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Some More Results on Fuzzy Pairwise r-Separation Axioms in Fuzzy Bitopological Spaces

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ABSTRACT: *Here we study fuzzy r-separation axioms in fuzzy bitopological spaces in detail. Some already existing definitions have been compared with each other. Several important results have been obtained.*

Keywords: *Fuzzy regularly open sets,Fuzzy r-seperation axioms,Fuzzy almost separation axioms.*

I. INTRODUCTION

The concept of fuzzy regularly open set was introduced by Azad [2] in 1981. In 1988, Arya and Nour [1] introduced the notion of r-separation axioms in a bitopological space .Using these concepts Srivastava et al.[9] in 2015 introduced fuzzy r-separation axioms in fuzzy topological spaces. Fuzzy pairwise r-separation axioms were introduced and studied by Srivastava et al.[8].Here a comparative study among various definitions of fuzzy pairwise r-separation axioms have been established..We have obtained some results relating to these axioms also.

PRELIMINARIES

We take I= [0, 1].For a fuzzy set $A \in I^x$, co A denotes its fuzzy complement. For $\alpha \in I$, α denotes α valued constant fuzzy set. We denote the characteristic function of $Y \subseteq X$ as Y and regard it as an element of I^x.A fuzzy point x_r in X is a fuzzy set in X taking value $r \in (0, 1)$ at x and 0 elsewhere, x and r are called the support and value of x_r respectively. A fuzzy point x_r is said to belong to $A \in I^x$ if $r < A(x)$. Two fuzzy points are said to be distinct if their support are distinct. Given a fuzzy topological space (in short fts) (X, T), $A \in I^x$ is called regularly open if int cl (A) =A . Also $A \in I^x$ is called regularly closed if co A is regularly open. Clearly A is regularly closed if and only if A=cl int A. All undefined concepts are taken from Lowen[3].

Proposition2.1: [2] Intersection of two fuzzy regularly open sets is regularly open.

Proposition2.2: [2] Closure of a fuzzy open set in a fts (X ,T) is fuzzy regularly closed and interior of a fuzzy closed set in X is fuzzy regularly open.

Definition2.1 [4] Let (X, T) be an fts and x_r be a fuzzy point in X. A fuzzy set A is called a fuzzy r-neighbourhood if there exists a fuzzy regularly open set U such that $x_r \in U \subseteq A$. A fuzzy set A is called quasi r-neighbourhood of a fuzzy singleton x_r in X if there exist a fuzzy regularly open set U such that $x_r\, qU \subseteq A$.

Definition2.2 [4] Let (X,T) be a fts ,then the set of all fuzzy regularly open sets forms a base for some topology on X. This fuzzy topology is called the fuzzy semi regularization topology of T and is denoted by T*, clearly $T^* \subseteq T$

. (X, T*) is called the fuzzy semi regularization of (X, T).

Definition2.3 [8] A fuzzy set A in fuzzy topological space is said to be a fuzzy δ-open set in X if it can be expressed as a union of fuzzy regularly open sets in X. The compliment of fuzzy δ-open set is called fuzzy δ-closed set.For a fuzzy set A in X, the δ-closure of A (in short, δ-clA) is defined as the intersection of all fuzzy δ-closed sets in X which contain A.

3. Fuzzy pairwise r-separation axioms in a fuzzy bitopological space.

We recall the definitions of fuzzy pairwise T_i axioms (i=0,1,2) from [6,7] and define fuzzy weakly pairwise T_i and fuzzy pairwise semi T_2 in the following lines.

Definitions 3.1 Let (X, τ_1, τ_2) be an fbts. Then it is called

(i) **Fuzzy pairwise** T_0 if $\forall x, y, \in X, x \neq y, \exists$ a $\tau_1 -$ fuzzy open set of a τ_2- fuzzy open set U such that $U(x) \neq U(y)$.

(ii) **Fuzzy pairwise** T_1 if $\forall x, y, \in X, x \neq y, \exists$ a $\tau_1 -$ fuzzy open set U and a τ_2- fuzzy open set V such that $U(x) = 1$, $U(y) = 0$ and V(x)=0, V(y)=1.

(iii) Fuzzy weakly pairwise T_1 if $\forall x, y, \in X, x \neq y, \exists$ a $\tau_1 -$ fuzzy open set or a τ_2- fuzzy open set U such that $U(x) = 1,\ U(y) = 0$.

(iv) Fuzzy weakly pairwise T_1 if $\forall$ pair of distinct fuzzy points $x_r, y_s \in X, \exists$ a τ_1- fuzzy open set U and a τ_2- fuzzy open set V such that $x_r \in U, y_s \in V$, or $x_r \in V, y_s \in U$, and $U \cap V = \emptyset$.

(v) Fuzzy pairwise T_2 $[SS_1]$ if $\forall$ pair of distinct fuzzy points $x_r, y_s \in X, \exists$ a τ_1- fuzzy open set U and a τ_2- fuzzy open set V such that $x_r \in U, y_s \in V$, and $U \cap V = \emptyset$.

Now replacing 'fuzzy open sets' by 'fuzzy regularly open sets' in (i), (iv) and (v) and by 'fuzzy δ-open sets' in (ii) and (iii) of definition 3.1, we give the following definitions.

Definitions 3.2[8]: An fbts (X, τ_1, τ_2) is said to be

(i) Fuzzy pairwise rT_0 if $\forall x, y, \in X, x \neq y, \exists$ a $\tau_1 -$ fuzzy regularly open set of a τ_2- fuzzy open set U such that $U(x) \neq U(y)$.

(ii) Fuzzy pairwise rT_1 if $\forall x, y, \in X, x \neq y, \exists$ a $\tau_1 -$ fuzzy δ-open set U and a τ_2- fuzzy open set V such that $U(x) = 1,\ U(y = 0$ and V(x)=0, V(y)=1.

(iii) Fuzzy weakly pairwise rT_1 if $\forall x, y, \in X, x \neq y, \exists$ either a $\tau_1 -$ fuzzy δ-open set or a τ_2- fuzzy δ-open set U such that $U(x) = 1,\ U(y) = 0$.

(iv) Fuzzy weakly pairwise rT_2 if $\forall$ pair of distinct fuzzy points x_r, y_s in $X, \exists$ a τ_1- fuzzy regularly open set U and a τ_2- fuzzy regularly open set V such that $x_r \in U, y_s \in V$, or $x_r \in V, y_s \in U$, and $U \cap V = \emptyset$.

(v) Fuzzy pairwise rT_2 if $\forall$ pair of distinct fuzzy points x_r, y_s in X, $\exists$ a τ_1- fuzzy regularly open set U and a τ_2- fuzzy regularly open set V such that $x_r \in U, y_s \in V$, and $U \cap V = \emptyset$.

The relation among the above definitions has been stated in the following theorem.

Theorem 3.1 Let (X, τ_1, τ_2) be an fbts. then we have the following imlication diagram.

$$\begin{array}{ccccc}
\text{fuzzy pairwise semi } T_2 & \Rightarrow & \text{fuzzy weakly pairwise } T_1 & & \\
\Uparrow & & \Uparrow & & \\
\text{fuzzy pairwise semi } rT_2 & \Rightarrow & \text{fuzzy weakly pairwise } rT_1 & & \\
\Uparrow & & \Uparrow \quad \Downarrow & & \\
\text{fuzzy pairwise semi } rT_2 & \Rightarrow & \text{fuzzy pairwise } rT_1 & \Rightarrow & \text{fuzzy pairwise } rT_0 \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{fuzzy pairwise semi } T_2 & \Rightarrow & \text{fuzzy pairwise } T_1 & \Rightarrow & \text{fuzzy pairwise } T_0
\end{array}$$

Proof : we show that fuzzy pairwise $rT_2 \Rightarrow$ fuzzy pairwise rT_1

Let $x, x, y, \in X, x \neq y$. Now xr, ys, $(0 < r < 1, 0 < s < 1)$ are distinct fuzzy points in X. Therefore, since (X, τ_1, τ_2) is fuzzy pairwise $rT_2 \exists$ a τ_1- fuzzy regularly open set U_r and a τ_2- fuzzy regularly open set V_s such that $x_r \in U_r$, $y_s \in V_s$ and $U_r \cap V_s = \phi$. Now consider the fuzzy δ-open sets $U = \sup_r U_r$ and $V = \sup_s V_s$. The U(x)=1, U(y)=0 and V(x)=0, V(y)=1 showing that (X, τ_1, τ_2) is fuzzy pairwise rT_1.

Other implication can be easily proved.

None of the above implications are reversible, as exhibited in the following counter examples.

(i) fuzzy weakly pairwise $T_1 \nRightarrow$ fuzzy pairwise semi T_1.

Counter-example 3.1 Let X be an uncountable set and τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{U \subseteq X: X - U \text{ is finite}\}$ and τ_2 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{U \subseteq X: X - U \text{ is countable}\}$.

Then (X, τ_1, τ_2) is fuzzy weakly pairwise T_1 but it is not fuzzy pairwise semi T_2 since we can not find non empty $U \in \tau_1, V \in \tau_2$ which are disjoint.

(ii) fuzzy pairwise semi $T_2 \nRightarrow$ fuzzy pairwise semi T_2.

Counter-example 3.2 Let X be an infinite set and τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{U \subseteq X: X - U \text{ is finite}\}$ and τ_2 be the discrete fuzzy topology on X.

Then (X, τ_1, τ_2) is fuzzy pairwise semi T_1 but it is not fuzzy pairwise semi rT_2 since there are no τ_1- fuzzy regularly open sets other than the constants.

(iii) fuzzy weakly pairwise semi $T_1 \nRightarrow$ fuzzy weakly pairwise rT_1.

Counter-example 3.3 Let X be an infinite set, τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{U \subseteq X$: X-U in finite$\}$ and τ_2

$\{\underline{\alpha}: \alpha \in [0,1]\}$.

Then $(X, \tau_1, \text{open } \tau_2)$ is fuzzy weakly pairwise T_1 but not fuzzy weakly pairwise rT_1 since there are no τ_1-fuzzy regularly open set or τ_2-fuzzy regularly sets other than the constants.

(iv) fuzzy weakly pairwise $rT_1 \nRightarrow$ fuzzy pairwise semi rT_1.

Counter-example 3.4 Let X be an infinite set, τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{B_{xy}: x, y \in X, x \neq y\}$ where B_{xy} is the fuzzy set in X such that B_{xy} (x)=1, B_{xy} (y)=0 and B_{xy}(z)=1/2 for $z \in X, z \neq x, y$ and τ_2 be the fuzzy topology generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{D_{xy}: x, y \in X, x \neq y\}$ where $D_{x,y}$ (x)=0, $D_{x,y}$ (y)=0 and $D_{x,y}$ (z)=1/3 for $z \in X, z \neq x, y$.
Then (X, τ_1, τ_2) is fuzzy weakly pairwise rT_1 but not fuzzy pairwise semi rT_2 since we can not find τ_1-fuzzy regularly open set U and τ_2-fuzzy regularly open set V such that $x_r \in U$, $y_s \in V$ or $x_r \in V$, $y_s \in U$ and $U \cap V = \phi$.
(v) fuzzy pairwise semi $rT_2 \nRightarrow$ fuzzy pairwise rT_2.

Counter-example 3.5 Let X be any set containing more than two points. Let τ_1 be the discrete fuzzy topology on X and τ_2 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{U \subseteq X: U(x_0)=0$ for some fixed point x_0 in $X\}$.
Then (X, τ_1, τ_2) is fuzzy pairwise semi rT_1 since if we take distinct fuzzy points x_r, y_s in X, where x, y are different from x_0 then $\{x\}$ is a τ_1-fuzzy regularly open set and $\{y\}$ is a τ_2-fuzzy regularly open set, satisfying the condition that $x_r \in \{x\}$, $y_s \in \{y\}$ and $\{x\} \cap \{y\} = \phi$. Further, if $x = x_0$ or $y = x_0$ say $x=x_0$ then we can take $\{x_0\}$ in τ_1 and $\{y\}$ in τ_2 satisfying the requirement. But (X, τ_1, τ_2) is not fuzzy pairwise rT_2 since if we take the pair $(x_0)_r, y_r$ of distinct fuzzy points in X, then $\nexists$ τ_2-fuzzy regularly open set U and τ_1-fuzzy regularly open set V such that $(x_0) \in U$, $y_r \in V, U \cap V = \phi$.
(vi) fuzzy weakly pairwise $rT_1 \nRightarrow$ fuzzy pairwise rT_1.

Counter-example 3.6. Let X={x,y,z}, τ_1 be the discrete fuzzy topology on X and τ_2 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{\{x\}, \{y\}\}$.
Then (X, τ_1, τ_2) is fuzzy weakly pairwise rT_1 but not fuzzy pairwise rT_1.
(vii) fuzzy pairwise $rT_0 \nRightarrow$ fuzzy weakly pairwise rT_1.

Counter-example 3.7. Let X={x,y,z}, τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{\{x\}, \{y,z\}\}$ and τ_2 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{\{x\}, \{y\}\}$.
Then the fbts (X, τ_1, τ_2) is fuzzy pairwise rT_0 but not fuzzy weakly pairwise rT_1 since for the pair y, z there does not exist a $\tau_1 - \delta$-fuzzy open set or a τ_2-fuzzy δ-open set U such that U(z)=1, U(t)=0.
(viii) fuzzy pairwise $rT_1 \nRightarrow$ fuzzy pairwise rT_2.
Here the counter example 3.4 again works.
(ix) fuzzy pairwise $rT_0 \nRightarrow$ fuzzy pairwise rT_1.
Here the counter example 3.2 works. The fbts (X, τ_1, τ_2) is fuzzy pairwise rT_0 but it is not fuzzy pairwise rT_1 since we cannot find a τ_2-fuzzy δ-open set U such that U(z)=1, U(y)=0.
(x) fuzzy pairwise $T_2 \nRightarrow$ fuzzy pairwise rT_2.
Here counter example 3.2works. For distinct fuzzy points x_r, y_s in consider X-{y} $\in \tau_1$ and $\in \tau_2$ then $x_r \in X - \{y\}$, $y_s \in \{y\}$ and $(X - \{y\} \cap \{y\} = \phi$ showing that (X, τ_1, τ_2) is fuzzy pairwise T_2 but it is not fuzzy pairwise rT_2 since there do not exist τ_1-fuzzy regularly open set other than the constants.
(ix) fuzzy pairwise $T_1 \nRightarrow$ fuzzy pairwise rT_1.
Here the counter example3.2again works.
(xi) fuzzy pairwise $T_0 \nRightarrow$ fuzzy pairwise rT_0.

Counter-example 3.8 Let X={x,y,z}, τ_1 be the fuzzy topology on X generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{\{x\}\}$ and τ_2 be generated by $\{\underline{\alpha}: \alpha \in [0,1]\} \cup \{\{x\}\}$.
Then (X, τ_1, τ_2) is fuzzy pairwise T_0 but not pairwise rT_0 since for there are no τ_1-fuzzy regularly open sets or τ_2-fuzzy regularly open sets other than the constants.
(xiii) fuzzy pairwise $T_1 \nRightarrow$ fuzzy pairwise T_2.
Here counter example 5.2.1 works.
(xiv) fuzzy pairwise $T_0 \nRightarrow$ fuzzy pairwise T_1.

Counter-example 3.9. Let X be any non empty set containing at least two points. Let τ_1 be the discrete fuzzy topology on X and τ_2 be the indiscrete fuzzy topology on X. Then (X, τ_1, τ_2) is fuzzy pairwise T_0 but not fuzzy pairwise T_1.
Now we prove some results related to fuzzy pairwise rT_i axioms (i=0, 1, 2), fuzzy weakly pairwise rT_1.

Proposition 3.1. An fbts (X, τ_1, τ_2) is fuzzy pairwise rT_0 if either (X, τ_1) or (X, τ_2) is rT_o.
Proof. Given that (X, τ_1) or (X, τ_2) is rT_o if (X, τ_1) is rT_0, then for $x, y \in X, x \neq y, \exists$ a τ_1-fuzzy regularly open set U such that U(x)≠U(y) and if (X, τ_2) is rT_o, then for $x, y \in X, x \neq y, \exists$ a τ_1-fuzzy regularly open set V such that V(x)≠V(y). Thus for $x, y \in X, x \neq y, \exists$ a fuzzy regularly open set U in $\tau_1 \cup \tau_1$ such that U(x)≠U(y) implying that (X, τ_1, τ_2) is fuzzy pairwise rT_0.
We also have,

Proposition 3.2 An fbts (X, τ_1, τ_2) is fuzzy pairwise rT_1 if (X, τ_1) and (X, τ_2) are fuzzy rT_1.
Proof. First, let us suppose that the fbts (X, τ_1, τ_2) is fuzzy pairwise rT_1. If Then for $x, y \in X, x \neq y, \exists$ a τ_1-fuzzy δ-open set U_1 and a τ_2-fuzzy δ-open set V_1 such that U_1 (x)=1, U_1(y)=0 and V_1 (x)=0, V_1 (y)=1. Further we take pair x, y then ∃ a τ_2-fuzzy δ-open set V_2 such that U_2(x)=0, U_2(y)=1 and V_2(x)=1, V_2(y)=0. Thus for $x, y \in X, x \neq y$, we have found τ_1-fuzzy δ-open sets U_1 and U_2 such that U_1 (x)=1, U_1(y)=0 and U_2 (x)=0, U_2(y)=1 showing that (X, τ_1) is fuzzy rT_1. Similarly (X, τ_2) is fuzzy rT_1.

Conversely, suppose that (X, τ_1) and (X, τ_2) are fuzzy rT_1. Then since (X, τ_1) is fuzzy rT_1, for $x, y \in X, x \neq y, \exists$ a τ_1-fuzzy δ-open set U (say) such that U (x)=1, U(y)=0 and further, as (X, τ_2) is fuzzy rT_1, choosing $y, x \in X, \exists$ a τ_2-fuzzy δ-open set V (say) such that V(x)=0, V(y)=1. Thus for $x, y \in X, x \neq y$, we have found τ_1-fuzzy δ-open sets U and a τ_2-fuzzy δ-open set V such that U(x)=1, U(y)=0 and V(x)=0, V(y)=1 showing that (X, τ_1, τ_2) is fuzzy pairwise r T_1.

Theorem 3.2[8]. In an fbts (X, τ_1, τ_1) the following statements are equivalent:
(a) (X, τ_1, τ_1) is fuzzy weakly pairwise rT_1.
(b) $\tau_1 - \delta cl\ \{x\} \cap \tau_2$ng characterization of fuzzy pairwise rT_2 axiom.

Theorem 3.3[8]: An fbts (X, τ_1, τ_2) is fuzzy pairwise rT_2 iff the diagonal set Δ_x is fuzzy closed in $(X \times X, \tau_1^* \times \tau_2^*)$.

Now we study fuzzy pairwise r-regular and fuzzy pairwise r-normal fuzzy bitopological spaces.
In case of fuzzy pairwise r-regular, we have the following characterization.

Theorem 3.4[8] In an fbts (X, τ_1, τ_2) the following statements are equivalent:
(a) The fbts (X, τ_1, τ_2) is fuzzy pairwise r-regular.
(b) For each fuzzy point x_r in X and every τ_i-fuzzy open set F such that $x_r \subseteq F, \exists$ a τ_i-fuzzy regularly open set U such that $x_r \subseteq U \subseteq \tau_i$-cl U⊆F.

Proof (a) ⇒ (b)

Let the fbts (X, τ_1, τ_2) be fuzzy pairwise r-regular. Then for every τ_i-fuzzy open set F and for each fuzzy point x_r such that $x_r \subseteq F, \exists$ a τ_i-fuzzy regularly open set U and τ_i-fuzzy regularly open set V such that $x_r \subseteq U$, coF⊆V and $U \subseteq$ coF⊆V. Thus $x_r \subseteq U \subseteq$ coV⊆F. Now U⊆coV and coV is a τ_j-fuzzy regularly closed set and hence a τ_j-fuzzy closed set therefore τ_j-cl $U \subseteq$ coV. Hence $x_r \subseteq U \subseteq \tau_j -$ cl U⊆F.

(b) ⇒ (a)

Let x_r be a fuzzy point and F be a τ_i-fuzzy closed set such that $x_r \subseteq$ coF. In view of (b), ∃ a τ_i-fuzzy regularly open set U such that $x_r \subseteq U \subseteq \tau_j -$ cl U⊆coF. Consider now the fuzzy set U_1 and V_1 where U_1=U and V_1=1-τ_j-cl U. Then U_1 is a τ_i-fuzzy regularly open set and using proposition 4.1.2. V_1 is a τ_j-fuzzy regularly open set such that $x_r \subseteq U_1$, F⊆V_1 and $U_1 \subseteq$ coV_1 i.e. $U_1\ \bar{q}\ V_1$, as for any x∃X, U_1(z)+V_1(z)=U(z)+1-τ_j-cl U (z) which is obviously ≤ 1.

Theorem 3.5 Every crisp bifuzzy regularly open subset of a fuzzy pairwise r-regular space is fuzzy pairwise r-regular.
Proof. Let (X, τ_1, τ_2) be fuzzy pairwise r-regular and let $Y \subseteq X$ be a crisp bifuzzy regularly open subset of X. To show that $(X, \tau_{1y}, \tau_{1y})$ is fuzzy pairwise r-regular, let F be any τ_{1y}-fuzzy closed set and $x_r \subseteq$ coF. Then there is a τ_i-fuzzy closed set A such that F=A∩Y. Since $(X, \tau_{1y}, \tau_{1y})$ is fuzzy pairwise r-regular and A is a τ_i-fuzzy closed set such that $x_r \subseteq$ coA, ∃⊆ a τ_i-fuzzy regularly open set U and a τ_j-fuzzy regularly open set V such that $x_r \subseteq U$, A⊆V and $U \subseteq$ coV. Take U_y = U∩Y and U_y = V∩Y. Then U_y and V_y are fuzzy regularly open sets is Y by using proposition 4.1.1. Hence x_r⊆U_y, F⊆V_y and U_y,⊆coV_y showing that $(Y, \tau_{1y}, \tau_{1y})$ is fuzzy pairwise r-regular.
Now we define fuzzy pairwise r-normality in a fbts.

Definition 3.3 [8] An fbts (X, τ_1, τ_2) is said to be fuzzy pairwise r-normal if for any τ_i-fuzzy regularly open set U and a τ_i-fuzzy regularly open set V such that A ⊆coB, ∃ a τ_j-fuzzy regularly open set U and a τ_i-fuzzy regularly open set V such that A ⊆U, B⊆V and U⊆coV.

In case of fuzzy pairwise r-normal space, we have the following characterization:

Theorem3.6[8]. In an fbts (X, τ_1, τ_2) the following statements are equivalent:

(a) The fbts (X, τ_1, τ_2) is fuzzy pairwise r-normal.

(b) For any τ_i-fuzzy closed A and τ_j-fuzzy open set B such that $A \subseteq B$. $\exists$ a τ_j-fuzzy regularly open set U such that $A \subseteq U \subseteq \tau_i$-cl $U \subseteq B$.

Proof (a) $\Rightarrow$ (b)

Let the fbts (X, τ_1, τ_2) be fuzzy pairwise r-normal. Then any τ_i-fuzzy closed set A and τ_j-fuzzy open set B such that $A \subseteq B$. $\exists$ a τ_j-fuzzy regularly open set U and a τ_i-fuzzy regularly open set V such that $A \subseteq U$, $coB \subseteq V$ and $U \subseteq coV$. Thus $A \subseteq U \subseteq coV \subseteq B$. Since coV is τ_i-fuzzy regularly closed set and hence a τ_i-fuzzy closed set containing U, we have $A \subseteq U \subseteq \tau_i$-cl. $U \subseteq B$.

(b) $\Rightarrow$ (a)

Let A be any τ_i-fuzzy closed set and B be any τ_j-fuzzy closed set such that $A \subseteq coB$. In view (b) $\exists$ a τ_i-fuzzy regularly open set U such that $A \subseteq U \subseteq \tau_i - \text{cl } U \subseteq B$. Consider the fuzzy sets U_1 and V_1 where U_1=U and V_1=1-τ_i-cl U. Then U_1 is a τ_j-fuzzy regularly open and V_1 is a τ_i-fuzzy regularly open using proposition 4.1.2 and are such that $A \subseteq U_1$ $B \subseteq V_1$ and $U_1 \subseteq coV_1$ i.e. $U_1 \bar{q} V_1$, as for any $z \in X$, $U_1(z)+V_1(z)$ = U(z)+1-τ_i-cl U (z) which is obviously ≤ 1.

Theorem 3.7 Every bifuzzy closed and bifuzzy regularly open subspace of a fuzzy pairwise r-normal space is fuzzy pairwise r-normal.

Proof. Let Y be a bifuzzy closed and bifuzzy regularly open subspace of fuzzy pairwise r-normal space (X, τ_1, τ_2). Let A and B be any two fuzzy sets in Y such that A is τ_{iy}-fuzzy closed and B is τ_{iy}-fuzzy clsoed and $A \subseteq coB$. Since Y is a bifuzzy closed subset of X, A is τ_i-fuzzy closed and B is τ_j-fuzzy closed. Therefore since (X, τ_1, τ_2) is fuzzy pairwise r-normal, $\exists$ a τ_j-fuzzy regularly open set V such that $A \subseteq U$, $B \subseteq V$ and $U \subseteq coV$. Take $U_y = U \cap Y$ and $V_y = V \cap Y$. Now U_y is τ_{iy}-fuzzy regularly open and U_y is τ_{jy}-fuzzy regularly open (using the fact that Y is bifuzzy regularly open and proposition 4.1.1), such that $A \subseteq U_y$, $B \subseteq V_y$ and $U_y \subseteq coV_y$. Hence (X, τ_1, τ_2) is fuzzy pairwise r-normal.

II. CONCLUSION

Here we have studied fuzzy pairwise r-separation axioms in fuzzy bitopological space using fuzzy regularly open sets.Interrelations among the various pairwise r-separation axioms have been established.Several important results related to these axioms have been obtained.

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