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Model Analysis of Some Fluid Properties and Viscoelasticity

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Abstract: *Theoretical study of some fluid properties and viscoelasticity was carried out by determining the solution of the resulting governing equations. Graphical analyses of the solutions showed that, increase heat sink term and Frank-Kamenestkii parameter result in a decrease in the temperature profile of the fluid while an increase in radiation result in a decrease in the temperature of the fluid. Similar results are also observed for dilatant fluids but increase in magnitude. Increase in heat sink term and radiation also reduces the Nusselt number but Frank-Kamnestkii parameter increase causes an increase in the Nusselt number. Increase in relaxation constant and friction/viscosity, reduces the distance of the fluid flow while increase in the natural frequency increases the distance of the fluid flow.*

Keywords: *Pseudoplatic materials, Viscosity, Dilatant fluids, Viscoelastic materials, Non- Newtonian fluids, Energy equation, Non homogeneous differential equation.*

I. INTRODUCTION

Fluids are substances that deforms continuously when subjected to shear stress no matter how small [1]. It consists of liquids and gases. Fluids in physics and engineering are classified into two broad categories. The first is the Newtonian fluid; they are fluids that obey the Newton's law of viscosity. They are characterized by having moderate value of viscosity and flow easily. Examples are polar liquids and gases. The second class of fluids does not obey the Newton's law of viscosity and possessed high viscosity as well as flow sluggish. They are referred to as non Newtonian fluids and examples are paints, polymer fluids, blood and many more. One important property of fluids is its viscosity. According to [1], viscosity is that fluid property by virtue of which fluids offer resistance to shear stress. Mathematically, it is also the ratio of the shearing stress to the velocity gradient in a fluid. Viscosity arises on molecular scales due to intermolecular cohesion and transfer of molecular momentum. The former is important in liquids for which molecules are relatively densely packed while the later is important in gases in which the molecules are far apart. The observations are useful in explaining the fact that viscosity of a liquid decreases as temperature increases while that of a gas increases with increasing temperature. The analysis of fluids cannot be exhausted owing to its importance to man and the environment. Several scholars [2-8] have investigated fluids flow, properties and uses. Viscoelastic fluids are generally non Newtonian and substances that flow like fluids and deform like solids are generally described as viscoelastic [9]. There are shear-thining fluid that falls into this category, mainly gels and pastes. They are characterized as having viscosity decreases under stress. The importance or uses of viscoelastic materials cannot be over emphasized. They are extensively used in isolating vibration, damping noise and absorbing shock. Others are in industries, particularly in polymer processing and in molten plastics and slurries. They give off the energy absorbed as heat. A typical example is the sorbothane. Studies reported such as [10-15], examined the effect of various parameters on viscoslastic fluids. Our aim is to consider the effect of heat source/sink term and radiation as well as damping and viscosity on the two flow configurations.

II. MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

The formulation of the problem under investigation is based on the relation that the velocity gradient is a function of temperature and the flow behaviour index is not constant. With these assumptions, the energy equation takes the form

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \varphi\left(\frac{\partial u}{\partial x}\right)^n - \frac{\partial q_x}{\partial x} + \varsigma_0\left(T - T_0\right) \quad (1)$$

where T(x,t) is temperature of the fluid, k is thermal conductivity, n is flow behaviour index (power law exponent) , φ is consistency index, T_0 is characteristic temperature, q_x is radiative term, ς is heat source/ sink term and u is fluid velocity.

$$\frac{\partial^2 q_x}{\partial x^2} - 3\sigma^2 q_x - 16\sigma T_0^3 \frac{\partial T}{\partial x} = 0 \quad (2)$$

where σ is the Stefan-Boltzman's constant. For optically thin medium with relatively low density in the spirit of Cogley et al [16], equation (2) reduces to

$$\frac{\partial q_x}{\partial x} = 4\delta^2 (T - T_0) \quad (3)$$

where $\delta^2 = \int_0^\infty (\alpha_{K^*} \frac{\partial \wedge}{\partial T}) dk^*$

It has been established by Hughes and Brighton [17] that

$$\varphi\left(-\frac{\partial u}{\partial x}\right)^n = -\frac{x}{2}\left(\frac{\partial p}{\partial x}\right) \quad (4)$$

where p is fluid pressure.

For Pseudoplastic material $n < 1$, we assume 0.5 and approximate $\left(\frac{\partial u}{\partial x}\right)^{0.5}$ by ignoring powers of $\left(\frac{\partial u}{\partial x}\right)^{0.5}$ greater than unity using Taylor's series expansion about 0, we get

$$\left(\frac{\partial u}{\partial x}\right) = \frac{x}{\varphi}\left(\frac{\partial p}{\partial x}\right) \quad (5)$$

The equation of state for an ideal fluid is given by

$$p = \rho RT \quad (6)$$

where ρ density of fluid and R is universal fluid constant.

Using equations (3), (5) and (6), we can rewrite equation (1) as

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \rho Rx\frac{\partial T}{\partial x} - 4\delta^2 (T - T_0) + \varsigma_0 (T - T_0) \quad (7)$$

With the boundary conditions $T(0) = 1, T(1) = 100$ [8]

For dimensional homogeneity of equation (3), using Buckingham $\pi -$ method, it is convenient to use the dimensionless variables

$$t' = \frac{t}{t_0},\ \theta = \frac{T}{T_0},\ y = \frac{ut}{x},\ \vartheta = \frac{\varphi}{k},\ \varsigma = \frac{\varsigma_0 x}{k\rho t},\ N = \frac{4\delta^2 x^2}{k} T^3\quad \rho' = \frac{\rho u^2 x}{p},\ R' = \frac{Rpx^2}{T}$$

We therefore write equation (7) as

$$\frac{\partial\theta}{\partial t'} = \frac{\partial^2\theta}{\partial y^2} + \beta_1 y\frac{\partial\theta}{\partial y} - N\theta + \varsigma\theta \qquad (8)$$

where $\beta_1 = \frac{\rho' R'}{\vartheta}$ and ϑ is Frank-Kamenestkii parameter

III. METHOD OF SOLUTION

We take steady state and use the Frobenius method of the form

$$\theta(x) = \sum_{i=0}^{\infty} a_i y^{i+C} \qquad (9)$$

If we substitute equation (9) into equation (8) and simplify, we get the solution of the indicial equation as

$$C = 0 \text{ or } 1 \qquad (10)$$

and the recurrence relation as

$$a_{i+2} = \frac{-(\beta_1(i+C) + (\varsigma - N))a_i}{(i+C+2)(i+C+1)} \qquad i = 0, 1, 2\ldots\ldots\ldots \qquad (11)$$

The complete solution is therefore

$$\theta(y) = a_0\left(1 + y - \left(\frac{\varsigma - N}{2}\right)y^2 - \left(\frac{\varsigma - N}{2}\right)y^3\right) + a_1\left(y + y^2 - By^3 - By^4\right) \ldots\ldots.. \qquad (12)$$

where $a_0 = 1$, $a_1 = \frac{3(100 - (2 - (\varsigma - N)))}{(6 - (6\beta_1 + (\varsigma - N)))}$, $B = \left(\frac{\beta_1 + (\varsigma - N)}{6}\right)$

For dilatant fluids, n > 1. we therefore take n = 2 and following the same procedure, our solution takes after equation (12) with $\beta_1 = \frac{\beta_1}{4}$

Nusselt Number

The dimensionless rate of heat transfer coefficient is the Nusselt Number (Nu) given as

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\left[a_0\left(1 - (\varsigma - N)y - \frac{3}{2}(\varsigma - N)y^2\right) + a_1\left(1 + 2y - B\left(3y^2 + 4y^3\right)\right)\right] \qquad (13)$$

Viscoelastic formulation

When a shear stress is applied to a material, the material may accelerate, flow like a fluid, deform or get stuck. The governing equation that describe the four physical consequences can be modeled as

$$m\frac{d^2x}{dt^2} - b\frac{dx}{dt} - k_0 x - f = 0 \qquad (13)$$

with the boundary conditions x(0) = 0, $x(\infty) = 1$ [5]

where, m, b, x, t, k_0 and f are respectively mass of the material, damping factor, displacement, time elapsed, spring constant and static friction/viscosity.

The complete solution of equation (13) is therefore

$$x(t) = A\exp\frac{\phi_1}{2}t + B\exp\frac{\phi_2}{2}t + \frac{\omega_1^2}{\omega_0^2} \qquad (14)$$

where, $\phi_1 = \frac{1}{\gamma} + \sqrt{\left(\frac{1}{\gamma}\right)^2 + 4\omega_0^2}$, $\phi_2 = \frac{1}{\gamma} - \sqrt{\left(\frac{1}{\gamma}\right)^2 + 4\omega_0^2}$, $\omega_0^2 = \frac{k_0}{m}$, $\omega_1^2 = \frac{f}{m}$

$$\frac{b}{m} = \frac{1}{\gamma},\ A = \frac{1}{\exp\ \phi_1\infty - 1},\ B = 1 - \frac{\omega_1^2}{\omega_0^2} - \frac{\exp\ \phi_1\infty}{\exp\ \phi_1\infty - 1}$$

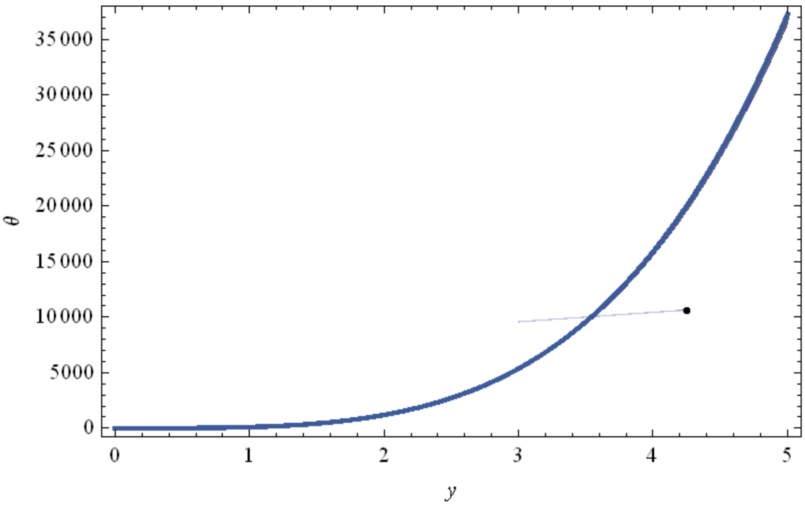

Figure 1: Temperature profile θ against boundary layer y for varying heat sink term (ς)

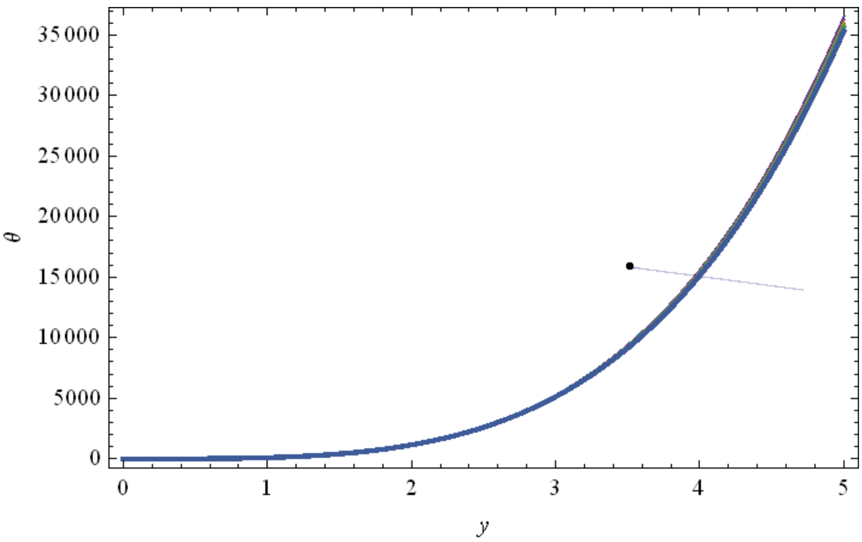

Figure 2: Temperature profile θ against boundary layer y for varying radiation (N)

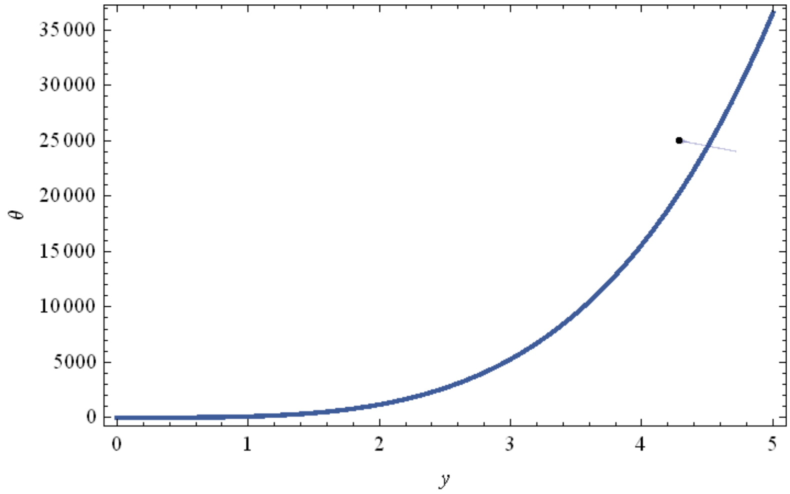

Figure 3: Temperature profile θ against boundary layer y for varying Frank-kamenestkii parameter (ϑ)

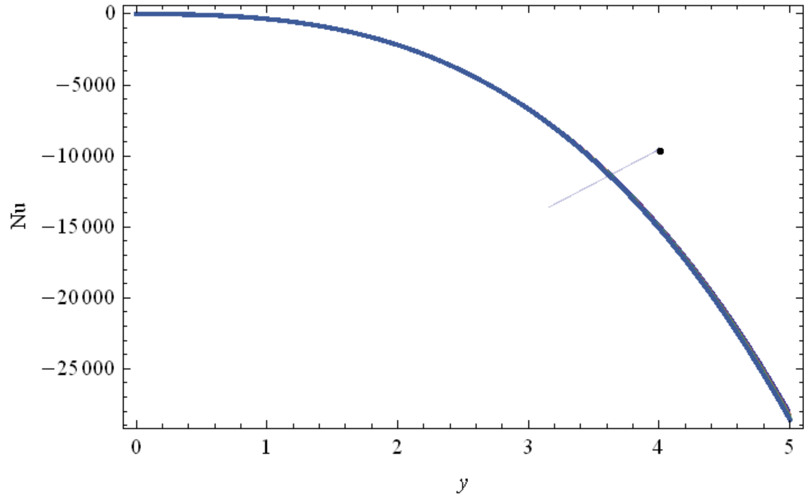

Figure 4: Nusselt number Nu against boundary layer y for varying heat source/ sink term (ς)

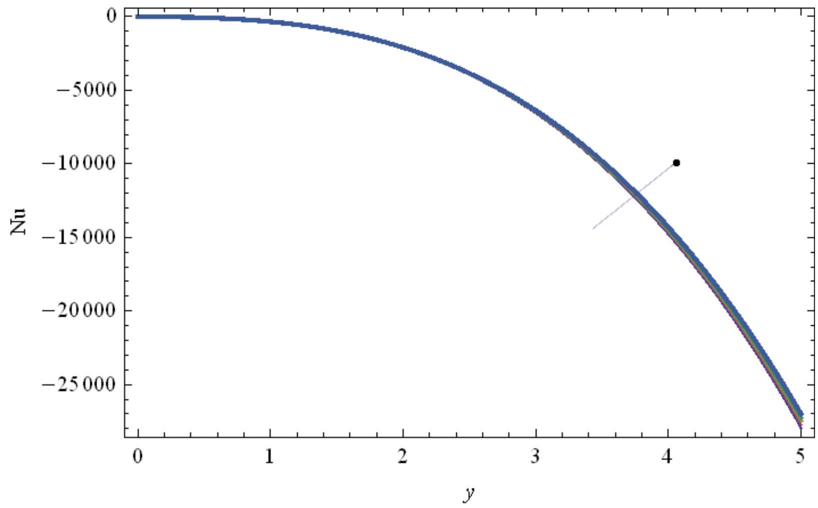

Figure 5: Nusselt number Nu against boundary layer y for varying Radiation term (N)

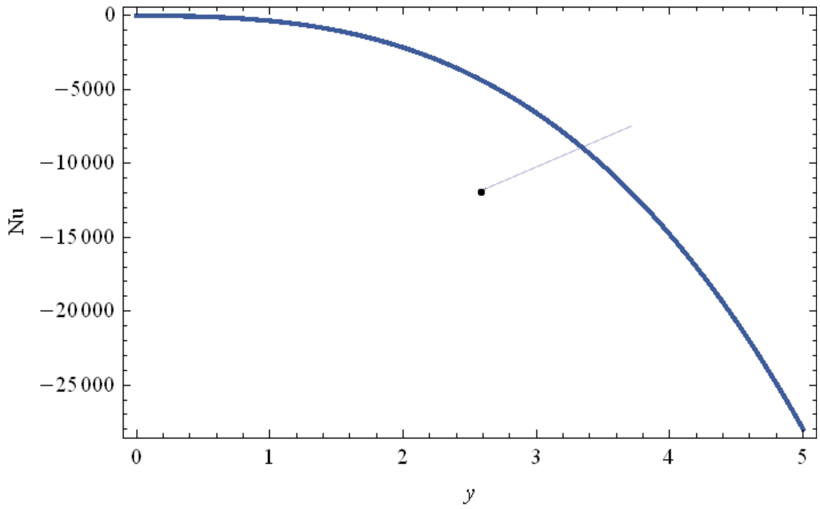

Figure 6: Nusselt number Nu against boundary layer y for varying Frank kamenestkii parameter (ϑ)

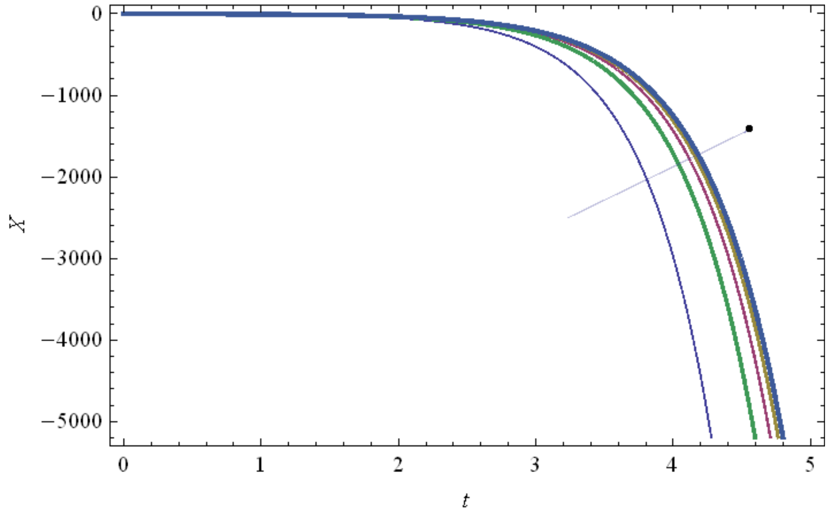

Figure 7: Displacement X against time t for varying relaxation constant (γ)

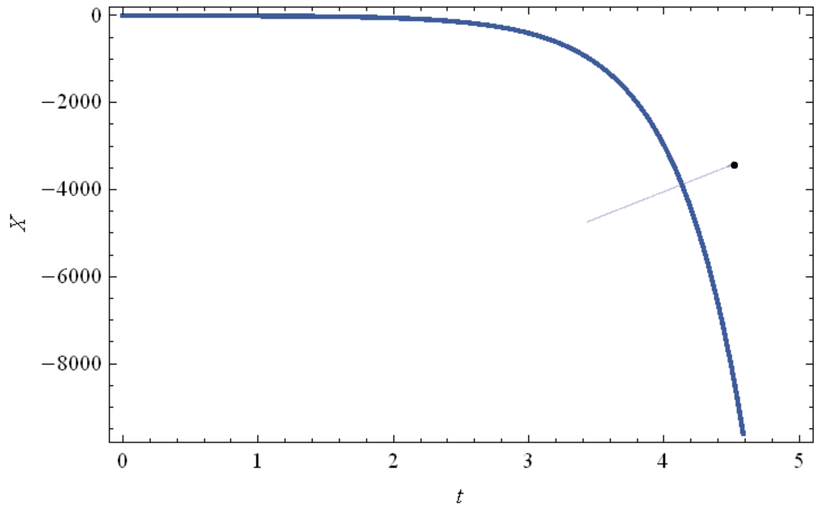

Figure 8: Displacement X against time t for varying friction/viscosity (ω_1^2)

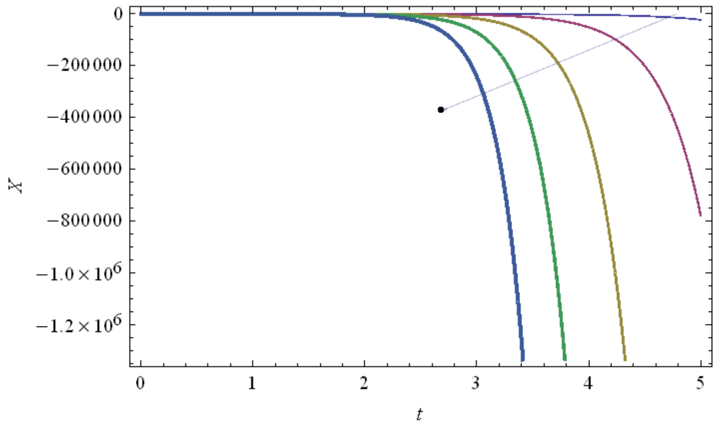

Figure 9: Displacement X against time t for varying natural frequency $\left(\omega_0^2\right)$

IV. RESULTS AND DISCUSSION

For numerical validation and physical consideration of the problem, an approximate value of the universal fluid constant is taken as 8.31 and density of non Newtonian fluids is 2500. Other parameters involved are

$\gamma = 2,4,6,8,10$

$\omega_0^2 = 3,6,9,12,15$

$\omega_1^2 = 0.94, 1.24, 1.54, 1.84, 2.14$

$\vartheta = 0.52, 1.52, 2.52, 3.52, 4.52$

$N = 1.3, 2.3, 3.3, 4.3, 5.3$

$\varsigma = 0.5, 1.5, 2.0, 2.5, 3.5$

Consideration of figure 1, shows that an increase in the heat sink term result in an increase in the temperature of the fluid and decreases the viscosity of the fluid. Figure 2 displays an increase in radiation which causes an increase in shear stress thereby reducing the temperature of the fluid. In figure 3, an increase in the Frank-kamenestkii parameter reduces the temperature of the fluid as a result of increase in viscosity. The heat transfer coefficient as explained by the Nusselt number also shows that the heat sink term and radiation as shown in figures 4 and 5 decreases the Nusselt number as they are respectively increased while the Frank-Kamenestkii parameter causes an increase in the Nusselt number as it is increased. For dilatant fluids, the observations are the same with pseudoplastic materials except in magnitude of its affected parameters. This is expected because its fluid flow index is greater than pseudoplastic fluids hence possessed higher viscosity. As the damping term increases, the relaxation term also increases which in turn increases the viscosity of the fluid thereby reducing its distance of flow as depicted in figure 7. It is evident that increase in viscosity/friction will definitely reduce the flow of fluid and this is clearly demonstrated in figure 8. Increase in the natural frequency will heat the fluid which will reduce the viscosity thereby increasing the distance of flow of the fluid as shown in figure 9.

V. CONCLUSION

Generally, fluid viscosity is affected by temperature and flow distance and these facts were elucidated in this work. Our observation was also in accord with the work of [6 and 10]. The solution for dilatant fluids was not shown because the results are just an increment in magnitude for the pseudoplastic materials.

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