American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-5, pp-301-311
www.ajer.org
Research Paper Open Access

Predicting Drillstring Buckling

Nwonodi Roland Ifeanyi[1], Adali Francis[2], Tswenma Tsokwa[3],
[1]*Federal University Wukari,*
[2]*University of Port Harcourt,*
[3]*Federal University Wukari.*

Abstract: *Drillstring buckling is one of the major challenges facing the drilling industry, which can increase non-productive time (NPT) by locking up the wellbore and it may require serious fishing operations to solve. The use of oversimplified assumptions, which neglects friction effect, torque and wellbore geometry, is one of the reasons for the poor prediction of drillstring buckling. The model governing the deflection of the drillstring can yield several integration constants requiring several accurate boundary conditions in order to solve. The difficulty in generating such boundary conditions makes it cumbersome to predict drillstring buckling. In this study, we overcome this difficulty by creating a particular solution from the total moment equation comprising bending moment and torque in the drillstring. The study includes the effects of wellbore geometry and coefficient of friction into the analysis while predicting critical force. The results obtained shows that for any wellbore geometry, the drillstring stiffness and unit weight are the most important factors affecting the critical force. In vertical wellbores, the radial clearance has little or no influence on the critical force. For horizontal and inclined wellbores, the radial clearance has a significant influence on the critical force with wellbore inclination an important factor for the later. The value of the critical force obtained using the model developed in this study was 11,272 Lbf which compares with 11, 725.3 Lbf obtained using the Lubinski`s model. In addition, the value obtained for horizontal well was 206,552.7 Lbf that compares favourably with the Huang et al model (2015a). Thus, the recommendation is to model properly the radial clearance and drillstring stiffness, and to introduce drilling packers in order to prevent tubular buckling of oil well drillstring.*

Keywords: *Drillstring buckling; drillstring stiffness; radial clearance; critical force; beam-column model; wellbore geometry.*

I. INTRODUCTION

The major concern when drilling a well is how to follow the well planning program in order to carry out a safe, usable and economic operation, (Adams 1985; Azar 2006). For the well to be usable, it must be stable and problem free. One of the integral components of a drilling rig is the drillstring, which is composed of drillpipe, drill collars, bottom hole assembly and other accessories, Mitchell and Miska(2011), the stability of which determines if the concern of the well planner will be successful or not. Excess loads acting on the drillstring render it unstable and result in buckling, when a critical force is exceeded. When a tubular string buckles, it will acquire a curved shape, Gao and Huang (2015).

Drillstring buckling, a boundary value problem, can incur huge cost on a drilling project because it may lead to an increase in non-productive time (NPT) and serious fishing operations may be required to solve the challenge. It can lead to drillstring lock up, casing wear and drillstring failure, Gao and Huang (2015). The Euler deflection model defines the buckling of a slender column as presented by Chapra and Canales (2007), and Mitchell and Miska (2011). Nevertheless, this model is not applicable to the buckling of drillstring because of downhole complexities arising from such factors as frictional forces, torque, weight of drill collars, size of the string and the wellbore geometry, (Mitchell and Miska 2011; Gao and Huang 2015).

In the process of solving equations governing the elastic behaviour of the drillstring, many boundary conditions are required because of the many constants of integration generated from the equations. This is the case because the equations are usually of fourth (4th) order and appear as inhomogeneous differential equations. The solutions of such differential equations are obtained by the combination of a complimentary solution and a particular solution, Chapra and Canales (2007)**.** The exact boundary conditions are required if any useful solution are to be obtained. Thus, research is incapacitated since only the boundary conditions at the extremes of deflection are known with precision. Prediction of drillstring buckling has been a challenge to drilling engineering, Mehdi and Jeremy (2014), and the use of dimensionless parameters has been successfully applied as presented by Mitchell and Miska (2011). One of the reasons for this difficulty is the inability to define a relationship between the deflection in the lateral directions and the length of the drillstring. Predicting the maximum displacement and location of the drillstring for a specific depth is essential to defining drillstring buckling, Abdul-Ameer (2012).The exact conditions of the drillstring in-situ are also hard to define. There are also weaknesses in the assumptions used to generate the governing equations.

While the determination of accurate model for predicting the critical force is important, more important is to have a model that is applicable in vertical or non-vertical wellbores. This will yield a cost effective way of predicting drillstring buckling.

The aim of this study is to overcome the difficulties in defining the critical force causing drillstring buckling by reducing the number of unknowns generated in the governing equations. The objective is to overcome the necessity of using many boundary conditions in trying to solve buckling models by developing a particular solution. This will be applicable in a straight wellbore.

Tubular Buckling Literature

Several models have been proposed for predicting tubular buckling. Some of the notable models for vertical wellbores are those presented by, (Lubinski 1950; W. Huang *et al* 2016). Huang, W., *et al* (2015a) worked on horizontal wells, while Huang, W., *et al* (2015b) worked on inclined wellbores. Apparently, none of these models appears to predicting the buckling force for a vertical, inclined or horizontal well. The results obtained from such models are quite different even for similar wellbore geometry mainly because of the different assumptions in their formulations. The most commonly used models for studying tubular buckling are, (Gao and Huang 2015), the beam-column model, the buckling differential equation, and the energy method.

The beam-column model assumes the drillstring to be in the form of a column having two possible ways of deflecting in the lateral directions. Only a small amount of deflection occurs in the tubular compared to the axial length of the drillstring. Thus, the deflection satisfies the linear elastic theory. The governing equation for such a deflection, which is a differential equation, is given as follows, Gao and Huang (2015):

$$\frac{d^4u}{dz^4} + \frac{M_T}{EI}\frac{d^3v}{dz^3} + \frac{d}{dz}\left(\frac{F - qzCosi}{EI}\frac{du}{dz}\right) - \frac{qSini}{EI} = 0$$

$$\frac{d^4v}{dz^4} - \frac{M_T}{EI}\frac{d^3u}{du^3} + \frac{d}{dz}\left(\frac{F - qzCosi}{EI}\frac{dv}{dz}\right) = 0$$

(1)

The integration of equation (1) will yield four integration constants that are obtainable only when accurate boundary conditions are specified. Equation (1) is a differential equation of the inhomogeneous form, solvable by the combination of a complimentary solution and particular solution, Chapra and Canales (2007)**,** the substitution of the particular solution into the general equation satisfies the equation. The Beam-Column model will be used in this study to generate the critical buckling force. This will be done in such a way as to make prediction for vertical, inclined or horizontal wellbore geometry.

In the buckling differential equation method, the well is constrained by the clearance radius along the lateral directions:

$$u = r_c Cos\ \theta$$

(2)

$$v = r_c Sin\ \theta$$

(3)

The substitution of equations (2) and (3) into the governing equation for the beam-column, equation yields the following, Gao (2006):

$$\frac{d^4\theta}{dz^4} - 6\left(\frac{d\theta}{dz}\right)^2\frac{d^2\theta}{dz^2} + 3\frac{M_T}{EI}\frac{d\theta}{dz}\frac{d^2v}{dz^2} + \frac{d}{dz}\left(\frac{F}{EI}\frac{d\theta}{dz}\right) + \frac{qSini}{EIr_c}Sin\ \theta = 0$$

(4)

Where $\theta =$ angular displacement from the buckling differential equation

The bending moment and torque make up the total moment vector in the drillstring as follows, Mitchell and Miska (2011):

$$M = EI\kappa + M_T$$

(5)

While in a state of compression, the drillstring responds to bending moment and there may be significant drag force depending on the contact made with the walls of the wellbore. The sinusoidal buckling solution for the deflection of tubular is presented in equation (6), Gao and Huang (2015):

$$\theta = ASinw_v z \quad (6)$$

The energy method considers the total potential energy of the tubular. The sinusoidal solutions are substituted into the expression for total potential energy. The calculated minimum value of the expression gives the required solution for buckling.

All three basic methods are applied to determine the critical buckling load. Lubinski (1950) was the earliest researcher to give a detailed description of tubular buckling in the drillstring. He used a vertical wellbore with no torque and substituted into the beam-column model to obtain the following expression:

$$F_1 = 1.94\, w_{bp} \sqrt[3]{\frac{EI}{w_{bp}}} \quad (7)$$

Mitchell (2013) used numerical analysis to solve the buckling differential equation for the helical buckling, while Parslay and Bogu (1964) used the energy method to obtain an expression for the critical load in an inclined wellbore. The mathematical expression for the critical force is as follows, (Parslay and Bogu, 1964):

$$F_{cr} = \frac{n^2 EI \pi^2}{L^2} + \frac{L^2 qSini}{n^2 \pi^2 r_c} \quad (8)$$

Equation (8) is much like the Euler model but has additional term. It is possible to use this model to obtain the critical force for vertical, inclined or horizontal wellbore, but in the current study, the Beam-Column model will employed for this purpose.

Menand *et al* (2011) and Dawson (1984) used the energy method to obtain the following expression for inclined wellbore:

$$F_1 = 2\sqrt{\frac{EIqSini}{r_c}} \quad (9)$$

The results produced using equation (9) and equation (8) are similar when applied in a vertical well, but evidently, the Menand et al (2011), and Dawson (1984) models do not work for vertical wells.

Huang, W., *et al* (2015a) derived a new model for predicting the behaviour of horizontal wellbore based on the assumptions that the wellbore is straight, the drillstring is in continuous contact with the wellbore, and frictional effects are negligible. The expression they obtained is given as follows:

$$F_z = 2\sqrt{\frac{EIq}{r_c}} \quad (10)$$

The result obtained using equations (9) and (10) yield similar results when applied to horizontal wells. Mitchell (2003) developed a similar equation for predicting the critical force in horizontal wells.

II. METHODOLOGY

This section contains the approach used in predicting the critical force to buckle drillstring in vertical, horizontal or inclined wellbores. The fundamental assumptions applicable are that the drillstring is elastic; with considerable friction effects acting on it, and that the radial clearance constrains the deflection. Thus, the governing equation applied is the beam-column model, which is fourth order. The difficulty of applying many boundary conditions to solving the fourth order differential equation was overcome by generating a particular solution for the deflections in the drillstring. This solution was obtained from the total moment equation, which comprised contributions from bending moment and torque in the drillstring. The substitution of the derivatives of the particular equation into the beam-column model resulted in a quadratic equation with respect to the critical force.

An MS excel programme was developed in order to compute the critical force for buckling, while the effects of the variables affecting the critical force was presented using the crystal ball software.

Analysis of study

The following are the assumptions applied in this study:

- The drillstring is an elastic material with significant friction effects
- Drillstring curvatures are neglected and wellbore inclination affects buckling
- Drillstring unit weight is significant.

Considering the sinusoidal buckling of a drillstring in a wellbore filled with drilling mud, the upper section of the drillstring exists in tension due to the hook load used in anchoring it. The tension of the upper section puts the lower section of the drillstring, below a point called the neutral point, in compression. The combined weight of the axial force and the drillstring weight results in the buckling of the tubular when a critical force is exceeded.

The expression for the deflection of a section of the drillstring in the y direction can be written as follows:

$$EI\frac{d^4v}{dz^4} + M_T\frac{d^3v}{dz^3} + \left(F - wzCosi\right)\frac{d^2v}{dz^2} = wSini$$

(11)

The bending moment in the section of the drillstring can be given as, (Chapra and Canales 2007; Mitchell and Miska (2011); Ibrahim 2003):

$$EI\frac{d^2v}{dz^2} = M$$

(12)

Where M = is the total moment in the drillstring section, which can be written as follows, Mitchell and Miska (2011):

$$M = M_b + M_{torque}$$

(13)

The integration of equation (12) yields the following:

$$EIv = \frac{Mz^2}{2} + zC_1 + C_2$$

(14)

Application of the boundary conditions that $v(z=0)=0$ and $v(z=L)=0$ results in the following equation:

$$v = \frac{Fr}{2EI}(zL - z^2)$$

(15)

Allowing the maximum deflection to occur at the middle of the drillstring section, the particular model for the deflection changes to the following expression:

$$v = \frac{ML^2}{8EI}$$

(16)

From equation (16), the following expressions are obtained:

$$\frac{dv}{dz} = \frac{ML}{4EI}$$

(17)

$$\frac{d^2v}{dz^2} = \frac{M}{4EI}$$

(18)

$$\frac{d^3v}{dz^3} = 0$$

(19)

The model for obtaining the torque acting in the drillstring is obtainable as follows, Aadnoy (2010):

$$M_{torque} = \mu rk_b wLSini$$

(20)

The bending moment equation is presented in appendix A

The substitution of equations (17), (18), (19) and (20) into equation (11), and carrying out further mathematical arrangements yields the following expression when applied to a straight inclined drillstring:

$$F_c^2 r_c - (\mu r k_b wLSini + r_c LwCosi)F_c + \mu r k_b w^2 L^2 CosiSini - 4EIwSini = 0$$

(21)

The negative sign of the friction coefficient has been incorporated while deriving equation (21), which is quadratic in terms of the critical force. The variable, L, is the length of the buckled section of the drillstring.

Equation (21) can be simplified to the following expressions:

$$AF_c^2 + BF_C + C = 0$$

(22)

$$A = r_c$$

(23)

$$B = -(\mu r k_b wLSini + r_c LwCosi)$$

(24)

$$C = \mu r k_b w^2 L^2 CosiSini - 4EIwSini$$

(25)

The positive value of the solution to equation (22) can be written as follows:

$$F_C = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

(26)

The axial length over which drillstring buckling occurs is a requirement. From the solution of sinusoidal buckling, the following expression was obtained for the buckled length:

$$L = \frac{n\pi}{p}$$

(27)

p is a unit length factor, which in this study is a function of drillstring unit weight, stiffness and length as follows:

$$p = f(w, L, EI)$$

(28)

From dimensional analysis shown in appendix **B**, the expression for the unit length factor is:

$$p = \sqrt{\frac{wL}{EI}}$$

(29)

Combining equations 27 and 29, and subsequently including a design factor for safe drilling purpose yields the expression below for the buckled length:

$$L = \frac{2.145}{D_F}\sqrt[3]{\frac{EI}{w}}$$

(30)

D_F is the design factor ranging from 1.1-1.2 (Mitchell and Miska 2011). In this study, the average value is assumed.

III. RESULTS AND DISCUSSION

The importance of equation (22) is that it is applicable in a vertical or non-vertical well, unlike most other models. For the buckling of drillstring in a straight vertical wellbore, the expression for the critical load can be given as follows:

$$F_c = 1.865\, w\sqrt[3]{\frac{EI}{w}}$$

(31)

Equation (31) shows that the radial clearance has no impact on the critical force causing buckling in a vertical wellbore. The main reason for this could be that prior to buckling of the drillstring, the tubular does not rest on any part of the wellbore walls except for some offsets. The unit weight of the drillstring has a very high impact on the buckling force in long vertical wells. The larger the unit weight of the drillstring the more difficult it is to buckle. That is, the use of heavy drillpipe in vertical sections will be favourable to drillstring stability. **Figure 1** is a display of the variables affecting buckling in a vertical wellbore. The result shows that the radial clearance has little or no impact on the model for obtaining the critical force. The unit weight of the drillstring has a higher impact on the critical force in vertical wells mainly because there is no support to the exposed length of the drillstring.

Using the data in **Table 4,** the value of the critical force for vertical wellbore is 11,272.02 Lbf as shown in **Table 1**. This compares very well with the value of 11,725.3 Lbf obtained using the Lubinski model. The coefficient of friction has little influence on the critical force for such wells mainly because there is no contact with the walls of the wellbore.

The critical force causing tubular buckling in a horizontal well is much higher than that of a corresponding vertical wellbore. One reason for this is that is, the drillstring rests on the faces of the wellbore wall, thus enabling a higher force to cause the buckling of the structure. Much frictional force has to be overcome for buckling to occur. The expression for the critical force in a horizontal well, derived from equation (22), is as follows:

$$F_{cr} = \sqrt{\frac{4wEI}{r_c}} \qquad (32)$$

Equation (32) is the same as that obtained by Mitchell (2003), Menand *et al* (2011), Dawson (1984), and Huang *et al* (2015a), for a horizontal well.

Figure 2 is a display of the variables affecting buckling in a horizontal well. It shows that for a horizontal well, the radial clearance, the unit weight and stiffness of the drillstring are all very important variables to consider. Each has almost the same influence on the critical force. There is a negative correlation for the radial clearance because the length at the bottom supports the lowest portion of the wellbore. The higher the stiffness the higher the critical force of the drillstring. That is, it becomes more difficult to cause the drillstring to buckle. The larger the radial clearance between the drillstring and the wellbore the lower the critical force. That is, it becomes easier to cause the drillstring to buckle. Computation of the critical force gives a value of 206,552.7 Lbf as presented in **Table 2**, which is quite large when compared to that of the vertical wellbore. This is the same value obtained when the Huang W. *et al* (2015a) model is applied. The radial clearance significantly affects the critical force in a non-vertical wellbore, (Dawson and Paslay 1984; Wu 1992; Paslay and Bogy 1964). Equation (22) approximates the Paslay and Bogy model when parameters A and C are introduced.

Figure 3 displays the variables affecting drillstring buckling of an inclined wellbore. The value of the critical force is quite lower than that of horizontal wellbore, but the difference gets smaller as the inclination approaches 90 degrees.

The radial clearance has a high impact on the critical force in an inclined wellbore. This is because of the eccentricity of the drill pipe due to the slanting of the well.

The critical force increases as the wellbore inclination increases. **Figure 4** is a display of the variation of critical force with wellbore inclination.

There is a positive correlation between the variables, and the result is similar to that obtained by Gong *et al* (2016) for the variation of critical force and inclination for helical buckling. The critical force increases with inclination since part of the wellbore bears the load on the string. Thus, it is more difficult to buckle a drillstring in a horizontal wellbore than in a vertical one. **Table 5** is a computation of the critical force for different wellbore geometries using several models. The significant variance in the critical force in an inclined wellbore maybe because of the non-curvature of the drillstring that was assumed.

IV. CONCLUSIONS

A new model for obtaining the critical force for buckling in a drillstring of any geometry was developed in this study. The equation accounted for the effect of bending moment and torque in the drillstring. The starting equation was the beam-column equation for deflection in the drillstring. The application of the total moment equation was used to develop a particular solution that was used to substitute for the differentials in the beam-column model.

From the analysis of the study, the following points are drawn:

- The radial clearance between the drillstring and wellbore, drillstring unit weight and drillstring stiffness significantly influence the critical force for buckling in non-vertical wellbores.
- The radial clearance has no influence on the critical force for buckling in vertical wellbores.

- The critical force increases with the wellbore inclination.
- In terms of buckling in non-vertical wells, friction enhances drillstring stability hence delaying the onset of buckling.

V. RECOMMENDATION

The proper modelling of the relationship between the radial clearance and drillstring stiffness is recommended. In addition, continues rotation of the drillstring should be ensured, and the use of drilling packers and centralizers in the annular space is recommended to help improves drillstring stability.

Nomenclature

A = Amplitude in the sinusoidal buckling solution

A, B, C = coefficients in quadratic equation of critical force in drillstring

C_1 = Constants of Integration, Lb / ft^2

C_2 = Constant of Integration, Lb / ft

D_F =Design factor, dimensionless

E =Young`s Modulus of drillstring, Psi

EI = Drillstring stiffness, product of Young`s modulus and moment of inertial, Lbf/sq.ft

F, F_1, F_c, F_z = Critical force to cause buckling, Lbf

i = Wellbore inclination, Degree

I = Moment of inertial, ft^4

κ = Wellbore Curvature of the drillstring centreline, ft

L_{dc} = Length of buckled drillstring, ft

M, M_T, M_b **=** Total moment, torque and bending moment of drillstring respectively, Lbf-ft

n = number of sinusoidal period of buckling, dimensionless

q, w_{bp} = Weight per unit Length of drillstring in mud, Lbf/ft

r_c = Radial Clearance, ft

Z = axial distance, ft

θ =Angular displacement of drillstring, Rad

w_v = angular velocity, $rad / \sec$

u, v = Lateral Displacements of the drillstring, ft

REFERENCES

[1] Aadnoy, B.S. 2010. Modern well design, 2nd edition, CRC Press, pp 244-245.

[2] Abdul Ameer. 2012. Drill String Modelling and Stress Analysis, Proc. World Congress on Engr & Computer Science, Volume II, Oct 24-26, San Francisco, USA.

[3] Adams, N.J. 1985. Drilling Engineering, A Complete Well Planning Approach, PennWell Publishing Company, Tulsa, pp 1-3, ISBN: 0-87814-269-7.

[4] Azar, J.J. 2006. Drilling Problems and Solutions, In Robert F. Mitchell (ed.), Petroleum Engineering Handbook Vol. 2-Drilling Engineering, SPE Publication, pp 433.

[5] Chapra, S. C., Canales, R. P. 2007. Numerical Methods for Engineers, 5th edition, Tata McGraw Hills, pp136-138, 644-646., ISBN: 0-07-063416-5.

[6] Dawson, R. 1984. Drill Pipe Buckling in Inclined Hole, SPE J, **36** (10): 1734-1738, SPE-11167-PA

[7] http://dx.doi.org/10.2118/11167-PA .

[8] Dawson, R., Paslay, P.R. 1984. "Drill pipe buckling in inclined hole", Journal of Petroleum Technology, 36, (10), pp 1734-8.

[9] Gao, D.L. 2006. Down-hole tubular mechanics and its applications. Dongying: China University of Petroleum Press.

[10] Gao, D., Huang, W. 2015. A review of down-hole tubular string in well engineering, Pet. Sci., Dec., pp 443-448. http://dx.doi.org/10.1007/s12182-015-0031-z

[11] Gong, Y., Ai, Z., Sun, X., Fu, B. 2016. The Effects of Boundary Conditions and Friction on the Helical Buckling of Coiled Tubing in an inclined Wellbore, PLoS ONE 11 (9), http://dx.doi.org/10.1371/journal.pone.0162741

[12] Huang, W., Gao, D.L., Liu, F. 2015a. "Buckling Analysis of Tubular Strings in Horizontal Wells", SPE Journals, 408. http://dx.doi.org/10.2118/171551-PA.

[13] Huang, W., Gao, D.L., Wei, S. 2015b. "Local mechanical model of down-hole tubular strings constrained in curve wellbores", JPSE. http://dx.doi.org/10.1016/j.petrol.2015.03.017

[14] Huang, W., Gao, D.L., Liu, Y. 2016. "A Study of tubular string buckling in vertical Wells", IJMS, 118, 231-253.

[15] Lubinski, A. 1950. A study of the buckling of rotary drilling strings. American Pet Inst., 224(1):123–65.

[16] Mehdi, H., Jeremy, S.D. 2014. Advances in critical buckling load assessment for tubular inside wellbores, JPSE, Vol 116, pp 136-144.http://dx.doi.org/10.1016/j.petrol.2014.02.019

[17] Menand, S., Bjorset, A., Macresy, L. Anew Buckling Model Successfully Validated with Full Scale Buckling Tests, 2011, AADE-11-NTCE-9, p1.
[18] Mitchell RF. 2003. Lateral buckling of pipe with connectors in horizontal wells. SPE J; 8(2):124–37.
[19] Mitchell, R.F. 2013. "Effects of well deviation on helical buckling", SPE Drill Eng & Compl, 12(01), pp 63-70. http://dx.doi.org/10.2118/29462-PA
[20] Mitchell, R.F., Miska, S.Z. 2011. FUNDAMENTALS OF DRILLING ENGINEERING, SPE TEXTBOOK SERIES, Vol 12. Pp 585-590. ISBN 978-1-55563-338-7
[21] Paslay, P.R., Bogy, D.B. 1964. The stability of a circular rod laterally constrained to be in contact with aninclined Circular cylinder, J Appl Mech., 31 (4), pp 605-610.http://dx.doi.org/10.1115/1.3629721.
[22] Wu, J.1992. Buckling behaviour of pipes in directional and horizontal wells, Texas, Texas A&M University, (Ph.D Dissertation 1992).

APPENDIX A: Dimensional Analysis for Drillstring Curvature

The equation of the bending moment of a drillstring is related to the curvature as follows:

$$M = EI\kappa$$

A-1

Where EI is the drillstring stiffness and κ is the drillstring curvature.

The curvature of tubular is a function of the axial force, unit weight of drillstring, stiffness and radial distance, Gao (2006), Huang *et al* (2015a).

Let the drillstring be constrained by the radial clearance between the drillstring and wellbore walls, $r = r_c$, and the net compressive axial force affects the bending moment, Thus:

$$\kappa = f(r_c, EI, F)$$

A-2

From the Rayleigh method of dimensional analysis, the following obtains:

$$\kappa = r_c^a (EI)^b F^c$$

A-3

Dimension of κ is L^{-1}

Dimension of r_c is L^1

Dimensions of (EI) is ML^3T^{-2}

Dimensions of F is MLT^{-2}

Equating the power of each variable, the following expressions are obtained:

$$a + 3b + c = -1$$

A-4

$$b + c = 0$$

A-5

$$-2b - 2c = 0$$

A-6

The feasible set of solution that defines the bending moment is

$$a = 1, b = -1, c = 1$$

Equation A-3 can be written as follows:

$$\kappa = \frac{r_c F}{EI}$$

A-7

Equation A-7 can be recast into the following:

$$\kappa = r_c^2 \left(\sqrt{\frac{F}{r_c EI}} \right)^2$$

A-8

The bending moment equation, A-1, can then be written as follows:

$$M = EIr_c^2 \varpi^2$$

A-9

Where $\varpi = \sqrt{\frac{F}{r_c EI}}$

APPENDIX B: Dimensional analysis for buckled length

The sinusoidal solution for a buckled drillstring is a function of length. From such solution is possible to have the following:

$$L = \frac{n\pi}{p}$$

B-1

Where p is a variable called the unit length, which is a function of unit weight, stiffness and length:

$$p = f(w, L, EI)$$

B-2

From the Rayleigh method of dimensional analysis, the following obtains:

$$p = w^{a} L^{b} (EI)^{c}$$

B-3

Dimension of κ is L^{-1}

Dimension of L is L^{1}

Dimensions of (EI) is $ML^{3}T^{-2}$

Dimensions of w is MT^{-2}

Equating the power of each variable, the following expressions are obtained:

$$b + 3c = -1$$

B-4

$$a + c = 0$$

B-5

$$-2a - 2c = 0$$

B-6

The feasible set of solution that defines the bending moment is

$$a = 0.5, b = 0.5, c = -0.5$$

Equation A-3 can be written as follows:

$$p = \sqrt{\frac{wL}{EI}}$$

B-7

APPENDIX C: List of Tables

Table 1: Critical Force For Vertical Well

			Inputs				Keys
Dbit	12.2	inch	γsteel	7.85	-	Dbit	Drill bit diameter
Didc	2.25	inch	E	4300000000	Psi	Didc	internal diameter of drill coll
Dodc	7	inch	wdc	95.53757962	Lbf/ft	Dodc	outside diameter of Drill Co
ρmud	12	ppg	rc	0.216666667	ft	Wds	unit weight of Drillstring
Wdc	117	Lbf/ft	i	0	degree	wds	unit weight of Drillstring in m
γmud	1.44	-	EI	24189094.18	Lbf/ft^2	ρmud	density of mud
			μ	0		Fc	critical buckling load
			Outputs			γmud	specific gravity of mud
I	0.005625	ft^4	A	0.216666667		γsteel	specific gravity of Steel
rc	0.216667	ft	B	-2442.271048		i	wellbore inclination
Kb	0.816561	-	C	0		Kb	buoyancy factor
Fc	11272.02	Lbf				E	Youngs modulus
						I	moment of inertia
						rc	radial clearance
						A, B, C	constants in quadratic equat

Table 2: Critical Force For Inclined Well

			Inputs				Keys
Dbit	12.2	inch	γsteel	7.85	-	Dbit	Drill bit diameter
Didc	2.25	inch	E	4300000000	Psi	Didc	internal diameter of drill coll
Dodc	7	inch	wdc	95.53757962	Lbf/ft	Dodc	outside diameter of Drill Co
ρmud	12	ppg	rc	0.216666667	ft	Wds	unit weight of Drillstring
Wdc	117	Lbf/ft	i	5	degree	wds	unit weight of Drillstring in m
γmud	1.44	-	EI	24189094.18	Lbf/ft^2	ρmud	density of mud
			μ	0		Fc	critical buckling load
			Outputs			γmud	specific gravity of mud
I	0.005625	ft^4	A	0.216666667		γsteel	specific gravity of Steel
rc	0.216667	ft	B	-2432.977469		i	wellbore inclination
Kb	0.816561	-	C	-805656359.5		Kb	buoyancy factor
Fc	66851.29	Lbf				E	Youngs modulus
						I	moment of inertia
						rc	radial clearance
						A, B, C	constants in quadratic equat

Table 3: critical force for horizontal well

			Inputs				**Keys**
Dbit	12.2	inch	Ysteel	7.85	-	Dbit	Drill bit diameter
Didc	2.25	inch	E	4300000000	Psi	Didc	internal diameter of drill colla
Dodc	7	inch	wdc	95.53757962	Lbf/ft	Dodc	outside diameter of Drill Co
ρmud	12	ppg	rc	0.216666667	ft	Wds	unit weight of Drillstring
Wdc	117	Lbf/ft	i	90	degree	wds	unit weight of Drillstring in m
Ymud	1.44	-	EI	24189094.18	Lbf/ft^2	ρmud	density of mud
			μ	0		Fc	critical buckling load
			Outputs			Ymud	specific gravity of mud
I	0.005625	ft^4	A	0.216666667		Ysteel	specific gravity of Steel
rc	0.216667	ft	B	-1.49607E-13		i	wellbore inclination
Kb	0.816561	-	C	-9243870043		Kb	buoyancy factor
Fc	206552.7	Lbf				E	Youngs modulus
						I	moment of inertia
						rc	radial clearance
						A, B, C	constants in quadratic equat

Table 4: Drillstring parameters used in the study

Drill bit diameter = 12.20 inch	Unit weight of drillstring in air = 117 Lbf/ft
Drillstring outer diameter = 7.0 inch	Inclination = 0 deg
Drillstring internal diameter = 2.25 inch	Buoyancy factor = 0.816561
Coefficient of friction = 0.2	Mud density = 12 ppg
Modulus of elasticity = 4300000000 Psi	Specific gravity of Steel = 7.85

Table 5: Comparison of Results

Wellbore geometry	Model	Critical Force (Lbf)
Vertical Well	Lubinski (1951)	11,725.3
	New model	11,272.02
Inclined Well	Menand et al (2011)	60,974.08
	Dawson (1984)	60,974.08
	New model	66,851.29
Horizontal Well	Huang et al (2015a)	206,552.7
	New model	206,552.7

APPENDIX D: Figures

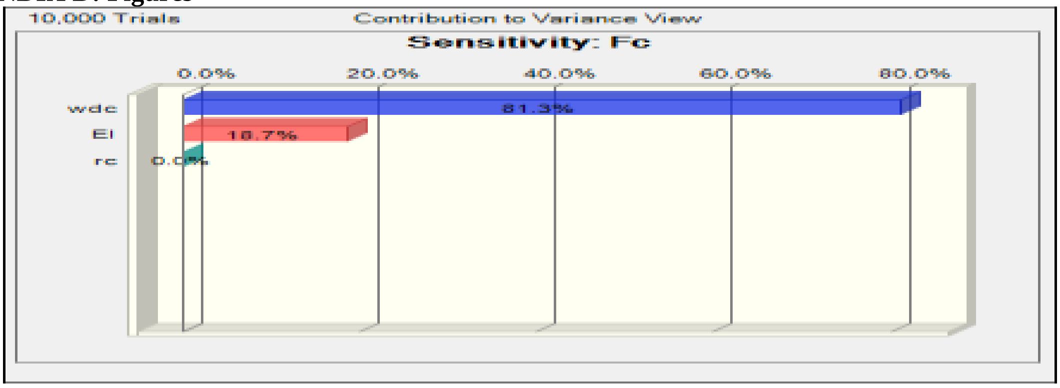

Fig. 1- Sensitivity of parameters in vertical wellbores

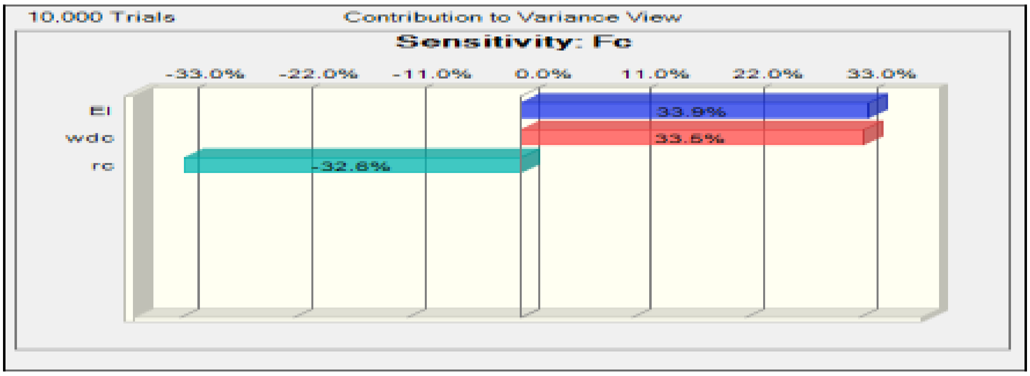

Fig. 2- Sensitivity of parameters for buckling in horizontal wells

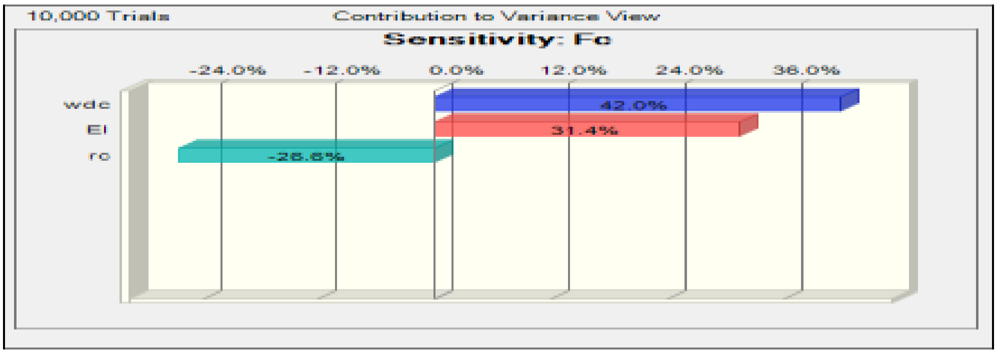

Fig. 3- Sensitivity of parameters to buckling in inclined wellbore

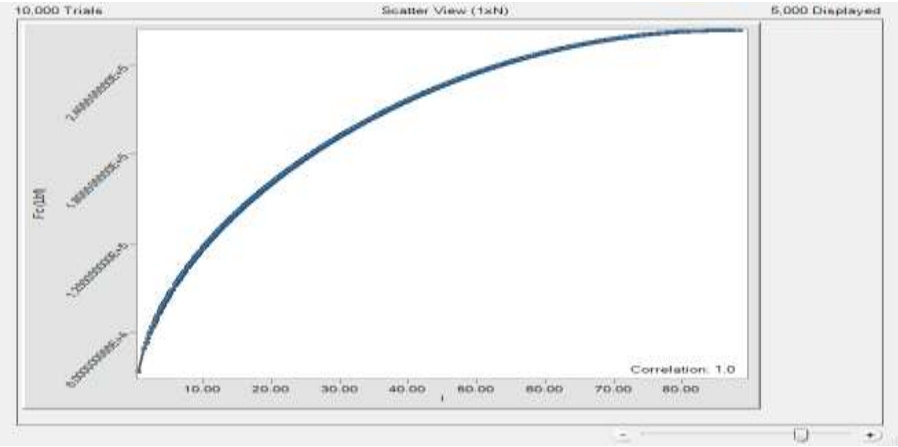

Fig. 4- Variation of critical force and wellbore inclination.

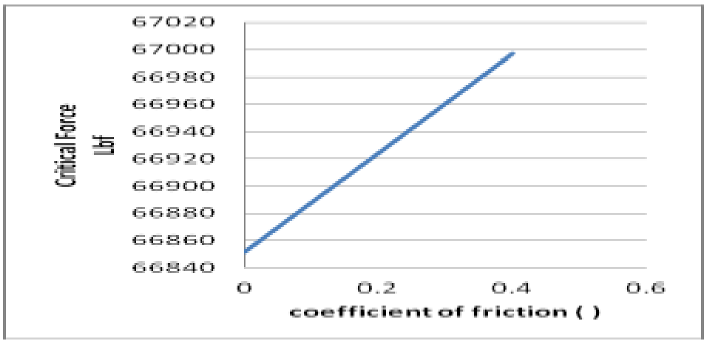

Fig. 5- Effect of friction on critical force

Authors Biography

Nwonodi Roland Ifeanyi works with Federal University Wukari. His research interest is in Petroleum Geomechanics and Well performance modelling. He holds bachelor`s degree in petroleum engineering and currently awaits MS degree in drilling engineering all from the University of Port Harcourt. Nwonodi is a member of SPE and a registered engineer of Nigeria.

Adali Francis Eromosele is a graduate student of the University of Port Harcourt. His research interest is in drilling and well Engineering and petroleum Geomechanics. He holds a bachelor`s degree in Petroleum engineering and currently awaits his MS degree in Drilling Engineering both from the University of Port Harcourt.

Tswenma Tsokwa works with Federal University Wukari. He is currently a master`s degree student of Modibbo Adama University of Technology, Yola.