American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-9, pp-207-215
www.ajer.org
Research Paper Open Access

DQ Modeling and Dynamic Characteristics of a Three-Phase Induction Machine

Emmanuel S. Oluwasogo[*1], Ignatus K. Okakwu[2], Olanike O. Ade-Ikuesan[3], Habeeb O. Suleiman, [3]

[1]*Department of Electrical & Computer Engineering, Kwara State University, Nigeria.*
[2]*Department of Elect./Elect. Engineering, University of Benin, Nigeria.*
[3] Department of Computer and Electrical Engineering, Olabisi Onabanjo University, Ago Iwoye,Ogun State.
[3]*Department of Electrical & Computer Engineering, Kwara State University, Nigeria.*
Corresponding Author: E. S. Oluwasogo

Abstract: *AC motors as a drive have become more popular over its DC motors counterpart (For example Ward Leonard System) in the variable speed drives applications. The latter uses three electrical rotating machines and very costly to maintain despite the low efficiency offered by them. The former offers superior dynamic performance. In this paper, the theory of reference frames is used to examine the dynamic performance of the induction machine. The reference frame considered is synchronously rotating DQ-reference frame. This is because this reference frame transforms the AC signals to the equivalent DC signals. The dynamic model provides detailed expressions for the study of transient and steady state behaviors of the machine. 4th Order Runge-Kutta (ode45) was used to solve the differential equations using MATLAB. Results shows that MATLAB a reliable and sophisticated tool to study the transient and steady state response of the machine.*

Keywords: *Reference Frame, Induction Machine, Dynamic Response, MATLAB, 4th Order Runge-Kutta*

Date of Submission: 01-09-2017 Date of acceptance: 17-09-2017

I. INTRODUCTION

With the huge success in the vector control of AC motors, Induction (AC) motors as drive have become more popular over its DC motors counterpart(For example Ward Leonard System) in the variable speed drives applications. The latter uses three electrical rotating machines and very costly despite the low efficiency offered by them.

Therefore, it is essential to analyze the behavior or performance of the induction motor. Although, in the past decades lots of research has been carried out on the steady state analysis of the induction motor which was proved to have neglected the performance of the motor under perturbation as a result of the load changes and fluctuations from the supply. Therefore, it became vital to study the dynamic response of the induction motor in order to determine the transients behavior of this motor.

To predict the dynamic performance of the induction motors [Krishnan] concluded that the dynamic model derived from two-phase fictitious winding in the direct and quadrature (dq) axes through the reference frames theory must be used. With this approach, an n-phase machine can be reduced to a set of two – phase windings which are at an angle 90^o to each other. Hence, the required variables (voltages, currents and flux linkages) of the induction motor is transformed to a reference frame rotating at an arbitrary angular speed ω known as generalized reference frame.

It is very important to mention here that there are three commonly used reference frames derived from the generalized (arbitrary) reference frame model of induction motor. They are briefly discussed below:

i. ***Stationary (Stator) Reference Frame Model:*** The reference frame speed is zero i.e. It is equivalent to the stator (stationary) speed. $\omega = 0$
Usually, it is adopted by the stator (scalar) controlled induction motor drives such as: Inverse-Parallel Thyristorized controller or Inverter Fed induction motor.

ii. ***Rotor Reference Frame Model:*** This normally used by the induction motors whose speed control is done from the rotor side. The reference frame speed of this model is equivalent to that of the rotor. i.e. $\omega = \omega_r$

Where ω_r is the rotor angular speed.

iii. ***Synchronous Rotating Reference Frame Model:*** The reference frame speed of this model is the same as that of the stator supply angular speed of the magnetic flux set-up in the air gap by the stator supply current. i.e. $\omega = \omega_s = 2\pi f_s$

Where f_s is the supply frequency in Hz

For the purpose of this paper, the reference frame considered is synchronous reference frame. This is because this reference frame transforms the AC signals to the equivalent DC signals since the difference between the reference frame angular speed and supply angular speed is zero which corresponds to the DC angular speed or frequency. This breakthrough achieved through synchronous reference frame model has led to superior speed control (Vector Control) of induction motor similar to the separately excited dc motor.

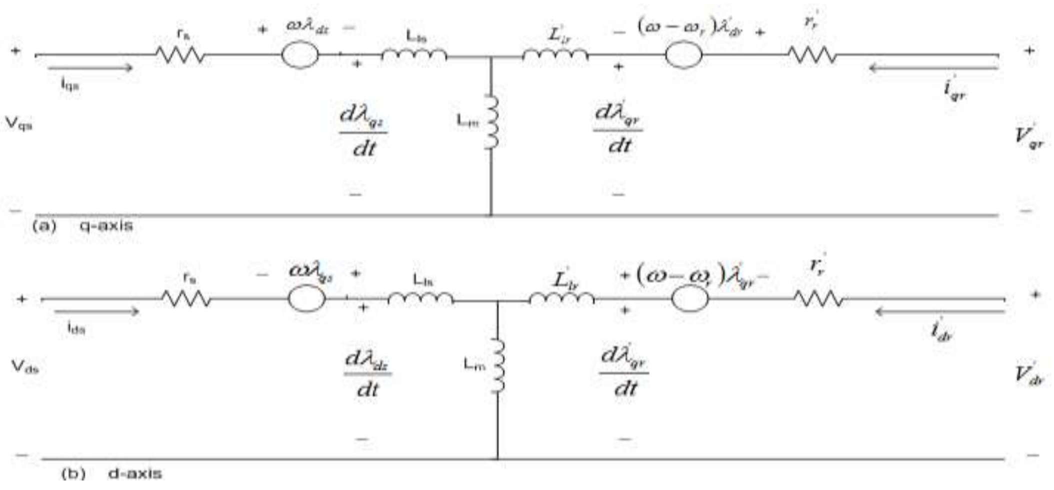
In [2], the dynamic behaviors of 3-φ induction motor were addressed based on a quality mathematical model and computer program simulation in a stationary reference frame to avoid the complexity involved in the course of solving time-varying differential equations obtained from the dynamic model. Fourth Order Runge-Kutta was adopted to give numerical solution of the dynamic model. Also, *Enemuoh et al* modeled induction motor by transforming "abc" coordinates to rotating "dqo" coordinates. The v/f speed control of the motor was simulated in MATLAB/SIMULINK environment using Pulse Width Modulation (PMW) technique. They concluded that with the use of reference frame transformation it becomes possible to control the speed of electric motor as well as to conserve the electrical energy (Enemuoh *et al*, 2013).. In the same year, *Hamdy et al.* presented a paper on analytical and experimental performance and control of a three-phase induction motor fed from a single-phase supply via a single capacitor. The transient and steady state analysis were predicted using d-q model representation whose frame was chosen stationary. The speed was controlled using a TRIAC coupled in series with the supply. Likewise, in a paper work presented by *Naresh et al*, where the three-phase AC quantities were reduces to two DC quantities by simplified calculations they carried out. Thereafter, the inverse transformation was done to recover the actual three-phase AC results.

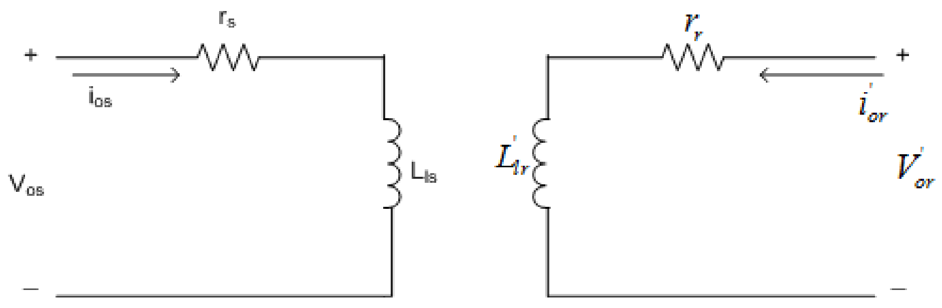
Therefore, it is necessary to model the machine parameters in dqo reference frame by transforming the alternating three-phase parameters of induction machine to its two- phase equivalent and deduce the dynamic response of the machine.

II. SYSTEM MODEL

The set of differential equations describing the induction motor have been stated explicitly in the literature to be nonlinear which makes its performance analysis intricate and the scalar linear control strategies adopted in the past unreliable. Therefore, it is necessary to transform these complicated equations into simpler and clearer set of equations from which the small-signal equations can be developed and the control technique becomes more apparent.

Consider the equivalent circuit of the induction motor in the dqo-reference frame as shown in Fig.1 below. The induction motor is assumed to have balanced windings, balanced inputs and the zero sequence circuit is eliminated from the discussions.





(b) o-axis

Fig. 1: The Equivalent Circuit of a Three-Phase Induction Motor in an Arbitrary dqo Reference Frame

Voltage Equation

Taking the stator and the rotor closed loop of the dq equivalent circuit of Fig. 1 (a), (b) and applying KVL and Faraday laws around the loop, we have:

Stator Loops:

$$\left.\begin{aligned} V_{qs} &= r_s i_{qs} + \omega\lambda_{ds} + \frac{d\lambda_{qs}}{dt} \\ V_{ds} &= r_s i_{ds} - \omega\lambda_{qs} + \frac{d\lambda_{ds}}{dt} \end{aligned}\right\} \quad \text{..........} \quad (1a)$$

Where

$$\left.\begin{aligned} \frac{d\lambda_{qs}}{dt} &= (L_{IS} + L_m)\frac{di_{qs}}{dt} + L_M \frac{di_{qr}}{dt} \\ \frac{d\lambda_{ds}}{dt} &= (L_{IS} + L_m)\frac{di_{ds}}{dt} + L_M \frac{di_{dr}}{dt} \end{aligned}\right\} \quad \text{..........} \quad (1b)$$

Apply KVL to rotor loop of figure 1(a) and figure 1(b) above, we have;

$$\left.\begin{aligned} V_{qr} &= r_s i_{qr} + (\omega - \omega_r)\lambda_{dr} + \frac{d\lambda_{qr}}{dt} \\ V_{dr} &= r_r i_{qr} - (\omega - \omega_r)\lambda_{qr} + \frac{d\lambda_{dr}}{dt} \end{aligned}\right\} \quad \text{..........} \quad (2a)$$

Where

$$\left.\begin{aligned} \frac{d\lambda_{qr}}{dt} &= (L_{Ir} + L_m)\frac{di_{qr}}{dt} + L_M \frac{di_{qs}}{dt} \\ \frac{d\lambda_{dr}}{dt} &= (L_{Ir} + L_m)\frac{di_{dr}}{dt} + L_M \frac{di_{ds}}{dt} \end{aligned}\right\} \quad \text{..........} \quad (2b)$$

Flux linkage equation

Removing the differential operator $\frac{d}{dt}$ from equation (1b) and (2b)

$$\left.\begin{aligned} \lambda_{qs} &= (L_{IS} + L_m)i_{qs} + L_M i_{qr} \\ \lambda_{ds} &= (L_{IS} + L_m)i_{ds} + L_M i_{drr} \\ \lambda_{qr} &= (L_{Ir} + L_m)i_{qr} + L_M i_{qs} \\ \lambda_{ds} &= (L_{Ir} + L_m)i_{dr} + L_M i_{ds} \end{aligned}\right\} \quad \text{..........} \quad (3a)$$

Replacing λ by ψ and L by X from equation (3a) to obtain (3b)

$$\left.\begin{aligned}\psi_{qs} &= (X_{IS} + X_m)i_{qs} + X_M i_{qr}\\ \psi_{ds} &= (X_{IS} + X_m)i_{ds} + X_M i_{drr}\\ \psi_{qr} &= (X_{ir} + X_m)i_{qr} + X_M i_{qs}\\ \psi_{ds} &= (X_{Ir} + X_m)i_{dr} + X_M i_{ds}\end{aligned}\right\}\text{..}(3b)$$

Where: $\lambda = \dfrac{\psi}{\omega_b}$

From equation 3a let

$$\left.\begin{aligned}L_{is} + L_m &= L_{sm}\\ L_{ir} + L_m &= L_{rm}\end{aligned}\right\}\text{..}(3ci)$$

$$\left.\begin{aligned}X_{is} + X_m &= X_{sm}\\ X_{ir} + X_m &= X_{rm}\end{aligned}\right\}\text{..}(3cii)$$

If f_b denote supply frequency used in calculating inductive reactance such that:

$$X = 2\pi f_b L = \omega_b L \quad \text{or} \quad L = \frac{X}{\omega_b}\text{..}(3ciii)$$

Substitute (3c) into (3a) and form the matrix (3d)

$$\begin{bmatrix}\psi_{qs}\\ \psi_{ds}\\ \psi'_{qr}\\ \psi'_{dr}\end{bmatrix} = \begin{bmatrix}X_{sm} & 0 & X_m & 0\\ 0 & X_{sm} & 0 & X_m\\ X_m & 0 & X'_{rm} & 0\\ 0 & X_m & 0 & X'_{rm}\end{bmatrix}\begin{bmatrix}i_{qs}\\ i_{ds}\\ i'_{qr}\\ i'_{dr}\end{bmatrix}\text{..}(3\ d)$$

Substitute (1b), (2b) and (3a) into (1a) and (2a) respectively and let V_{qr} and V_{dr} equal zero since the motor rotor side is locked i.e. the rotor side is short circuited.

$$V_{qs} = r_s i_{qs} + L_{sm}\frac{di_{qs}}{dt} + \omega L_{sm} i_{ds} + L_m \frac{di_{qr}}{dt} + \omega L_m i_{dr}\text{..}(4ai)$$

$$V_{ds} = r_s i_{ds} + L_{sm}\frac{di_{ds}}{dt} - \omega L_{sm} i_{qs} + L_m \frac{di_{dr}}{dt} - \omega L_m i_{qr}\text{..}(4aii)$$

$$0 = r_r i_{qr} + L_m \frac{di_{qs}}{dt} + (\omega - \omega_r) L_m i_{ds} + L_{rm}\frac{di_{qr}}{dt} + (\omega - \omega_r) L_{rm} i_{dr}\text{..}(4aiii)$$

$$0 = r_r i_{dr} + L_{rm}\frac{di_{dr}}{dt} - (\omega - \omega_r) L_m i_{qs} + L_m \frac{di_{ds}}{dt} - (\omega - \omega_r) L_{rm} i_{qr}\text{..}(4aiv)$$

Representing the voltage equation in matrix form

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ 0' \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_{sm}\frac{d}{dt} & \omega L_{sm} & L_m \frac{d}{dt} & \omega L_m \\ -\omega L_{sm} & r_s + L_{sm}\frac{d}{dt} & -\omega L_m & L_m\frac{d}{dt} \\ L_m\frac{d}{dt} & -(\omega-\omega_r)L_m & r_{rs} + L_{rm}\frac{d}{dt} & -(\omega-\omega_r)L'_{rm} \\ -(\omega-\omega_r)L_m & L_m\frac{d}{dt} & -(\omega-\omega_r)L'_{rm} & r_r + L_{rm}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} \Bigg\} \text{.......... ....}(4av)$$

The model for d-q currents of the three phase induction motor are obtained as follows:

Multiplying equation (4ai) by L_{rm} and (4aii) by L_m and subtracting the results to obtain (4bi)

$$\frac{di_{qs}}{dt} = \frac{\left[L_{rm}V_{qs} - r_s L_{rm} i_{qs} - (\omega L_{rm} L_{sm} - (\omega - \omega_r)L_m^2)i_{ds} + r_r L_m i_{qr} - (\omega_r L_m L_{rm})i_{dr}\right]}{L_{sm}L_{rm} - L_m^2} \text{.......... .}[A]$$

Multiplying equation (4aii) by L_{rm} and (4aiv) by L_m and subtracting the results to obtain (4bii)

$$\frac{di_{ds}}{dt} = \frac{\left[L_{rm}V_{ds} + (\omega L_{rm} L_{sm} - (\omega - \omega_r)L_m^2)i_{qs} - r_s L_{rm} i_{ds} - \omega_r L_m L_{rm} i_{qr} + r_r L_m i_{dr}\right]}{L_{sm}L_{rm} - L_m^2} \text{........}[B]$$

Multiplying equation (4ai) by L_m and (4aii) by L_{sm} and subtracting the results to obtain (4biii)

$$\frac{di_{qr}}{dt} = \frac{\left[-L_m V_{qs} + r_s L_m i_{qs} - \omega_r L_m L_{sm} i_{ds} - r_r L_{sm} i_{qr} - ((\omega - \omega_r)L_{rm}L_{sm} - \omega L_m^2)i_{dr}\right]}{L_{sm}L_{rm} - L_m^2} \text{........}[C]$$

Multiplying equation (4aii) by L_m and (4aiv) by L_{sm} and subtracting the results to obtain (4biv)

$$\frac{di_{dr}}{dt} = \frac{\left[-L_m V_{ds} - \omega_r L_m L_{sm} i_{qs} + r_s L_m i_{ds} + ((\omega - \omega_r)L_{rm}L_{sm} - \omega L_m^2)i_{qr} - r_r L_{sm} i_{dr}\right]}{L_{sm}L_{rm} - L_m^2} \text{.......... .}[D]$$

Torque Equation

An electromagnetic torque of power equivalence transformation between the three-phase and two-phase model is expressed as:

$$\tau_e = \left(\tfrac{3}{2}\right)\left(\tfrac{P}{2}\right)L_m\left(i_{qs}i'_{dr} - i_{ds}i'_{qr}\right) \quad \text{.......... ....[E]]}$$

Where P is the number of machine pole.

Let the retarding torque produced by the ball bearing be $f\omega$ and the load torque be τ_L. Therefore, Newton's equation becomes:

$$J\frac{d\omega}{dt} = \tau_e - f\omega - \tau_L \quad \text{.......... ....}(i)$$

Also, $\omega = \frac{d\theta}{dt}$

Equation [F] below is deduced from [E] and (vi)

$$\frac{d\omega}{dt} = \frac{\left[\left(\tfrac{3}{2}\right)\left(\tfrac{P}{2}\right)(L_m)(i_{qs}i'_{dr} - i_{ds}i'_{qr}) - f\omega - \tau_L\right]}{J} \text{.......... ....}[F]$$

The instantaneous three-phase motor variables (stator and rotor) can be obtained using synchronously rotating reference frame as follows:
Let the balanced three-phase supply voltages to the stator of the induction motor be defined as:

$$v_a = V_m \cos(\omega t)$$

$$v_b = V_m \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = V_m \cos\left(\omega t + \frac{2\pi}{3}\right)$$

These three-phase set of voltages transformation to d-q synchronously rotating reference frame is done using the two following equations:

$$\left.\begin{array}{l} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \\ \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4ci)$$

Likewise, the three phase stator and rotor currents are finally best obtained using inverse transformation as given below:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4ciii)$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4civ)$$

These gives:

$$\left.\begin{array}{l} i_a = \frac{2}{3}\left(i_d\cos(\omega t) - i_q\sin(\omega t)\right) \\ i_b = \frac{1}{3}(i_d(\sqrt{3}\sin(\omega t) - \cos(\omega t)) + (i_q(\sin(\omega t) - \sqrt{3}\cos(\omega t))) \\ i_c = \frac{1}{3}(i_d(-\sqrt{3}\sin(\omega t) - \cos(\omega t)) + (i_q(\sin(\omega t) - \sqrt{3}\cos(\omega t))) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(4cv)$$

In order to obtain the dynamic characteristics of the induction machine on dq-axis, equations [A] to [F] are solved using 4th Order Runge Kutta [Oluwasogo *et. al.*]

III. COMPUTER SIMULATION AND RESULTS

The operating parameters of the three-phase squirrel cage induction motor used for this study are given below:
Vm = 179.63V; Rr = 0.816Ω; Lm = 0.0346H; Lr = 0.0713H; P = 4; Rs = 0.435Ω; f = 50Hz
Ls = 0.0713H; ω_r *= 179.07rpm;* ω *= 376.99rpm; J = 0.089kg/m*^{2}*;*
The MATLAB M-file developed to obtain the performance characteristic of the three-phase induction motor was achieved using the algorithm stated in the flow-chat of fig.2 below.

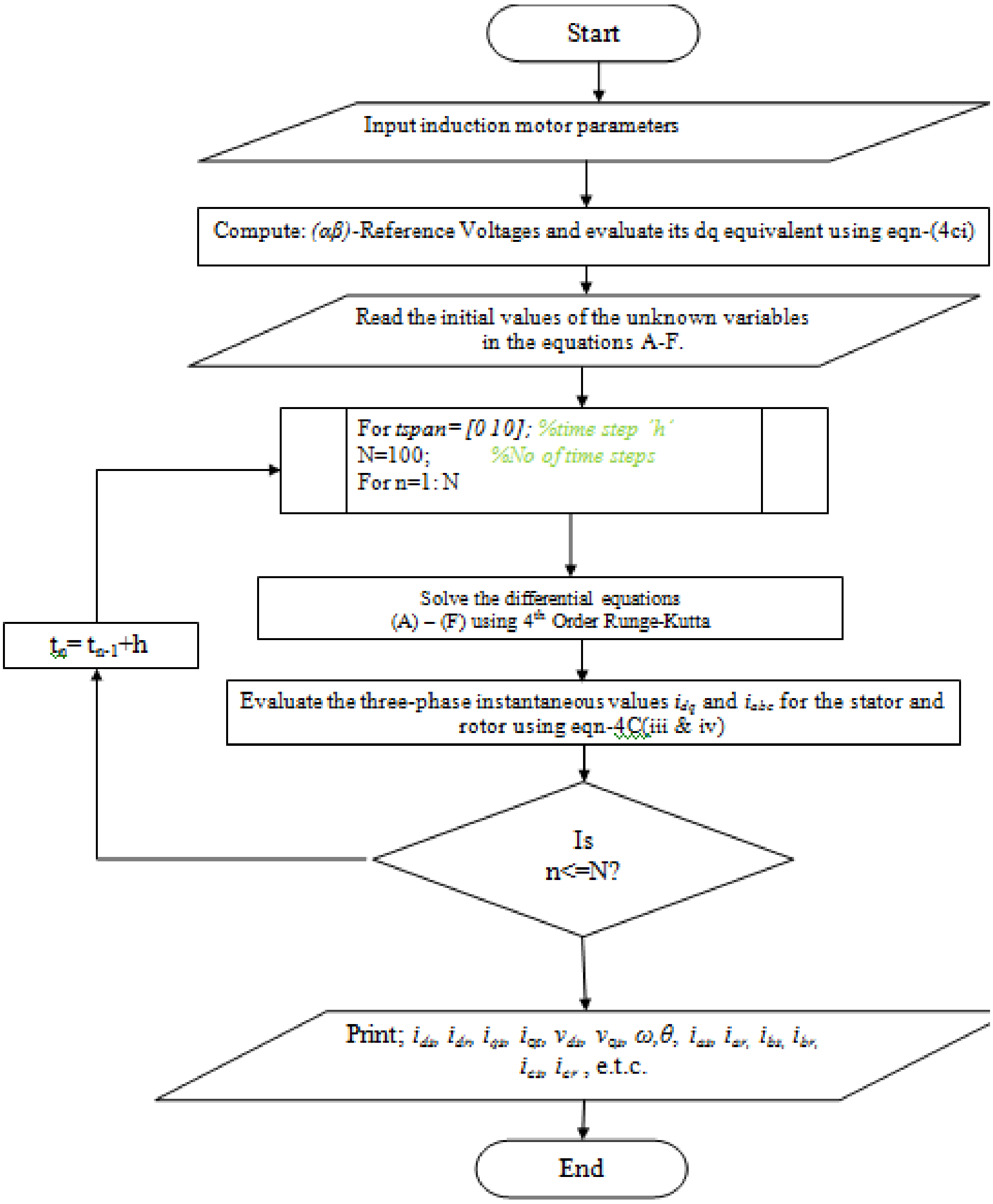


Fig. 2: Program Flow Chart

Figure 3 below shows the result obtained from the MATLAB m-file program

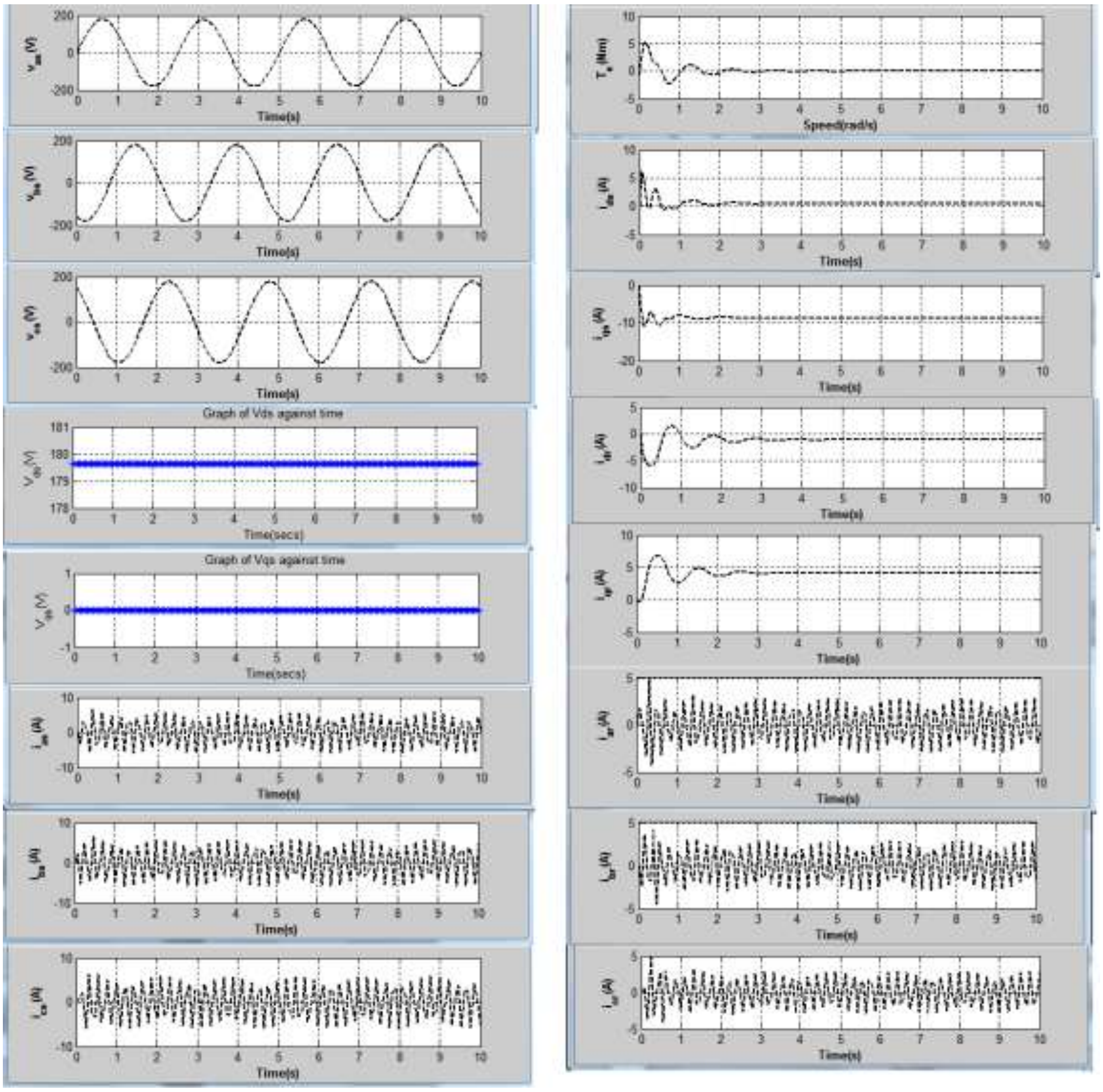


Fig. 3: Simulation Result.

IV. RESULT DISCUSSION

The transformation of *abc* supply voltage (cosine) to two-phase dq equivalent gives V_m and zero voltages respectively. The two-phase equivalent variables (stator and torque) and torque oscillated for few seconds after which it remained stable. Whereas, the inversed currents of two-phase machine dq-equivalent is sinusoidal stable.

V. CONCLUSION

This paper gives a step-by-step mathematical model of a three-phase induction machine in a synchronously rotating reference frame. The model provides detailed expressions for the study of transient and steady state behaviors of the machine. The response obtained from the numerical solution to the model using 4^{th} Order Runge-Kutta executed by the MATLAB M-file program is satisfactory.

This concludes that 4^{th} Order Runge-Kutta (ode45) is a reliable and sophisticated tool to solve differential equations.

REFERENCES

[1] P. C. Krause, O. Wasynczuk, S. D. Sudhoff "Analysis of Electric Machinery and Drive Systems", IEEE Press, A John Wiley & Sons, Inc. Publication Second Edition, 2002.

[2] E.S. Oluwasogo, O.D. Osunde, A.K. Babarinde, I.K. Okakwu, (2015). "The Dynamic Modelling and Electrical Profiles of Three Phase Induction Motor Using MATLAB Program", Innovative Systems Design and Engineering. Vol.6, No.1. www.iiste.org

[3] R. Krishnan, (2001). ELECTRIC MOTOR DRIVES: Modeling, Analysis, and Control. Prentice Hall, Inc. New Jersey.

[4] Enemuoh F. O., O. E. (2013). Modelling, Simulation and Performance Analysis of A Variable Frequency Drive in Speed Control Of Induction Motor . International journal of engineering invention , Vol.1 and No.7.
[5] K. Naresh, V. R. (2014). Simulations for Three Phase to Two Phase Transformation. internationaljournal of emerging trendin Electrical and Electronic .

ABOUT THE AUTHORS

E.S. Oluwasogo graduated from University of Ilorin, Nigeria, in 2008. He received the M.Sc. degree from the University of Lagos, Nigeria, all in Electrical Engineering. He is a lecturer in the Department of Electrical & Computer Engineering at Kwara State University, Nigeria. He is currently pursuing a PhD degree in Electrical / Electronics Engineering, University of Benin, Nigeria. His research interests include Power System Stability and Control, Control of Electrical Machines, Optimization and Robust Control, Automation etc. He is a member of IEEE and a registered Engineer with COREN.

I.K. Okakwu graduated from Ambrose Alli University, Ekpoma (Nigeria) in 2008. He received M.Sc degree from University of Lagos in 2012 all in Electrical/Electronics Engineering. He is currently pursuing a PhD degree in Electrical/Electronics Engineering University of Benin, Nigeria. His area of interest include power systems stability and control, electrical machines, power systems reliability, economic dispatch, FACTS and its applications, Deregulation

E. S. Oluwasogo, M.Sc. "DQ Modeling and Dynamic Characteristics of a Three-Phase Induction Machine ." American Journal of Engineering Research (AJER), vol. 6, no. 9, 2017, pp. 207–215.