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Analysis of a New Hyper Chaotic System with six cross-product nonlinearities terms

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ABSTRACT: *In this paper, a new continuous-time four-dimensional autonomous hyper chaotic system was introduced, the new system completely different from well-Known hyper chaotic systems , the system employs ten terms include six quadratic cross-product nonlinearities terms and generate two-scroll hyper chaotic attractor at same time ,the dynamical behaviors investigated with dissipativity , symmetry , Jacobian matrix , Lyapunov Exponents, Kaplan-Yorke dimension, presence of hyper chaotic attractor and waveform analysis, where the maximum non-negative Lyapunov exponents (MLE) for system obtain as 1.9342 and Kaplan-Yorke dimension obtain as 3.0243, and the new system characteristics with ,unstable , high complexity ,and unpredictability .*

Keywords: *autonomous, high dimensional, hyper-chaotic, unstable, waveform analysis*

I. INTRODUCTION

The Chaotic systems is one class of dynamical systems, that characteristics with some unique Features like complexity, high sensitivity to tiny change in initial conditions , unpredictability and ergodicity[1]. Chaotic behavior obtains when dynamical system evolve with some Specific values for initial conditions and parameters, the first system show chaotic behavior discovered by accident in 1963 when professor Lorenz worked to construct model for weather patterns, the mathematical model of system describe by three first order differential equation with seven terms, include two cross-product nonlinearities terms, chaotic systems attracted much attention due to its wide application and desirable properties, many chaotic systems with three dimension introduced like LÜ system, Chen system, Chua system, Qi system, etc.[2], In 1979 RÖssler introduced a new high dimensional chaotic system, that has two non-negative Lyapunov Exponents in different with normal chaotic system that has one positive Lyapunov Exponents, where the dynamics of new system widened in more than one orientation at the same time, and that lead to more unpredictability, complexity and randomness in comparison with normal chaotic systems, these systems know as hyper chaotic systems [3], several hyper chaotic system presented such as Chen system, Wang system ,Jia system , Hu et al system, etc.[4], A new and different hyper chaotic system presented in this paper, with four state variables, eight parameters and six cross-product nonlinearities terms, the dynamical behavior of the proposed system studied and analyzed its basic dynamical characteristics and behaviors briefly via MATHEMATICA program.

II. CONSTRUCTION OF THE TEN TERMS HYPER CHAOTIC SYSTEM

The new ten terms hyper chaotic system describe with six quadratic nonlinearities terms, given by four first order differential equations

$$\begin{aligned} \frac{dx}{dt} &= ayz - bxz - cw \\ \frac{dy}{dt} &= dx - xz - y \\ \frac{dz}{dt} &= exy - fz \\ \frac{dw}{dt} &= gxz + hzy \end{aligned} \qquad (1)$$

The hyper chaotic system assumed that x, y, z, w, are variables and a, b, c, d, e, f, g, h. Are system parameters, where the system display hyper chaotic attractor when a = 18, b = 3.1, c=2, d=10, e=3, f=2.6, g=5, h =13 and for numerical simulation the initial values assumed as x(0)=0.2, y(0)=0.4, z(0)=0.6, w(0)=0.8, where Fig's 1-4 show the chaotic strange attractor 3-D view for the proposed system (1) in (x, y, z) ,(x, z, y),(y, x, z) and (z, y, x) spaces .

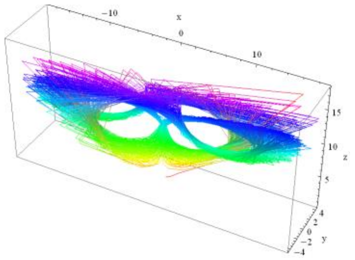


Fig.1. phase portrait of system in (x, y, z)

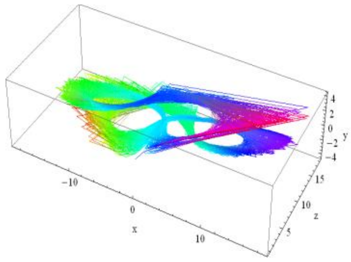


Fig.2.phase portrait of system in (x, z, y)

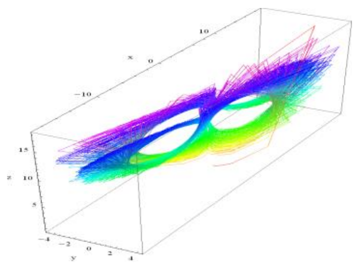


Fig.3. phase portrait of system in (y, x, z)

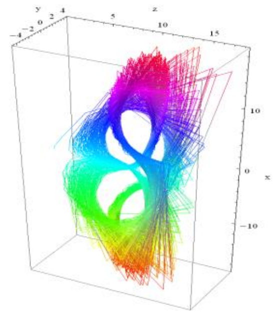


Fig.4.phase portrait of system in (z, y, x)

III. DYNAMICAL BEHAVIOR ANALYSIS OF THE NEW HYPER CHAOTIC SYSTEM

In this section essential and intricacy dynamics of the new hyper chaotic system (1) are invested, and the system has the following basic Characteristics.

3.1 System dissipativity

When we describe the system (1) in vector Formula as:

$$f=\begin{bmatrix} f_1 = ayz - bxz - cw \\ f_2 = dx - xz - y \\ f_3 = exy - fz \\ f_4 = gxz + hzy \end{bmatrix} \qquad (2)$$

And the control parameter values chosen as:

a = 18, b = 3.1, c=2, d=10, e=3, f=2.6, g=5, h =13

The divergence of the vector field $\boldsymbol{f}$ **on** $\mathbb{R}^4$ gained as

$$\nabla\cdot\boldsymbol{f} = \frac{\partial f_1}{\partial x}+\frac{\partial f_2}{\partial y}+\frac{\partial f_3}{\partial z}+\frac{\partial f_4}{\partial w} = -(bz+f+1) < 0 \qquad (3)$$

Where

$$(b+f+1) > 0$$

Let Ω be any region in $\mathbb{R}^4$ with a smooth boundary, and $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector filed$\boldsymbol{f}$.

Let V (t) indicate the volume of $\Omega(t)$, by Liouville s theorem, we get that

$$\frac{dV(t)}{dt} = \int_{\Omega(t)}(\nabla\cdot f)\, dxdydzdw \qquad (4)$$

And when substituting the value of $\nabla\cdot\boldsymbol{f}$ in (4) leads to obtain the first order ODE

$$\frac{dv}{dt} = -(bz+f+1)V(t) \qquad (5)$$

Where the integrating the first order differential equation (5) , V(t) is obtain as

$$V(t) = V(0)e^{-(bz+f+1)t} \qquad (6)$$

From (6), the volume $V(t)$ shrinks to zero exponentially as $\rightarrow \infty$.

The new hyper chaotic system (1) is dissipative when $z <= 0$, Due to limit set of system (1) are eventually confined into fixed limit set of zero volume.

3.2 Symmetry

When the coordinate is transformed from (x, y, z, w) into$(-x, -y, z, -w)$, the new dynamic system is symmetry and invariant about the z-axis, and it is easily to seen that the z-axis is invariant for the flow of the new hyper chaotic system and all orbits of new system (1) starting from the z-axis stay in the z-axis for all values of time [6].

3.3 Equilibria

To obtain the equilibrium points of system (1), must be solving the nonlinear equations as follow:

$$\begin{aligned} & ayz - bxz - cw = 0 \\ & dx - xz - y = 0 \\ & exy - fz = 0 \\ & gxz + hzy = 0 \end{aligned} \tag{7}$$

And chosen the system parameter values as in the chaotic case

$$a = 18, b = 3.1, c = 2, d = 10, e = 3, f = 2.6, g = 5, h = 13 \tag{8}$$

Solving the system (7) with (8), we get three equilibrium points for the new hyper chaotic system as

E_1=(0,0,0,0) (9)

$E_2 = (\,0.+4.8373i\,, 0.-1.8605i\,, 10.3846\,, 0.-251.7499i\,)$ (10)

$E_3 = (\,0.-4.8373i\,, 0.+1.8605i\,, 10.3846\,, 0.+251.7499i\,)$ (11)

And when linearizing the system (1) around the E* equilibrium point, with the aim to determine The Jacobian matrix, we obtain:

$$J(E^*) = \begin{bmatrix} -bz & az & ay-bx & -c \\ d-z & -1 & -x & 0 \\ ey & ex & -f & 0 \\ gz & hz & gx+hy & 0 \end{bmatrix} \tag{12}$$

Now, Find the Jacobian matrix at each equilibrium points to confirm it's unstably.

The Jacobian matrix at E_1 equilibrium point obtain as:

$$J_1 = J\,(E_1) = \begin{bmatrix} 0 & 0 & 0 & -2 \\ 10 & -1 & 0 & 0 \\ 0 & 0 & -2.6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The matrix J_1 has four eigenvalues:

$$\lambda_1 = -2.6, \quad \lambda_2 = -1, \quad \lambda_3 = 0, \quad \lambda_4 = 0 \tag{14}$$

The eigenvalues results is real number and E_1 considered as saddle point, hence, E_1 is unstable.

Next, the Jacobian matrix for E_2 is obtain as:

$$J_2 = J(E_2) = \begin{bmatrix} -3.1*10.3846 & 18*10.3846 & 18*\text{-}1.8605i - 3.1*{+}4.8373i & -2 \\ 10-10.3846 & -1 & -4.8373i & 0 \\ 3*-1.8605i & 3*4.8373i & -2.6 & 0 \\ 5*10.3846 & 13*10.3846 & 5*4.8373i + 13*-1.8605i & 0 \end{bmatrix} \tag{15}$$

Where eigenvalues for matrix J_2 are obtained as:

$$\lambda_{1,2} = 16.2588 \pm 57.6623i, \; \lambda_3 = 4.2047 + 5.4802 \times 10^{-17}i, \; \lambda_4 = -0.9302 - 2.6415 \times 10^{-17}i \tag{16}$$

Due to result values, the E_2 is a saddle- focus, and unstable.

Next, the Jacobian matrix at E_3 is obtain as:

$$J3 = J(E3) = \begin{bmatrix} -3.1*10.3846 & 18*10.3846 & 18*1.8605i-3.1*-4.8373i & -2 \\ 10-10.3846 & -1 & 4.8373i & 0 \\ 3*1.8605i & 3*-4.8373i & -2.6 & 0 \\ 5*10.3846 & 13*10.3846 & 5*-4.8373i+13*1.8605i & 0 \end{bmatrix} \quad (17)$$

Where eigenvalues for matrix J_3 are obtained as:

$$\lambda_{1,2} = 16.2588 \mp 57.6623i,\ \lambda_3 = 4.2047 - 5.4802 \times 10^{-17} i,\ \lambda_4 = -0.9302 + 2.6415 \times 10^{-17} i \quad (18)$$

Due to result values, the E_3 is a saddle- focus, and unstable [7].
Hence, all equilibrium points of the four dimension new chaotic system (1) is unstable where E_1 is a saddle point and E_2, E_3 are saddle- focus points.

3.4 Lyapunov Exponents and Lyapunov dimension

When the values of control parameters and initial condition for the system (1) , selected as a = 18, b = 3.1, c=2, d=10, e=3, f=2.6, g=5, h =13 and x(0)=0.2 , y(0)=0.4 , z(0)=0.6 , w(0)=0.8 , the Lyapunov Exponents for the new hyper chaotic system obtain as :L_1= 1.4904, L_2= 1.9342, L_3= -2.6706, L_4= -31.0164, due to two positive Lyapunov Exponents the system is hyper chaotic.
Where the maximal positive Lyapunov Exponents (MLE) of the new system is L_2= 1.9342 and the Lyapunov dimension or Kaplan-Yorke dimension for the new system calculated as [8]:

$$D_{KY} = 3 + \frac{1}{|L_{j+1}|}\sum_{i=1}^{3} L_i = 3 + \frac{L_1 + L_2 + L3}{L_4}$$

$$= 3 + \frac{1.4904 + 1.9342 + -2.6706}{31.0164}$$

$$= 3.0243$$

The Lyapunov dimension for system (1) has a fractional nature and that refer to existence of hyper chaos behavior in the system.

3.5 waveform analysis

The waveform for a chaotic system characteristic with non-periodic shape, where the waveforms of x(t),y(t),z(t),w(t) for the new system , show in Figures 5-8 and has non-cyclical properties , which is one of basic chaotic dynamical characteristic .

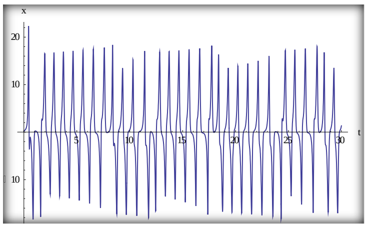


Fig.5. Time versus *x* of the new chaotic system.

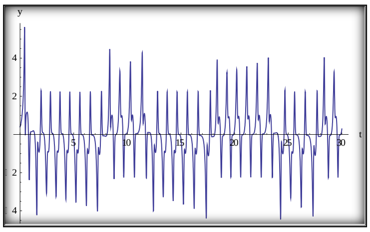
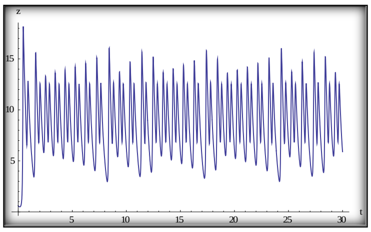


Fig.6. Time versus *y* of the new chaotic system.



‘**Fig.7.** Time versus *z* of the new chaotic system.

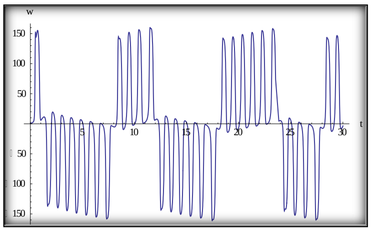


Fig.8. Time versus *w* of the new chaotic system.

IV. CONCLUSION

In this paper, a new hyper chaotic system with six cross-product nonlinearities terms presented and the basic features of dynamical system analyzed by means of dissipativity , symmetry , Jacobian matrix , Lyapunov Exponents, fractal dimension, presence of hyper chaotic attractor and waveform analysis, the system generate the hyper chaos behavior when a = 18, b = 3.1, c=2, d=10, e=3, f=2.6, g=5, h =13 and the Lyapunov Exponents for the system L_1= 1.4904, L_2= 1.9342, L_3= -2.6706, L_4= -31.0164 , which means the system is hyper-chaotic ,the system has three unstable equilibrium points , fractal dimension of system as 3.0243 , and system characteristics with high sensitivity and generate complex hyper chaotic attractor with two-scroll, the new system a suitable for many applications and can be used for encryption information , due to large set of keys that can generated when system variants .

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