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A New Defuzzification Technique for Forecasting Temperature and TAIFEX Based on Two Factors High-Order Fuzzy Time Series

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Abstract:*In the real world, one factor can be depended on by several factors. To improve the forecasting accuracy of fuzzy time series, all these factors can be combined in forecasting models. Therefore, if we consider more factors for prediction, the forecasting results are more improved. In this paper, an improved forecasting model based on the high - order two factors fuzzy time series with the new defuzzification rule is presented. In order to deal with factors together and improve the forecasting accuracy, we build a forecasting model based on establishing the two - factors high – order fuzzy relationship groups in the stage of determination of fuzzy logical relationships and propose the new fuzzy solving of rules in the defuzzification stage. The daily temperature data set and the Taiwan Futures Exchange(TAIFEX) data in Taipei, Taiwan are employed for the examined purpose, which consists of two factors, viz.,"temperature", "cloud density" factors and "TAIFEX index", "TAIEX index" factors, respectively. The experimental results showed that the proposed model is more precise than the existing models based on the high-order fuzzy time series.*

Keywords: *Fuzzy time series, two factors high – order fuzzy logical relationship, temperature, TAIFEX*

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I. INTRODUCTION

Fuzzy time series forecasting models have applied the fuzzy theory set to deal various domain forecasting problems, such as: enrollments of the next year [1]- [3], tomorrow's temperature [4], [5], [6], [7], rice production [8], financial forecasting [9], etc. Based on these forecasting results, we can prevent damages to occur or get benefits from the forecasting activities. However, in order to deal with the forecasting problems with complex influencing factors such as weather forecasting [7] and stock price forecasting [10], [6], these factors should be considered to derive more accurate results. Along with this, we need to discover some intelligent forecasting models to solve the forecasting problems. Song and Chissom proposed the concepts of fuzzy time series based on the fuzzy set theory [11]. They also presented the time-invariant fuzzy time series model [1] and the time-variant time series model [2] to deal with forecasting problems in which the historical data are represented by linguistic values. Both of them used the Max–Min operations to forecast the enrollments of the University of Alabama. However, the drawback of these two models is that they take a lot of time to deal with Max–Min composition operations. Therefore, Chen [3] changed into a more efficient model which is generally accepted by researchers and is the common form of fuzzy time series. Huarng[12] pointed out that the effective length of the intervals in the universe of discourse can affect the forecasting accuracy rate. In other words, the choice of the length of intervals can improve the forecasting results. In order to get a higher forecasting accuracy rate, Chen [13] presented a high-order fuzzy time series model for forecasting the enrolments of the University of Alabama. Yu [14] presented a new model which can refine the lengths of intervals during the formulation of fuzzy relationships and hence capture the fuzzy relationships more appropriately. Singh [8] proposed an improved and versatile method of forecasting based on the concept fuzzy time series. He utilizes various difference parameters being implemented on current state for forecasting the next state values to accommodate the possible vagueness in the data in a better way and making it a robust method. Ref. [6] presented a method to handle forecasting problems using two factors high-order fuzzy time series for temperature prediction and for forecasting the TAIFEX in Taipei, Taiwan. In recent years, some methods have been presented using optimization techniques to select proper intervals and adjust their interval lengths. For example, Particle Swarm Optimization (PSO) is used to find proper intervals and adjust interval lengths [15] - [18]. Lee et al. [7] presented a method based on two factor high-order fuzzy logical relationships and genetic simulated annealing techniques for forecasting the temperature and the TAIFEX. In this paper, we present a new method to forecast temperature and the TAIFEX, based on the two factors high-order fuzzy time series. The proposed forecasting method is built by establishing two factors high-order fuzzy relation groups and the content of forecast rules and based on the two historical data to increase the forecasting accuracy rate. The proposed method gets a higher forecasting accuracy rate than the existing methods.The rest of this paper is organized as follows. In Section 2, a briefly review the basic concepts of fuzzy time series and algorithm are introduced. In Section 3, the details of the proposed forecasting model to forecast the temperature of Taipei is presented. In Section 4, we make a comparison of the experimental results of the proposed model with the existing models, where the historical data of the daily average temperature and TAIFEX in Taipei, Taiwan are used for the experiments. The conclusions are discussed in Section 5.

II. BASIC CONCEPTS OF FUZZY TIME SERIES AND ALGORITHM

2.1 Basic Concepts of Fuzzy Time Series

This section briefly summarizes the basic fuzzy time series concepts. The main difference between the fuzzy time series and traditional time series is that the values of the fuzzy time series are represented by fuzzy sets rather than real value. Let $U = \{u_1, u_2, \dots, u_n\}$ be an universal set; a fuzzy set A_i of U is defined as:

$A_i = \left\{ f_{Ai}(u_1)/_{u_1} + f_{Ai}(u_2)/_{u_2} + \cdots + f_{Ai}(u_n)/_{u_n} \right\}$;where f_{Ai} is a membership function of a given set A, f_{Ai}:U→[0,1], $f_{Ai}(u_i)$ indicates the grade of membership of u_i in the fuzzy set A, $f_{Ai}(u_i) \in [0,1]$, and $1 \le i \le n$. Here, the symbol "+" indicates the operation of union and the symbol "/" indicates the separator rather than the commonly used summation and division in algebra, respectively. General definitions of FTS are given as follows:

Definition 1: Fuzzy time series [1], [2]

Let $Y(t)(t = .., 0, 1, 2 ..)$, a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2 ...) are defined and if F(t)be a collection of $f_1(t), f_2(t), \dots$, then F(t)is called a fuzzy time series on Y(t)(t . . ., 0, 1,2 . ..).

***Definition 2*: Fuzzy logical relationship (FLR) [3]**

The relationship between F(t) and F(t-1) can be denoted by $F(t-1) \to F(t)$. Let $A_i = F(t)$ and $A_j = F(t-1)$, the relationship between F(t) and F(t -1) is denoted by fuzzy logical relationship $A_i \to A_j$ where A_i and A_j refer to the current state or the left - hand side and the next state or the right-hand side of the fuzzy relationship.

***Definition 3*:** m- order fuzzy logical relationships[13]

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1),\ F(t-2), \dots, F(t-m+1)\ F(t-m)$ then this fuzzy relationship is represented by by$F(t-m), \dots, F(t-2),\ F(t-1) \to F(t)$ and is called an m - order fuzzy time series.

Definition 4: Fuzzy logical relationship group (FLRG)[3]

All fuzzy logical relationships in the training dataset can be further grouped together into different fuzzy logical relationship groups according to the same left-hand sides of the fuzzy logical relationship. Suppose there are relationships such that

$A_i \to A_{j1},\ A_i \to A_{j2},\ \dots\dots$

So, based on [3], these fuzzy logical relationship can be grouped into the same FRG as : $A_i \to A_{j1}, A_{j2} \dots$

Definition 5: Two- factors $\boldsymbol{m}$- order fuzzy relations [6]

Suppose $F_A(t)$ và $F_B(t)$ are two fuzzy time series. If $F_A(t)$ is caused by $(F_A(t-1), F_B(t-1))$, $(F_A(t-2), F_B(t-2))$,…, $(F_A(t-m), F_B(t-m))$ then the two-factor m - order fuzzy relationship is represented by $(F_A(t-m), F_B(t-m))$,…, $(F_A(t-2), F_B(t-2))$, $(F_A(t-1), F_B(t-1)) \to F_A(t)$, trong đó $F_A(t)$ are called the main factor fuzzy time series and, $F_B(t)$ the second-factor fuzzy time series. $(F_A(t-m), F_B(t-m))$,…, $(F_A(t-2), F_B(t-2))$, $(F_A(t-1), F_B(t-1))$ and $F_A(t)$ are called the current state or the left - hand side and the next state or the right-hand side of the fuzzy logical relationship.

2.2 Two Factors High – Order FTS Algorithm

The two factors high – order FTS algorithm contains the main steps as follows:

Step 1: Define two universes of discourse $U_A(t)$ and $U_B(t)$ of two fuzzy time series $F_A(t)$ and $F_B(t)$, respectively;

Step 2: Divide the universes $U_A(t)$ and $U_B(t)$ into proper length of intervals, respectively;

Step 3: Define the linguistic terms A_i and B_i represented by fuzzy sets in accordance with the intervals;

Step 4: Fuzzify all historical data of the two factors fuzzy time series;

Step 5: Identify all two factors m-order fuzzy relationships;

Step 6: Establish and Calculate the forecasting value all two factors m-order fuzzy relationship groups;

Step 7: Defuzzify the forecasting output.

III. FORECASTING MODEL BASE ON TWO FACTORS HIGH – ORDER FTS

To verify the effectiveness of the proposed model, all historical data of the daily average temperature and the daily cloud density [6] from June-1-1996 to June- 30-1996 in Taipei, Taiwan (Central Weather Bureau, 1996) are used to illustrate the two factors high - order fuzzy time series forecasting process shown in Table 1. The step-wise procedure of the proposed model is detailed as follows:

Step 1: Define two universes of discourse $U_A(t)$ and $V_B(t)$

Let $U_A(t) = [I_{minA}, I_{maxA}]$ is universes of discourse of the main-factor, be historical data of the temperature on day t and $V_B(t) = [I_{minB}, I_{maxB}]$ is universes of discourse of second-factor, be historical data of the cloud density. For defining the universe of discourse, at first find the minimum value D_{minA} , the maximum value D_{maxA} and the minimum value D_{minB} , the maximum value D_{maxB} of the historical time series data set, respectively. We set $I_{minA} = D_{minA} - N_{1A}$; $I_{maxA} = D_{maxA} + N_{2A}$ and $I_{minB} = D_{minB} - N_{1B}$; $I_{maxB} = D_{maxB} + N_{2B}$. In order to ensure the forecasting values bounded in the universe of discourse $U_A(t)$ and $U_B(t)$, respectively. Where N_{1A}, N_{2A} and N_{1B}, N_{2B} are proper positive integers to tune the lower bound and upper bound of the $U_A(t)$, $V_B(t)$, respectively.From Table 1, it is obvious that the daily minimum temperature value and maximum temperature value are D_{minA} = 23.3 ^{0}C and D_{maxA} = 31.6 ^{0}C, respectively. For convenience of illustrating the forecasting example here, we set N_{1A}= 0.3 and N_{2A}= 0.4 C. So get the universe of discourse on $U_A(t)$ = [23.0, 32.0]. The same way, get the universe of discourse of the second-factor $V_B(t)$ = [0.0, 100].

Table 1: Historical data of the daily average temperature and the daily cloud density from June 1996 to September 1996 in Taipei, Taiwan (Taiwan Central Weather Bureau, 1996).

Date	***Temperature (unit ^{0}C) / month***				***Cloud density(unit %)/ month***			
Days	**June**	**July**	**August**	**September**	**June**	**July**	**August**	**September**
1	26.1	29.9	27.1	27.5	36	15	100	29
2	27.6	28.4	28.9	26.8	23	31	78	53
3	29	29.2	28.9	26.4	23	26	68	66
4	30.5	29.4	29.3	27.5	10	34	44	50
5	30	29.9	28.8	26.6	13	24	56	53
6	29.5	29.6	28.7	28.2	30	28	89	63
7	29.7	30.1	29	29.2	45	50	71	36
8	29.4	29.3	28.2	29	35	34	28	76
9	28.8	28.1	27	30.3	26	15	70	55
10	29.4	28.9	28.3	29.9	21	8	44	31
11	29.3	28.4	28.9	29.9	43	36	48	31
12	28.5	29.6	28.1	30.5	40	13	76	25
13	28.7	27.8	29.9	30.2	30	26	50	14
14	27.5	29.1	27.6	30.3	29	44	84	45
15	29.5	27.7	26.8	29.5	30	25	69	38
16	28.8	28.1	27.6	28.3	46	24	78	24
17	29	28.7	27.9	28.6	55	26	39	19
18	30.3	29.9	29	28.1	19	25	20	39
19	30.2	30.8	29.2	28.4	15	21	24	14
20	30.9	31.6	29.8	28.3	56	35	25	3
21	30.8	31.4	29.6	26.4	60	29	19	38
22	28.7	31.3	29.3	25.7	96	48	46	70
23	27.8	31.3	28	25	63	53	41	71
24	27.4	31.3	28.3	27	28	44	34	70
25	27.7	28.9	28.6	25.8	14	100	29	40
26	27.1	28	28.7	26.4	25	100	31	30
27	28.4	28.6	29	25.6	29	91	41	34
28	27.8	28	27.7	24.2	55	84	14	59
29	29	29.3	26.2	23.3	29	38	28	83
30	30.2	27.9	26	23.5	19	46	33	38
31		26.9	27.7			95	26	

Step 2: Partition the universes $U_A(t)$ and $V_B(t)$ into equal lengths of intervals.

The data of each factor is divided separately and number of the intervals for each factor may be different. The universes of discourse on $U_A(t)$ of the main factor and the universes of discourse on $V_B(t)$ of the second-factor are cut into the pre-defined number of intervals. Compared to the previous models in[5], [7], [18], we divide $U_A(t)$ into 9 intervals $u_1, u_2, \dots, u_9$, respectively. The length of each interval is $LA = \frac{I_{maxA} - I_{minA}}{9} = \frac{32-23}{9} = 1$. Thus, the nine intervals are defined as follows:

$U_i = (I_{minA} + (i-1)*LA, I_{minA} + i*LA]$, with $(1 \le i \le 9)$ gets seven intervals as: U_1 = (23, 24], U_2 = (24,25], …, U_8 = (30,31], U_9 = (31, 32]. In the same way, we divide $V_B(t)$ into 7 intervals with equal lengths, get seven intervals as: V_1 = (0, 14.29], V_2 = (14.29, 28.57], …, V_6 = (71.43,85.71], V_7 = (85.71,100.0].

Step 3: Define the fuzzy sets for two factors fuzzy time series

Each interval in Step 2 represents a linguistic variable. Firstly, defining the main-factor represented by fuzzy sets A_i. Each linguistic variable represents a fuzzy set A_i and its definitions is described in (1)and (2) as follows:

$$A_i = \frac{a_{i1}}{u_1} + \frac{a_{i2}}{u_2} + \cdots + \frac{a_{ij}}{u_j} + \cdots + \frac{a_{i9}}{u_9} \quad (1)$$

$$a_{ij} = \begin{cases} 1 & j == i \\ 0.5 \quad \text{if} & j == i-1 \text{ or } j == i+1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $a_{ij} \in [0,1]$, $1 \le i \le 9, 1 \le j \le 9$ and u_j is the j-th interval of $U_A(t)$. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set A_i. For simplicity, the different membership values of fuzzy set A_i are selected by according to Eq.(1). According to (1) and (2), a fuzzy set contains 9 intervals. On the contrary, an interval belongs to all fuzzy sets with different membership degrees. For example, u_1 belongs to A1 and A2 with membership degrees of 1 and 0.5 respectively, and other fuzzy sets with membership degree is 0.

By the same way, we define fuzzy sets B_i for the second-factor, Each linguistic variable represents a fuzzy set B_i and its definitions is described in (3)and (4) as follows.

$$B_i = \frac{b_{i1}}{v_1} + \frac{b_{i2}}{v_2} + \cdots + \frac{b_{ij}}{v_j} + \cdots + \frac{b_{i7}}{v_7} \quad (3)$$

$$b_{ij} = \begin{cases} 1 & j == i \\ 0.5 \quad if & j == i-1 \; or \; j == i+1 \\ 0 & otherwise \end{cases} \quad (4)$$

where $b_{ij} \in [0,1]$, $1 \le i \le 7, 1 \le j \le 7$ and v_j is the j-th interval of $V_B(t)$. The value of b_{ij} indicates the grade of membership of v_j in the fuzzy set B_i

Step 4: Fuzzify all historical data of the two - factors fuzzy time series

In order to fuzzify all historical data, it's necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set that each interval belongs to with the highest membership degree.

For example, the historical data on June 1, 1996, the actual daily average temperature and daily cloud density are 26.1 ^{0}C and 36%, respectively and they belong to interval U_4 = (26.0 , 27] and interval V_3= [28.57, 42.86], respectively. Hence, we assign the fuzzy set A4 corresponding to interval U_4 of the main - factor and assign the fuzzy set B3 corresponding to interval V_3 of the second-factor, respectively. From Table1, the results of fuzzification are listed in Table 2 by using Eqs. (1), (2), (3) and (4) to fuzzify the historical data of the daily average temperature and the daily cloud density.

Table 2: Fuzzified historical data of the main-factor (*the daily average temperature*) and the second-factor (*the daily cloud density*) in june 1996 in Taipei

Month	Data of main - factor	Fuzzified value of main - factor	Data of second - factor	Fuzzified value of second - factor
June-1-96	26.1	A4	36	B3
June-2-96	27.6	A5	23	B2
June-3-96	29	A7	23	B2
June-4-96	30.5	A8	10	B1
June-5-96	30	A8	13	B1
June-6-96	29.5	A7	30	B3
June-7-96	29.7	A7	45	B4
June-8-96	29.4	A7	35	B3
June-9-96	28.8	A6	26	B2
June-10-96	29.4	A7	21	B2
June-11-96	29.3	A7	43	B4
June-12-96	28.5	A6	40	B3
June-13-96	28.7	A6	30	B3
June-14-96	27.5	A5	29	B3
June-15-96	29.5	A7	30	B3
June-16-96	28.8	A6	46	B4
June-17-96	29	A7	55	B4
June-18-96	30.3	A8	19	B2
June-19-96	30.2	A8	15	B2
June-20-96	30.9	A8	56	B4
June-21-96	30.8	A8	60	B5
June-22-96	28.7	A6	96	B7
June-23-96	27.8	A5	63	B5
June-24-96	27.4	A5	28	B2
June-25-96	27.7	A5	14	B1
June-26-96	27.1	A5	25	B2
June-27-96	28.4	A6	29	B3

June-28-96	27.8	A5	55	B4
June-29-96	29	A7	29	B3
June-30-96	30.2	A8	19	B2

Step 5: Identify all two factors m-order fuzzy logical relationships

After two fuzzy time series $F_A(t)$ and $F_B(t)$ have been created, we can find out all fuzzy relationships under different orders. The way to create all two-factor m-order fuzzy relationship is to find any relationship consisting of the type $(F_A(t-m), F_B(t-m)), \ldots, (F_A(t-2), F_B(t-2)), (F_A(t-1), F_B(t-1)) \rightarrow F_A(t)$, where $(F_A(t-m), F_B(t-m)), \ldots, (F_A(t-2), F_B(t-2)), (F_A(t-1), F_B(t-1))$ and $F_A(t)$ are called the current state and the next state, respectively. Then m - order fuzzy relationship can be obtained by replacing $(F_A(t-m), F_B(t-m)), \ldots, (F_A(t-2), F_B(t-2)), (F_A(t-1), F_B(t-1))$ and $F_A(t)$ with the corresponding fuzzy set.

For instance, with m = 3, based on Table 2, we get a 3rd – order fuzzy relationship (A4, B3), (A5, B2), (A7, B2) → A8 by replacing $(F_A(\text{June}-1-96), F_B(\text{June}-1-96)), \ldots, (F_A(June-2-96), F_B(June-2-96)), (F_A(June-3-96), F_B(June-3-96)) \rightarrow F_A(\text{June}-4-96)$. From Table 2, we complete two factors 3rd-order fuzzy relationships are listed in Table 3, where there are 27 fuzzy relations and each fuzzy relation of two factors 3rd - order fuzzy relationship is fuzzified from the historical data of the daily average temperature and the daily cloud density ranged from June-1-1996 to June- 30-1996.

Table 3: Two factors 3rd-order fuzzy logical relationships

No	Month	Fuzzified value of main - factor	Fuzzified value of second - factor	Fuzzy logical relationships
	June-1-96	A4	B3	
	June-2-96	A5	B2	
	June-3-96	A7	B2	
1	June-4-96	A8	B1	(A4, B3), (A5, B2), (A7, B2) → A8
2	June-5-96	A8	B1	(A5, B2), (A7, B2), (A8, B1) → A8
3	June-6-96	A7	B3	(A7, B2), (A8, B1), (A8, B1) → A7
4	June-7-96	A7	B4	(A8, B1), (A8, B1), (A7, B3) → A7
5	June-8-96	A7	B3	(A8, B1), (A7, B3), (A7, B4) → A7
6	June-9-96	A6	B2	(A7, B3), (A7, B4), (A7, B3) → A6
7	June-10-96	A7	B2	(A7, B4), (A7, B3), (A6, B2) → A7
8	June-11-96	A7	B4	(A7, B3), (A6, B2), (A7, B2) → A7
9	June-12-96	A6	B3	(A6, B2), (A7, B2), (A7, B4) → A6
10	June-13-96	A6	B3	(A7, B2), (A7, B4), (A6, B3) → A6
11	June-14-96	A5	B3	(A7, B4), (A6, B3), (A6, B3) → A5
12	June-15-96	A7	B3	(A6, B3), (A6, B3), (A5, B3) → A7
13	June-16-96	A6	B4	(A6, B3), (A5, B3), (A7, B3) → A6
14	June-17-96	A7	B4	(A5, B3), (A7, B3), (A6, B4) → A7
15	June-18-96	A8	B2	(A7, B3), (A6, B4), (A7, B4) → A8
16	June-19-96	A8	B2	(A6, B4), (A7, B4), (A8, B2) → A8
17	June-20-96	A8	B4	(A7, B4), (A8, B2), (A8, B2) → A8
18	June-21-96	A8	B5	(A8, B2), (A8, B2), (A8, B4) → A8
19	June-22-96	A6	B7	(A8, B2), (A8, B4), (A8, B5) → A6
20	June-23-96	A5	B5	(A8, B4), (A8, B5), (A6, B7) → A5
21	June-24-96	A5	B2	(A8, B5), (A6, B7), (A5, B5) → A5
22	June-25-96	A5	B1	(A6, B7), (A5, B5), (A5, B2) → A5
23	June-26-96	A5	B2	(A5, B5), (A5, B2), (A5, B1) → A5
24	June-27-96	A6	B3	(A5, B2), (A5, B1), (A5, B2) → A6
25	June-28-96	A5	B4	(A5, B1), (A5, B2), (A6, B3) → A5
26	June-29-96	A7	B3	(A5, B2), (A6, B3), (A5, B4) → A7
27	June-30-96	A8	B2	(A6, B3), (A5, B4), (A7, B3) → A8
28	July -01-96	#		(A5, B4), (A7, B3), (A8, B2) → #

Step 6: Establish and calculate the forecasting value all two factors m-order fuzzy relationship groups.

According to[3], [15]all the fuzzy relationships having the same fuzzy sets on the left-hand side or the same current state can be put together into one fuzzy relationship group. For example: Suppose there are three – order fuzzy relationships such that : (A4, B3), (A5, B2), (A7, B2) → A8 , (A4, B3), (A5, B2), (A7, B2) → A7; ….

These fuzzy logical relationships can be grouped into a fuzzy relation group (FRG) as: (A4, B3), (A5, B2), (A7, B2) → A8, A7, … Based on this viewpoint, we can construct two-factor three-order fuzzy logical relationships groups in Table 3 into all groups are listed in Table 4. In Table 4, group 28 consists of the fuzzy relationship (A5, B4), (A7, B3), (A8, B2) → # as it is created by the relationship $(F_A(June-28-96), F_B(June-28-96))$, $(F_A(June-29-96), F_B(June-29-96))$, $(F_A(June-30-96), F_B(June-30-96)) \rightarrow F_A(July-01-96)$ and called the untrained pattern. Since the linguistic value of $F_A(July-01-96)$ is unknown within the historical data from June 1, 1996 to June 30, 1996. Hence, we use symbol ‘‘#” to denote the unknown value.

Table 4: The complete forecasted values for all the two factors 3rd-order fuzzy relationship groups

No	Month	Fuzzy set of main factor	Fuzzy set of second factor	Fuzzy relation groups	Forecasted value
	June-1-96	A4	B3		
	June-2-96	A5	B2		
	June-3-96	A7	B2		
1	June-4-96	A8	B1	(A4, B3), (A5, B2), (A7, B2) → A8	30.62
2	June-5-96	A8	B1	(A5, B2), (A7, B2), (A8, B1) → A8	30.12
3	June-6-96	A7	B3	(A7, B2), (A8, B1), (A8, B1) → A7	29.62
4	June-7-96	A7	B4	(A8, B1), (A8, B1), (A7, B3) → A7	29.62
5	June-8-96	A7	B3	(A8, B1), (A7, B3), (A7, B4) → A7	29.38
6	June-9-96	A6	B2	(A7, B3), (A7, B4), (A7, B3) → A6	28.88
7	June-10-96	A7	B2	(A7, B4), (A7, B3), (A6, B2) → A7	29.38
8	June-11-96	A7	B4	(A7, B3), (A6, B2), (A7, B2) → A7	29.38
9	June-12-96	A6	B3	(A6, B2), (A7, B2), (A7, B4) → A6	28.62
10	June-13-96	A6	B3	(A7, B2), (A7, B4), (A6, B3) → A6	28.62
11	June-14-96	A5	B3	(A7, B4), (A6, B3), (A6, B3) → A5	27.62
12	June-15-96	A7	B3	(A6, B3), (A6, B3), (A5, B3) → A7	29.62
13	June-16-96	A6	B4	(A6, B3), (A5, B3), (A7, B3) → A6	28.88
14	June-17-96	A7	B4	(A5, B3), (A7, B3), (A6, B4) → A7	29.12
15	June-18-96	A8	B2	(A7, B3), (A6, B4), (A7, B4) → A8	30.38
16	June-19-96	A8	B2	(A6, B4), (A7, B4), (A8, B2) → A8	30.12
17	June-20-96	A8	B4	(A7, B4), (A8, B2), (A8, B2) → A8	30.88
18	June-21-96	A8	B5	(A8, B2), (A8, B2), (A8, B4) → A8	30.88
19	June-22-96	A6	B7	(A8, B2), (A8, B4), (A8, B5) → A6	28.62
20	June-23-96	A5	B5	(A8, B4), (A8, B5), (A6, B7) → A5	27.88
21	June-24-96	A5	B2	(A8, B5), (A6, B7), (A5, B5) → A5	27.38
22	June-25-96	A5	B1	(A6, B7), (A5, B5), (A5, B2) → A5	27.62
23	June-26-96	A5	B2	(A5, B5), (A5, B2), (A5, B1) → A5	27.12
24	June-27-96	A6	B3	(A5, B2), (A5, B1), (A5, B2) → A6	28.38
25	June-28-96	A5	B4	(A5, B1), (A5, B2), (A6, B3) → A5	27.88
26	June-29-96	A7	B3	(A5, B2), (A6, B3), (A5, B4) → A7	29.12
27	June-30-96	A8	B2	(A6, B3), (A5, B4), (A7, B3) → A8	30.12
28	July-01-96	A4	B3	(A5, B4), (A7, B3), (A8, B2) → #	31.5

In the following, we calculate the forecasting value for all groups in the training phase and testing phase according to principles as follows:

Principle 1. If there is a fuzzy logical relationship in the two factors mth-order fuzzy logical relationship groups, shown as follows: $(A_{i\,m}, B_{im}), (A_{i\,m-1}, B_{im-1}), \ldots, (A_{i\,1}, B_{i1}) \rightarrow A_{j1}, A_{j2}, \ldots, A_{jp}$.

where the maximum membership level of A_{j1}, A_{j2}, …, and A_{jp} occur at intervals U_{j1}, U_{j2}, . . ., and U_{jp}, respectively, and the midpoints of U_{j1}, U_{j2}, . . ., and U_{jp} are m_{j1}, m_{j2}, . . ., and m_{jp} respectively. To make forecasted value for each group of principle 1, we propose a new fuzzy forecasting rule for each group of fuzzy relations. Particularly, for each group in Table 4, we divide each corresponding interval of fuzzy sets on the right-hand side of fuzzy relation groups into q sub-intervals with equal length, and calculate forecasted value for each group according to Eq. (5).

$$\text{Forecasted} = \frac{1}{n}\sum_{k=1}^{n} sub\_mid_{kj} \quad (5)$$

where, n is the total number of next states or the total number of fuzzy sets on the right-hand side within the same group;

- ✓ q is the total number of sub-intervals corresponding to k-th fuzzy set on the right-hand side within the same group;
- ✓ sub_mid_{kj} is the midpoint of one of q sub-intervals (*or means the midpoint of j-th sub-interval)* corresponding to k-th fuzzy set on the right-hand side where the highest level of A_{kj} occurs at this sub-interval, with $(1 \leq k \leq n;\ 1 \leq j \leq q)$.

For example, in Table 4, Group 1 has only one fuzzy set on the right-hand side as $(A4, B3), (A5, B2), (A7, B2) \rightarrow A8$, where the highest membership level belongs to interval $U_8 = [30, 3)$. From Eq.(5), we divide the interval U_8 into q sub-intervals (assumed q = 4) which are $SU_{8,1} = [30, 30.25)$, $SU_{8,2} = [30.25, 30.5)$, $SU_{8,3} = [30.5, 30.75)$, $SU_{8,4} = [30.75, 31)$. In Table 3, the 3rd-order fuzzy relationship (A4, B3), (A5, B2), (A7, B2) → A8 is got as $(F_A(June-01-96), F_B(June-01-96))$,$(F_A(June-02-96), F_B(June-02-96))$, $(F_A(June-03-96), F_B(June-03-96)) \rightarrow F_A(June-04-96\ )$; where the historical data on June 4, 1996, the actual daily average temperature is 30.5 and it belong to sub-interval $SU_{8,3} = [30.5, 30.75)$ and then the midpoint $subm_{8,3}$ of sub-interval $SU_{8,2}$ is 30.62. The finally, forecasted value for Group 1 according to Eq. (5) is 30.62.

Principle 2. If there is a fuzzy logical relationship in the two factors mth-order fuzzy relationship groups, shown as follows:$(A_{i\,m}, B_{im})$, $(A_{i\,m-1}, B_{im-1})$, …, $(A_{i\,1}, B_{i1}) \rightarrow \#$
where the symbol ‘‘#” denotes an unknown value, then the forecasted value of day i is calculated according to [18] as follows:

$$\text{Forecasted\_for\#} = m_{i1} + \frac{\sum_{k=2}^{m} m_{i(k-1)} - m_{ik}}{2^{k-1}} \quad (6)$$

where, $m_{i\,1}, m_{i2}, \dots, m_{ik}$ is midpoints of U_{i1}, U_{i2}, . . ., and U_{ik} $(2 \leq k \leq m)$, respectively.
For example, In Table 4 there is the 3rd – order fuzzy relationship group in which the right hand side of the fuzzy relationship is an unknown value "#," shown as follows: (A5, B4), (A7, B3), (A8, B2) → #
Following the above example, the complete forecasted values for all groups are listed in Table 4. From Table 4 and based on Table 2, the forecasting the daily average temperature of June 1996 in Taipei based on the two factors 3rd-order FTS can be obtained, as shown in Table 5, where the average forecasting error rate is defined by (7) and (8).

Table 5: The complete forecasted results based on the two factors third-order fuzzy time series daily temperature of June 1996

Month	main - factor	Fuzzified value	second - factor	Fuzzified value	Forecasted temperature
June-1-96	26.1	A4	36	B3	-----
June-2-96	27.6	A5	23	B2	-----
June-3-96	29	A7	23	B2	-----
June-4-96	30.5	A8	10	B1	30.62
June-5-96	30	A8	13	B1	30.12
June-6-96	29.5	A7	30	B3	29.62
June-7-96	29.7	A7	45	B4	29.62
June-8-96	29.4	A7	35	B3	29.38
June-9-96	28.8	A6	26	B2	28.88
June-10-96	29.4	A7	21	B2	29.38
June-11-96	29.3	A7	43	B4	29.38
June-12-96	28.5	A6	40	B3	28.62
June-13-96	28.7	A6	30	B3	28.62
June-14-96	27.5	A5	29	B3	27.62
June-15-96	29.5	A7	30	B3	29.62
June-16-96	28.8	A6	46	B4	28.88
June-17-96	29	A7	55	B4	29.12
June-18-96	30.3	A8	19	B2	30.38
June-19-96	30.2	A8	15	B2	30.12
June-20-96	30.9	A8	56	B4	30.88
June-21-96	30.8	A8	60	B5	30.88
June-22-96	28.7	A6	96	B7	28.62
June-23-96	27.8	A5	63	B5	27.88
June-24-96	27.4	A5	28	B2	27.38
June-25-96	27.7	A5	14	B1	27.62
June-26-96	27.1	A5	25	B2	27.12
June-27-96	28.4	A6	29	B3	28.38
June-28-96	27.8	A5	55	B4	27.88
June-29-96	29	A7	29	B3	29.12
June-30-96	30.2	A8	19	B2	30.12
July-01-96	N/A	#	N/A	N/A	31.5

MSE					0.007
MAPE					**0.27%**

To inspect forecasting performance of proposed model in two factors high – order the fuzzy time series, the mean square error (MSE) and the mean absolute percentage error (MAPE) are employed as an evaluation criterion to represent the forecasted accuracy. The MSE and MAPE value are calculated according to (7) and (8) as follows:

$$MSE = \frac{1}{n}\sum_{i=m}^{n}(F_i - R_i)^2 \quad (7)$$

$$MAPE = \frac{1}{n}\sum_{i=m}^{n}\left|\frac{F_i - R_i}{R_i}\right| * 100\% \quad (8)$$

Where, $R_i$and F_i note the actual and forecasted value of day i, respectively, n is the total number of days to be forecasted, m is the order of fuzzy logical relationship.

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this section, two illustrative examples are examined to compare proposed model with some other popular forecasting models. Where, all historical data of the daily average temperature and the daily cloud density from June 1, 1996 to September 30, 1996 shown in Tables 1 and TAIFEX data set [6] are used for the experiment. In the first experiment, the "temperature" is chosen as the main forecasting objective. In the second case, we perform to forecast the TAIFEX from August 3, 1998 to September 30, 1998, where the TAIFEX is called the main-factor and the TAIEX (Taiwan stock exchange capitalization weighted stock index) is called the second-factor.

4.1 Forecasting for Temperature

In order to verify the forecasting effectiveness of the proposed model under different number of intervals and different high - order FTS , four FTS models in the model [19], the model [6], the model [7] and the model MTPSO [18] are examined and compared. The forecasted accuracy of the proposed method is estimated using the equation in (8). A comparison of the forecasting results are listed in Table 6 – 9. During simulation, the number of intervals of the main- factor and the second – factor are kept fix of 9 intervals and 7 intervals for the proposed model, respectively. From Table 6-9, it can see that the proposed method gets smaller forecasting error rate than the methods compared in the same table. That is, the proposed method outperforms than the existing models under various high-order FLRs for temperature prediction from June 1996 to September 1996 in Taipei, Taiwan.

Table 6: A comparison of the average forecasting accuracy of the proposed method with those of the existing methods in June 1996 in the training phase.

Model		**Order of FLRs**						
		2nd-order	3rd-order	4th-order	5th-order	6th-order	7th-order	8th-order
Model [19]		2.88%	3.16%	3.24%	3.33%	3.39%	3.53%	3.67%
Model [6]		0.8%	0.76%	0.79%	0.76%	0.79%	0.79%	0.81%
	α=0.25	0.44%	0.42%	0.42%	0.42%	0.44%	0.40%	0.40%
Model [7]	α =0.5	0.50%	0.45%	0.42%	0.38%	0.43%	0.39%	0.46%
	α =0.9	0.46%	0.42%	0.44%	0.42%	0.41%	0.46%	0.39%
MPSO [18]		0.36%	0.34%	0.32%	0.31%	0.31%	0.28%	0.29%
Our model		**0.27%**	**0.27%**	**0.26%**	**0.25%**	**0.25%**	**0.25%**	**0.26%**

Table 7: A comparison of the average forecasting accuracy of the proposed method with those of the existing methods in July 1996 in the training phase

Model		**Order of FLRs**						
		2nd-order	3rd-order	4th-order	5th-order	6th-order	7th-order	8th-order
Model [19]		3.04%	3.76%	4.08%	4.17%	4.35%	4.38%	4.56%
Model [6]		0.96%	0.96%	0.98%	0.97%	1.00%	0.98%	0.99%
	α=0.25	0.44%	0.42%	0.42%	0.44%	0.40%	0.40%	0.40%
Model [14]	α =0.5	0.50%	0.42%	0.38%	0.43%	0.39%	0.46%	0.46%
	α =0.9	0.46%	0.44%	0.42%	0.41%	0.46%	0.39%	0.39%
MPSO [18]		0.34%	0.33%	0.33%	0.32%	0.32%	0.34%	0.33%
Our model		**0.42%**	**0.16%**	**0.16%**	**0.17%**	**0.17%**	**0.17%**	**0.17%**

Table 8: A comparison of the average forecasting accuracy of the proposed method with those of the existing methods in August 1996 in the training phase

Model		**Order of FLRs**						
		2nd-order	3rd-order	4th-order	5th-order	6th-order	7th-order	8th-order
Model [19]		2.75%	2.77%	3.30%	3.40%	3.18%	3.15%	3.19%
Model [6]		1.07%	1.06%	1.08%	1.08%	1.09%	1.07%	1.07%
	α=0.25	0.44%	0.42%	0.42%	0.44%	0.40%	0.40%	0.40%
Model [7]	α =0.5	0.50%	0.42%	0.38%	0.43%	0.39%	0.46%	0.46%
	α =0.9	0.46%	0.44%	0.42%	0.41%	0.46%	0.39%	0.39%

MPSO [18]	0.31%	0.34%	0.33%	0.33%	0.33%	0.34%	0.35%
Our model	**0.37%**	**0.31%**	**0.25%**	**0.25%**	**0.25%**	**0.24%**	**0.24%**

Table 9: A comparison of the average forecasting accuracy of the proposed method with those of the existing methods in September 1996 in the training phase

Model		**Order of FLRs**						
		2^{nd}-order	3^{rd}-order	4^{th}-order	5^{th}-order	6^{th}-order	7^{th}-order	8^{th}-order
Model [19]		3.29%	3.10%	3.19%	3.22%	3.39%	3.38%	3.29%
Model [6]		1.01%	0.90%	0.94%	0.96%	0.95%	0.95%	0.95%
	α=0.25	0.44%	0.42%	0.42%	0.44%	0.40%	0.40%	0.40%
Model [7]	α =0.5	0.50%	0.42%	0.38%	0.43%	0.39%	0.46%	0.46%
	α =0.9	0.46%	0.44%	0.42%	0.41%	0.46%	0.39%	0.39%
MPSO [18]		0.54%	0.56%	0.54%	0.50%	0.51%	0.52%	0.41%
Our model		**0.44%**	**0.26%**	**0.25%**	**0.26%**	**0.26%**	**0.26%**	**0.25%**

4.2 Forecasting for TAIFEX

In this subsection, we apply the proposed model to handle forecasting the TAIFEX, where the universe of discourse of the main-factor "TAIFEX" and the universe of discourse of the second-factor "TAIEX" are cut into 16 intervals. To verify the superiority of the proposed model under various two - factors high-order fuzzy logical relationships and different numbers of intervals, existing forecasting model, viz., C96 model in [3] , H01a model in[12], L06 model in[6], L07 model in[5], L08 model in [7] and MTPSO model in [18] are selected for comparison. The forecasted accuracy of the proposed method is estimated using the MSE technique in (7). A comparison of the forecasted accuracy among the existing models and the proposed model is listed in Table 10.

Table 10. A comparison of the forecasting results of the proposed model with the existing models based on the two- factor high – order FTS under number of intervals of 16

Date	Actual TAIFEX	C96	H01a	L06	L07	L08	MTPSO	Our model
8/3/1998	7552							
8/4/1998	7560	7450	7450					
8/5/1998	7487	7450	7450					
8/6/1998	7462	7500	7450	7450				
8/7/1998	7515	7500	7500	7550				
8/10/1998	7365	7450	7450	7350				
8/11/1998	7360	7300	7350	7350				
8/12/1998	7330	7300	7300	7350	7348	7329	7325.28	7326.58
8/13/1998	7291	7300	7350	7250	7301.5	7289.5	7287.48	7282.82
8/14/1998	7320	7183.33	7100	7350	7311.5	7329	7325.28	7326.58
8/15/1998	7300	7300	7350	7350	7301.5	7289.5	7287.48	7304.7
8/17/1998	7219	7300	7300	7250	7226.5	7215	7221.26	7217.2
8/18/1998	7220	7183.33	7100	7250	7226.5	7215	7221.26	7217.2
8/19/1998	7285	7183.33	7300	7250	7301.5	7289.5	7287.48	7282.82
8/20/1998	7274	7183.33	7100	7250	7256.5	7289.5	7287.48	7282.82
8/21/1998	7225	7183.33	7100	7250	7226.5	7215	7221.26	7217.2
8/24/1998	6955	7183.33	7100	6950	6952	6949.5	6952.02	6954.7
8/25/1998	6949	6850	6850	6950	6952	6949.5	6952.02	6954.7
8/26/1998	6790	6850	6850	6750	6783.5	6796	6781.01	6779.7
8/27/1998	6835	6775	6650	6850	6852	6848	6842.05	6845.32
8/28/1998	6695	6850	6750	6650	6713	6698.5	6696.17	6692.2
8/29/1998	6728	6750	6750	6750	6713	6726	6726.5	6735.94
8/31/1998	6566	6775	6650	6550	6561	6569.5	6580.45	6560.94
9/1/1998	6409	6450	6450	6450	6406	6417	6409.24	6407.82
9/2/1998	6430	6450	6550	6450	6406	6417	6409.24	6429.7
9/3/1998	6200	6450	6350	6250	6198.5	6205	6213.94	6210.94
9/4/1998	6403.2	6450	6450	6450	6406	6417	6409.24	6407.82
9/5/1998	6697.5	6450	6550	6650	6703	6698.5	6696.17	6692.2
9/7/1998	6722.3	6750	6750	6750	6713	6726	6726.5	6714.08
9/8/1998	6859.4	6775	6850	6850	6852	6848	6864.96	6867.2
9/9/1998	6769.6	6850	6750	6750	6783.5	6763	6781.01	6779.7
9/10/1998	6709.75	6775	6650	6750	6713	6726	6696.17	6714.08
9/11/1998	6726.5	6775	6850	6750	6713	6726	6726.5	6735.94
9/14/1998	6774.55	6775	6850	6817	6783.5	6763	6781.01	6779.7
9/15/1998	6762	6775	6650	6817	6783.5	6763	6781.01	6757.82
9/16/1998	6952.75	6775	6850	6817	6953	6949.5	6952.02	6954.7
9/17/1998	6906	6850	6950	6950	6952	6904.5	6906.7	6910.94
9/18/1998	6842	6850	6850	6850	6852	6848	6842.05	6845.32
9/19/1998	7039	6850	6950	7050	7089	7064	7039.2	7042.2
9/21/1998	6861	6850	6850	6850	6852	6848	6864.96	6867.2

9/22/1998	6926	6850	6950	6950	6952	6904.5	6906.7	6932.82
9/23/1998	6852	6850	6850	6850	6852	6848	6842.05	6845.32
9/24/1998	6890	6850	6950	6850	6893	6904.5	6906.7	6889.08
9/25/1998	6871	6850	6850	6850	6852	6848	6864.96	6867.2
9/28/1998	6840	6850	6750	6850	6852	6848	6842.05	6845.32
9/29/1998	6806	6850	6750	6850	6792.5	6796	6781.01	6801.58
9/30/1998	6787	6850	6750	6750	6783.5	6796	6781.01	6779.7
MSE		**9668.94**	**7856.5**	**1364.56**	**249.61**	**105.02**	**92.17**	**37.2**

As shown in Table 10, the proposed model has the smallest forecasting error rate by the MSE value among seven forecasting models compared. Namely, by using the proposed forecasting rules combined with the two - factors 7th - order fuzzy relationships, our model received an MSE value of **37.2**. That is, the proposed model outperforms than the existing models for forecasting TAIFEX from August 3, 1998 to September 30, 1998 in Taipei, Taiwan

V. CONCLUSION

In this paper, a novel forecasting model based on two - factors high - order fuzzy time series is presented. The proposed method constructs two factors high-order fuzzy logical relationship groups based on the historical data and utilizes proposed defuzzification rules for forecasting Temperature and TAIFEX to increase the forecasting accuracy rate. The experimental results show that, in many cases, the proposed method gets better forecasting performance than the existing ones. The detail of comparison was presented in Table 6-10. Even though this study shows the superior forecasting capability compared with the previous some of forecasting models. But, authors always expect that in future studies, authors can concentrate on an optimization technique for finding the proper lengths of intervals used in the stage of determining intervals of universe of discourse and may use different artificial intelligence techniques in fuzzification stage to deal various forecasting problems.

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