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A Model for Slow Flow of Water through Porous Channel

[1] Alalibo T Ngiangia and [2] Mercy A Orukari

[1] *Department of Physics University of Port Harcourt, P M B 5323 Choba, Port Harcourt, Nigeria*

[2] *Department of Mathematics, Niger Delta University Bayelsa state*

ABSTRACT: *A model for slow flow of water through porous channel using Darcy's equation and smoothed continuity equation was carried out. Solution and analysis of the governing equations showed that at Reynolds number regime for laminar flow, increase in Reynolds number leads to a decrease in the pressure profile of water. Similar results were also observed as a result of increase in viscosity and porosity but an increase in pressure is observed as a result of increase in permeability of water.*

Keywords*: Water, Darcy's equation, Smoothed continuity equation, Porous media, Permeability, Fluid pressure*

I. INTRODUCTION

Darcy's law is a phenomenological expression for slow flows through porous medium. It states that the velocity of the fluid flow is proportional to the hydraulic gradient. It holds when the fluid (water) moves in a smooth, orderly procession in the direction of flow that is unidirectional [1].To effectively study fluids in porous media, a combination of smoothed continuity equation and Darcy's equation for fluid through porous media is of necessity [2]. The study assumed that the fluid is Newtonian, continuum, steady, fully developed and incompressible. Water as one of the most abundant chemical compound is a common name given to hydrogen-oxygen compound (H_2O). Water as a fluid is an indispensable material or substance which constitutes a greater percentage of fluids in animals, therefore its discourse cannot be over emphasized. The interaction of water with man and its environment, its physical and chemical properties as well as its flow and use by man makes it an important material for virtually every facet of human endeavour. Some of the properties of water such as its viscosity, density, solubility, colour, freezing point and boiling point as well as its ionizing capability are used as standards to define calorific and specific latent heat of substances. According to hydrologist as reported by Redmond [3], water occurs as moisture in the upper portion of the soil profile in which it is held by capillary action to the particles of soil. Under the influence of gravity, water accumulates in rocks interstices beneath the surface of the earth as a vast ground water. Some unique properties of water are that the solid is less than the liquid and has an abnormally low volatility because its molecules are associated with each other by means of hydrogen bonds in both the solid and liquid states. Slow motion of fluids through porous media are abounded using Darcy's law and its modifications [4-10]. As a universal solvent, water can easily transport food, nutrients and mixture of rocks and cement for buildings. Studies of modeling of water are also available [11-12] and experimental analysis of water [13] is also key. Our aim is to study some physical properties of water as it flows through a porous medium. This we believe will further highlight some of the properties and uses of water.

II. FORMALISM

For slow flow of water through porous medium, the smoothed equation of continuity and the Darcy's equation respectively are

$$\xi \frac{\partial \rho}{\partial t} = -(\nabla . \rho v_0) \qquad (1)$$

$$v_0 = -\frac{k}{\mu}(\nabla p - \rho g) \qquad (2)$$

where $\xi, k, \rho, \mu, v_0, p, t, g$ are respectively the porosity, permeability, density of fluid, fluid viscosity, superficial velocity, pressure of fluid, time and acceleration due to gravity.

III. SOLUTIONS AND DISCUSSION

We solve equation (2) for one dimension of pressure by integration and ignoring the constant, the result is

$$P(z) = \left(-\frac{v\mu}{k} + \rho g \right) z \qquad (3)$$

From [14], the viscosities of water at $0^0 C$, $20^0 C$ and $100^0 C$ are respectively $0.018\ Nsm^{-2}$, $0.001\ Nsm^{-2}$ and $0.0003\ Nsm^{-2}$. The effect of decreasing viscosity as a result of increased temperature using equation (3) is shown in Figure (1) and that of permeability in Figure (2).

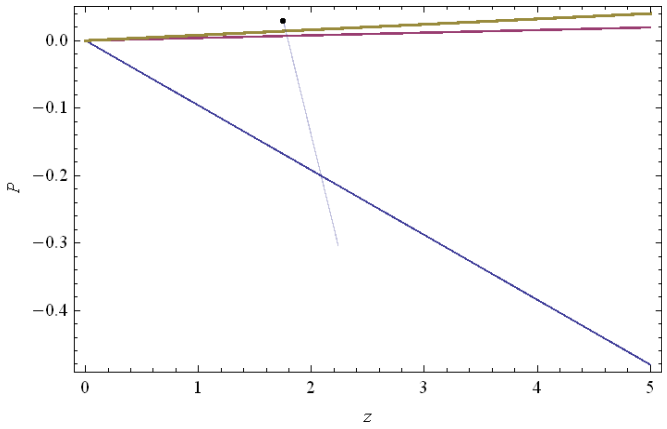

Figure 1: Pressure profile p against distance z for varying viscous term μ

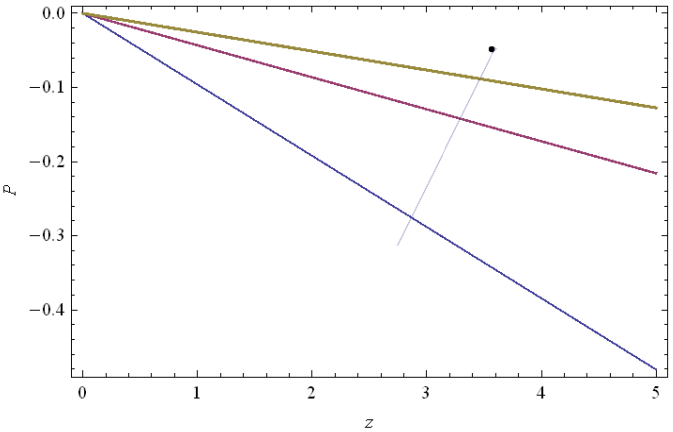

Figure 2: Pressure profile p against distance z for varying permeability term (k)

From experimental result as reported in [14], it is shown that increase in temperature, reduces the viscosity of water but increase in viscosity as shown in Figure 1, reduces the pressure of water owing to the increase in the intermolecular forces of the water. The ease with which fluid passes through a porous medium is the permeability. Its increase as depicted in Figure 2 corresponds to an increase in the pressure of water because more void spaces is available or size of pore spaces is increased which will result in an increase in the pressure of water.

Equation (3) can be rewritten as

$$P(z) = -\frac{\mu^2}{\rho k}\mathrm{Re} + \rho g z \qquad (4)$$

where Re is Reynolds number

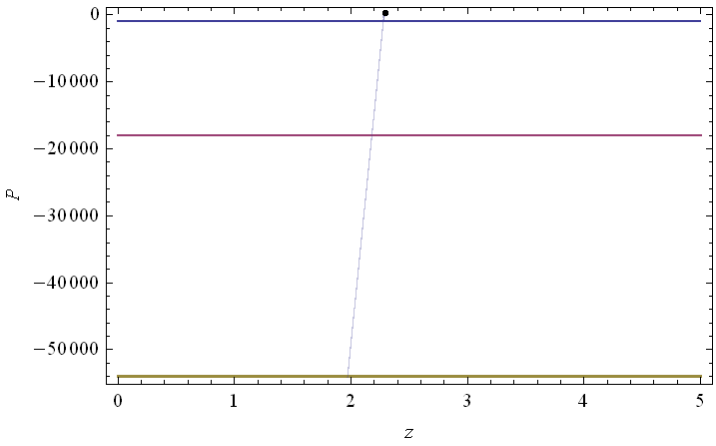

Figure 3: Pressure profile p against distance z for varying Reynolds number (Re)

Reynolds number defines the transition from laminar flow to turbulence flow regime. From Figure 3 the Reynolds number regime for laminar flow $(\mathrm{Re} \leq 3000)$ is used and it is shown that its increase, reduces the pressure of water since it is a ratio of inertia force to viscous force. Therefore, its increase will lead to a decrease in the flow of water.

Equations (1) and (2) can be combined to form

$$\left(\frac{\xi\mu}{k}\right)\frac{\partial\rho}{\partial t} = \left(\nabla.\rho\left(\nabla p - \rho g\right)\right) \qquad (5)$$

The equation of state following the argument of [2] is

$$\rho = \rho_0 p^m e^{\beta p} \qquad (6)$$

where ρ_0 is the fluid density at unit pressure, m and β are integers.

For water $m = 0$, $\beta \neq 0$, and substitution of equations (6) into equation(5), yield

$$\left(\frac{\xi\mu\beta}{k}\right)\frac{\partial\rho}{\partial t} = \nabla^2\rho - \left(\nabla.\rho^2\beta g\right) \qquad (7)$$

With the boundary conditions

$$\rho(0,t) = 0, \quad \rho(1,t) = 1, \quad \rho(z,0) = 0 \qquad (8)$$

We approximate ρ^2 by ignoring powers of ρ greater than unity using Taylor series expansion about 0 and equation (7) transforms into the form

$$\left(\frac{\xi\mu\beta}{\kappa}\right)\frac{\partial\rho}{\partial t} = \frac{\partial^2\rho}{\partial z^2} - 2\beta g\frac{\partial\rho}{\partial z} \qquad (9)$$

To solve equation (9), we assume a solution of the form

$$\rho(0,t) = Z(z)T(t) \qquad (10)$$

We substitute equation (10) into equation (9) and discarding the time part, results into

$$Z''(z) - 2\beta g Z'(z) - \alpha\theta Z(z) \qquad (11)$$

where α is a constant and $\theta = \dfrac{\xi\mu\beta}{k}$

The solution of equation (11) and the imposition of the boundary conditions as well as substituting in equation (10) for water, we get

$$\rho(z,t) = A\exp\lambda_1 z + B\exp\lambda_2 z \qquad (12)$$

where $\lambda_1 = \dfrac{2\beta g + \sqrt{4\beta^2 g^2 + 4\alpha\theta}}{2}$ $\qquad$ $\lambda_2 = \dfrac{2\beta g - \sqrt{4\beta^2 g^2 + 4\alpha\theta}}{2}$

$$A = -\frac{1}{\exp\lambda_2 - \exp\lambda_1} \qquad B = \frac{1}{\exp\lambda_2 - \exp\lambda_1}$$

Use of equation (6) in equation (12), we get

$$p(z) = \frac{1}{\beta}\log\left[\frac{1}{\rho_0}\left(A\exp\ \lambda_1 z + B\exp\ \lambda_2 z\right)\right] \qquad (13)$$

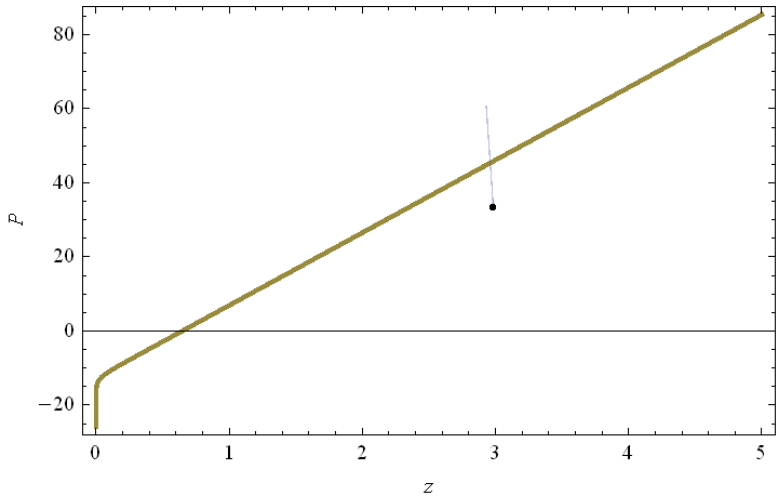

Figure 4: Pressure profile p against distance z for varying porosity term. ξ

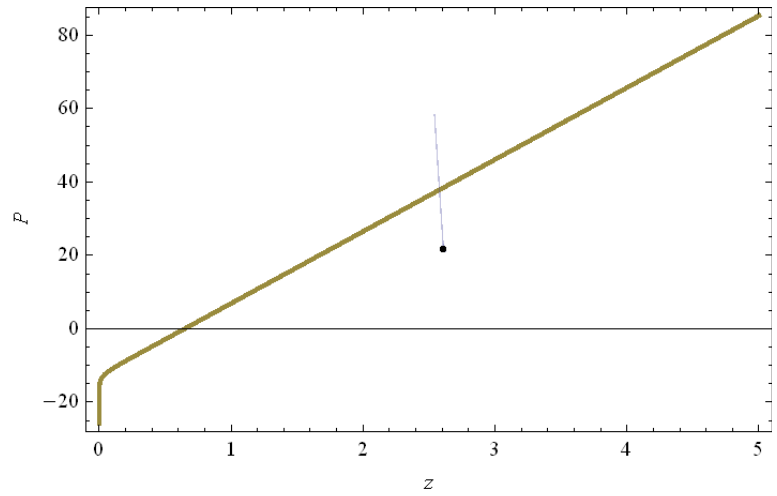

Figure 5: Pressure profile p against distance z for varying viscous term μ

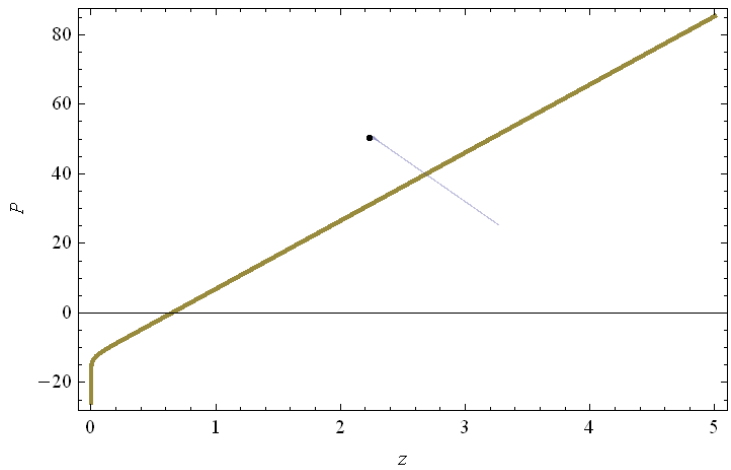

Figure 6: Pressure profile p against distance z for varying permeability term (κ)

In order to get physical insight and numerical validation of the problem, an approximate value of acceleration due to gravity $\left(g = 9.8 ms^{-2}\right)$, constant $\left(\alpha = 0.35\right)$ is chosen. The values of other parameters made use of are

$\beta = 1$

$\xi(m^2) = 0.9, 1.8, 2.7$

$k(m^2) = 0.85, 1.70, 2.55$

$\rho_0 = 1.0 x 10^{-3} kgm^{-3}$

Porosity decreases with depth in a given soil profile. Figure 4, shows that, as the height of a given rocks layer increases, the compact of the porosity decreases, hence increase in porosity will lead to a decrease in the pressure profile of the channel. In practice, an ideal or inviscid fluid is a mathematical expression; however, experiments show that increase in temperature decreases the viscosity of a fluid. Figure 5, depicts that an increase in the viscosity of water which result in the increase in internal friction, brings about a decrease in the pressure. Permeability is the ease with which fluid flow through a formation. As shown in Figure 6, increase in permeability parameter, results in an increase in the pressure exerted by the water.

IV. CONCLUSION

For slow flow of fluid in porous media, the use of Darcy's equation and its modification cannot be overemphasized. The results of our study is in agreement with the results [1], [5] and [11] in spite of our model and parameter approximations.

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