) of the pertinent parameters is quantified as:

$$\delta = \frac{Q_1 - Q_2}{Q_2} \times 1\,00 \qquad (42)$$

In Eq. (42), δcan be regarded as the error incurred by the exclusion of thermal radiation heat loss from the fin surface.

The analytical solution results of the aforementioned case study using Eqns. (37) and (38) and the relative deviation (in percentage) value obtained with the use of Eq. (42) are presented in Table1.The results displayed in Table 1 clearly show that by considering thermal radiation effect in fin design, a significant enhancement in the fin thermal performance is achievable. These results justify the reason for better performance of the fin for the case of considering thermal radiation effect (Model 1) compared to the case of excluding thermal radiation effect (Model 2) as observed with the numerical solution results discussed in section 3.1. Also, it can be seen that under the same operating temperature condition of 100 °C, about 20% of the total heat dissipated might be contributed by radiation, a result which is similar to the numerical results obtained in this study and that observed in the study of [19]. Moreover, the error value incurred by the exclusion of thermal radiation heat loss is observed to be more pronounced with increasing the system operating temperature as shown in Table 1and agreeing with **Rea and West** [23], these authors whose research work involved thermal radiation from finned heat sinks claimed that, depending on the heat sink design, operating temperature and ambient environment, 25% of the total heat dissipated from the heat sink might be contributed by radiation.

Table 1. Shows the results of the fin performance evaluation and the analytical solutions approach

Temperature [°C]	Q_1	Q_2	δ[%]
100	3.1	3.7	19.6
200	7.3	9.4	28.6
300	11.4	15.9	39.3
400	15.6	23.7	51.8
500	19.7	32.7	65.8

IV. CONCLUSION

Numerical analysis of heat flow through fin of a rectangular profile surface with and without considering radiation heat loss was carried out in this work. The effects of physical parameters which include: length, L, thickness, t, fin metal type and emissivity, ε, on the fin thermal performance are comparatively studied. The following conclusions can be drawn from the present study:

1. It is observed that heat dissipation rate for the fin with thermal radiation is higher than those without thermal radiation independently of the fin type metal considered in this study. The cases of excluding thermal radiation actually underrate the fin thermal performance in the range of 15.6%, 21.5% and 21.8% respectively for the stainless steel, the aluminium and the copper materials,
2. For the effect of increasing the fin material emissivity subjected to the cases of considering radiation heat loss, appreciable enhancement of the fin thermal performance was observed for aluminium and copper materials compared to stainless steel material. It is argue that this phenomenon is related to the high emissivity property possess by the stainless steel which render the expected enhancement of the fin material emissivity for the case with thermal radiation less notable.
3. By comparing the fin with thermal radiation and those without thermal radiation, it was observed a better heat transfer performance with increasing the fin length and the fin operating temperature. This implies that higher power dissipation can be achieved by considering thermal radiation effect and consequently improving the efficiency and safety of the thermal systems.

4. Enhancement of heat dissipation rate with reducing the fin thickness was observed for high operating temperature regardless of the two numerical models considered in this study. This behaviour was observed to be more pronounced for the fin with thermal radiation heat loss consideration.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the assistance of Mr O. Z. Ayodejiof the Department ofMechanical Engineering, Federal University of Technology Akure, Ondo State, Nigeria and the contribution of Mr N.O. Adewunmiand Mr.O.D.Owoseniof the Department ofMechanical Engineering, University of Lagos, Akoka, Yaba, Lagos in supplying the needed information used in the present study.

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