American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-12, pp-334-349
www.ajer.org
Research Paper Open Access

Systems Dynamics and Control, Proposed Course Overview and Education Oriented Approach for Mechatronics Engineering Curricula

Farhan A. Salem[1, 3, *], Ahmad A. Mahfouz[2, 3]
[1]*Mechatronics Engineering Program, Department of Mechanical Engineering, College of Engineering, Taif University, Taif, Saudi Arabia*
[2]*Department of Automatic and Mechatronics Systems, Vladimir State University, Vladimir, Russian Federation*
[3]*Alpha Center for Engineering Studies and Technology Researches, Amman, Jordan*

ABSTRACT: *Mechatronics engineer is expected to design engineering systems with synergy and integration toward constrains like higher performance, speed, precision, efficiency, lower costs and functionality. To meet such integrated abilities and knowledge requirements, it is desired that Mechatronics engineering curricula, to include a proper integrated courses' description, with specific topics, lab sessions, student projects and methods of integrated abilities and knowledge delivery. This paper proposes, a proper for Mechatronics education, Systems Dynamics and Control course detailed description, topics with specific learning objectives, prerequisites, administration, simple but effective teaching approach supported by simple and easy to memorize education oriented steps and tables, that integrate course outcomes, to solve control problems, all intended to support educators in teaching process, help students in concepts understanding, maximum knowledge and skills transfer / gaining in solving controller/algorithm selection and design problems as a stage of Mechatronics system design stages, to equip students with the key abilities and knowledge, required for further courses in Mechatronics curricula.*

Keywords: *Mechatronics Education, Teaching Approach, Course Description, Dynamics Control Design, Modeling*

I. INTRODUCTION

The continuous progress in information technology and the synergetic implementation of different engineering aspects caused the engineering problems to be harder, scientific problems are normally multidisciplinary and to solve them we require a multidisciplinary engineering systems procedures, such systems are used to be called Mechatronic systems. In the same time engineers affront hard challenges and in competitive market they must provide high attendance by presenting their selves as innovative, integrative, conceptual, and multidisciplinary. Engineers must be capable of treating in depth different engineering disciplines with a balance between theory and practice, therefore, they must have breadth in business and human values, an engineer with such qualifications is called Mechatronics engineer. Mechatronics engineer is hoped to design engineering systems with synergy and integration toward constrains like higher performance, speed, precision, efficiency, lower costs and higher functionality.

Role of Control Subsystem in Mechatronics System and Its Design; Mechatronics can be defined as a multidisciplinary concept, where mechanical engineering, electric engineering, electronic systems, information technology are integrated, morover intelligent control system, and computer hardware and software are involved to manage complexity, uncertainty, and communication through the design and manufacture of products and processes from the very start of the design process, thus enabling complex decision making. Modern products are used to be called Mechatronics products, when the comprehensive systems are fully integrated. Today for improving development processes in industry two top drivers are considered: shorter product-development schedules and increased customer demand for better performing products, The *Mechatronic system design process* is a modern interdisciplinary design procedure, it is the concurrent selection, evaluation, synergetic integration, and optimization of the whole system and all its sub-systems and components as a whole and concurrently, where all the design procedures should work in parallel and collaborative manner throughout the design and development process to produce an overall *optimal* design [1, 3].

Integration refers to combining disparate data or systems so they work as one system. The integration within a Mechatronics system can be performed in two kinds, *a)* through the integration of components (hardware integration) and *b)* through the integration by information processing (software integration) based on advanced control function. The integration of components results from *designing* the Mechatronics system as *an overall system*, and *embedding* the sensor, actuators, and microcomputers into the mechanical process, the microcomputers can be integrated with actuators, the process, or sensor or be arranged at several places. Integrated sensors and microcomputers lead to smart sensors, and integrated actuators and microcomputers developed into smart actuators. For large systems bus connections will replace the many cable. Hence, there are several possibilities to build up an integrated overall system by proper integration of the hardware**.** *Synergy* refers to the creation of a whole final products that is better than the simple sum of its parts, the principle of synergy in Mechatronics means, an integrated and concurrent design should result in a better product than one obtained through an uncoupled or sequential design, synergy can be generated by the right combination of parameters, [1, 14]

Mechatronics systems are supposed to be designed with synergy and integration toward constrains like higher performance, speed, precision, efficiency, lower costs and functionality and operate with exceptional high levels of accuracy and speed despite adverse effects of system nonlinearities, uncertainties and disturbances, Therefore, one of important decisions in Mechatronics system design process are, two directly related to each other subsystems, the control unit (physical-unit) and control algorithm subsystems selection, design and synergistic integration. During the concurrent design of Mechatronic systems, it is important that changes in the mechanical structure and other subsystems be evaluated simultaneously; a badly designed mechanical system will never be able to give a good performance by adding a sophisticated control system, therefore, Mechatronic systems design requires that a mechanical system, dynamics and its control system structure be designed as an integrated system (this desired that (sub-) models be reusable), modelled and simulated to obtain unified model of both, that will simplify the analysis and prediction of whole system effects, performance, and generally to achieve a better performance, a more flexible system, or just reduce the cost of the system. Possible physical-control subsystem and algorithm options are shown in Figure 1. As shown, three components can be identified at this level; the control system, control algorithm and the electronic unit subsystems. The control unit is the central and most important part (brain) of Mechatronic system, it commands, controls and optimizes the process, by reading the input signals representing the state of the system and environment, compares them to the desired states, and according to control algorithm, outputs signals to the actuators to control and optimise the physical system and meeting specifications. Control subsystem must ensure excellent steady-state and dynamic performance [1, 7].

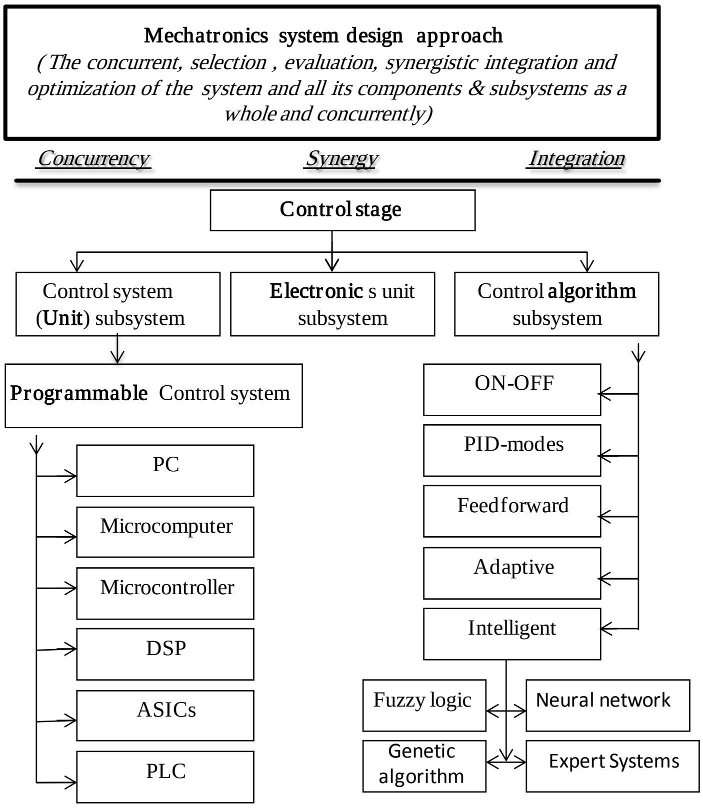


Figure 1a. Components at control stage; Control system, algorithm and electronic unit subsystems.

(a) (b) (c) (d)

Figure 1b. Some of physical control units options; *a)* PLC, *b)* Microcontroller, *c)* Computer control, *d*) analog controllers.

II. ''CONTROL SYSTEMS DESIGN AND ANALYSIS'' COURSE

This is a basic course, consisting of two parts; system dynamics and their control process. It focuses on gaining adequate abilities and knowledge in mathematical modelling of dynamic systems and corresponding selection and design of control system to meet and maintain desired performance. The course is taught in all mechanical, electric and Mechatronics engineering curricula and tracks; including; General mechanical engineering, Mechatronics engineering, Industrial engineering, Control engineering, Automation engineering, also can be found taught in other departments such as science/math. Departments. Depending on institution, department, minor's specific requirements and educators, it may have different description and titles, also is taught from different points of view and applying different approaches. Titles such as: Controlled differential equations, System dynamics and control, Dynamic systems and control, Control system design and analysis, Feedback control system, Control and engineering, introduction to control systems, and others [11].

A unified course description, with specific learning objectives/outcomes, correct prerequisites, other courses to which, this course is a prerequisites, also, simple but effective teaching approach supported with tables, that can help in achieving learning objectives, is highly required. This paper proposes, a proper for Mechatronics education, course detailed description, topics with specific learning objectives, correct prerequisites, administration, simple but effective teaching approach supported by simple and easy to memorize education oriented steps and tables, that integrate course outcomes, to solve control problems, all intended to support educators in teaching process, help students in concepts understanding, maximum knowledge and skills transfer /gaining in solving controller/algorithm selection and design problems as a stage of Mechatronics system design stages, and prepare them for other further courses applied in Mechatronics curricula including; Mechatronics fundamentals, Process control, Mechatronics systems design, Embedded systems design, Robotics, PLC, CNC and others [4, 13].

2.1. Proposed Course Description, Audience, and Course Learning Objectives

2.1.1. Course Learning Objectives

Course learning objectives (CLO) are the key abilities and knowledge that to be assessed in a course. One of main aims of Mechatronics curricula is to equip the students with multidisciplinary capabilities to design Mechatronics systems, the course is required, and is a basic course in the control of dynamical systems, intended to provide students with abilities and knowledge in control system/algorithm, selection and design. It is prerequisite for a group of further subjects/courses, mentioned above, in Mechanical/Mechatronics engineering program. By analysing what abilities and knowledge are desired for the student to have before attending each of these courses, it can be clarified what CLOs are desired. In particular, after taking this course, students should be able to:

a) Understand fundamentals associated with control theory; analysis, design, performance, response, types and role of; control, control loops, control loop components, control units, control algorithms there mathematical models, their effects upon process performance and selection criteria (summarized in Table 6). *b)* Apply fundamentals associated with representation of physical systems and related concepts; Represent a plant (process) mathematically, using block diagrams, transfer function, flow graphs, state equation *(Build control-oriented models of dynamic systems; electrical, mechanical, hydraulic and pneumatic). c)* Develop engineering and physical insights into analysis and evaluation (interpretation) of a plant's performance (or how systems respond to an input?), in terms of key characteristics of developed mathematical model (summarized in Tables 3, 4, 5).To analyze whether a given control system is stable or note?, what needs to be done to make it stable (*analyze*)?, how this can (*should*) be done (*synthesis*)? And how his solution will affect the system performance (*evaluation*)?, *Also to anticipate system's stability and response, based on poles (zeros) nature, location, damping ratio* (summarized in Table 3). *d)* Understand the conceptual selection and design of a control unit/algorithm, in time/frequency/state space domains, and apply principles and tools of feedback and control to select and design a control system/algorithm to design a control system to meet and maintain desired performance specification, despite adverse effects of system nonlinearities, uncertainties and disturbances. *e)* Apply fundamentals associated with the use of control systems analysis and design software (e.g. MATLAB, Labview) with facility to aid in the analysis, design and simulation of control systems. All these are described according to ABET in next section. For the next courses, to which course is a prerequisite, the following "control course" based abilities and knowledge given by (*a* to *e*) are desired: Mechatronics fundamentals (*a, b, c*), Mechatronics systems design: (*a, b, c, d, e*), Process control (*a, b, c, d, e*), Embedded systems design (*a, b*), Robotics (*a, b, c*, d), PLC (*a, b*, c), CNC (*a, b, c*). [2, 6].

2.1.2. Course Prerequisites

The course is intended to be taken by students with a diverse mathematics background, [2, 8**]**. To gain the key abilities and knowledge, associated with mentioned CLOs, and based on institution, Department and minor requirements, the following courses could be general prerequisites: Differential equations, Laplace transformations, Linear Algebra, Mechanical Vibrations, engineering dynamics. For Mechatronics engineering students, the required prerequisites are: Differential equations, Mechanical Vibrations, Basic electrical circuits.

2.1.3. Course Outcomes (ABET*)

(a): Ability to apply the knowledge of mathematics, science, and engineering, (have the knowledge and the ability to apply intermediate and advanced mathematics; differential calculus, Laplace transformations, and linear algebra). (b): Ability to design and conduct experiments, as well as to analyze and interpret data, (able to identify the measurable parameters. able to identify different methods for measuring the phenomenon. able to identify the relationship between the phenomenon and the measured parameters. able to demonstrate general lab safety. able to follow experimental procedures for the experiment while maintaining all safety precautions. able to collect and record_data using appropriate units of measurements and identify the dependent and independent variables in the experiments. ability to analyze the data to generate the required parameters. ability to discuss the raw and derived data/graphs and assess the validity of the results. ability to relate how experimental result can be used to improve a process). (c): An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability, (able to identify the customer and the needs, able to identify and list the design objectives. able to identify the design constraints. able to define the design strategy and methodology. to identify the types of information needed for a complete understanding of all aspects of the project. able to define functional requirements for design. able to transform functional requirements into candidate solution concepts/ mathematical modelling, able to evaluate candidate solutions to arrive at feasible. able to develop final design specifications). (d): An ability to function on multidisciplinary teams. (e): An ability to identify (understand), formulate, and solve engineering problems (f): An understanding of professional and ethical responsibility (use-apply of handbooks, codes, and standards) in obtaining, reporting, analyzing data or in design). (g): An ability to communicate effectively, (able to: demonstrate knowledge and understanding of the subject. Able to: organize presentation in well structured logical sequence making it easy for audience to follow the content with clear understanding. Able to: stay within time limits). (h): The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.(i): Recognition of the need for, and the ability to engage in life-long learning (able to identify and take advantage of learning opportunities available on internet and elsewhere such as seminars, conferences, workshops, and tutorials. Able to independently acquire additional knowledge and data needed for solving the problem).(k): An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice, (able to solve problems using current software used in the discipline, such as Matlab) [9].

2.1.4. Recommended Textbooks and Other Course Materials/References

The following are recommended worldwide textbooks and references: Norman S. Nise, "Control Systems Engineering", 4th edition, Wily and sons, 2008. K. Ogata, "Modern Control Engineering", 5th edition Prentice Hall of India, New Delhi, 1995. Richard C. Dorf, Robert H Bishop, "Modern Control Systems", Prentice Hall, 10th edition, 2004. Farid Golnarghi, Benjamin Kuo, "Automatic, Control System", 9th edition, Wily and sons, 2010., Franklin, Powell and Emami Naieni "Feedback Control of Dynamic Systems", 4th Edition, Prentice Hall, 2002 / Gene Franklin, J. David Powell, Abbas Emami-Naeini, "Feedback Control of Dynamic Systems", Prentice Hall, NJ, 5th edition. The Student Edition of MATLAB, by The Math Works Inc., Prentice Hall, 1992.

2.1.5. Proposed Description

"Introduction to control theory and related concepts (Control, control systems, control algorithm, types of control, classification of control loops, components/role of control system, response, performance,…). Mathematical foundation: Review of related mathematical theories; (Differential equations, Complex-variable theory and Laplace & z-transforms). System representation; mathematical modelling represent physical systems (electrical, mechanical, hydraulic and pneumatic) using the following forms; differential equations, Block diagrams (and corresponding algebra), transfer function (poles, zeros, pole zero map), state space equation, and signal flow graphs (Mason's gain formula). Analysis and evaluation of system (*plant*) performance (transient and steady state response measures of I and II order system, dominant poles of higher order systems). Selection and design of control system/compensators in time domain, to meet and maintain desired overall system performance. Analysis and design in frequency domain. Analysis and design in state space domain". Control systems analysis and design software (e.g. MATLAB, Labview) with facility to aid in the analysis, design and simulation of control systems, (MATLAB built-in function for analysis and plotting systems' response, Introduce control system toolbox *sisotool* and *rltool* and corresponding design and analysis) [9].

2.1.6. Proposed Simple and Easy to Memorized and Follow Control System Design Teaching Approach,

To support educators in knowledge delivery, and help students in achieving CLOs abilities and knowledge and solve control problems, the course topics and CLOs/outcomes are organized and integrated in simple and easy to memorized and follow steps, to select and design a control system to meet and maintain a desired performance. These steps are shown in Figure 2. Depending on institution, Department and minor, educator's back ground, these steps are given in different forms/details, shown in Figures 2a, b, c.

To evaluate concepts and gain required associated integrated abilities and knowledge desired for further courses, the description and the teaching approach, are supported with tables (1c: 7) and graphs, that are recommended to provide students with, and ask to bring on every lecture, where: In Table 1, the proposed course description and topics is explained in details, in particular, main topics mapped with their specific subtitles, objectives, number of lectures and weeks. In Table 2: Some basic rules of block diagram algebra. In Table 3a,b first and second order systems modelling, response measures and general forms of transfer function. In Table 3c the nature of second order system poles (roots) and the effect of changing damping and undamped natural frequency on systems response. In Table 4: the steady state error dependence on input signal and system type. In table 5 Control systems/algorithms transfer function, actions, selection criteria and root locus sketching rules. Table 6 analysis and design in Frequency domain [12]. Table 7 analysis and design- the modern State space (variable) approach. [10].

2.1.7. Recommend Course Administration

The course is taught in 14/15 weeks, with two (I and II) midterm exams, 3 Lab session (to convey concepts when possible, along with simulations and interactive MATLAB sessions), Course Project, Homework sets, and a final exam.

2.1.8. Class Schedule

4 Credits hours, (4: 2, 1, 1): 100-minutes lecture per week, 100-minutes tutorial per week, and 150-minutes laboratory hours every three weeks:

Table 1a. Class schedule.

Activity Name	Hours per Week	Sessions per Week	Weeks per Semester
Lecture	2	1	14
Tutorial	2	1	14
Labs	3/every three weeks	1/every *three* weeks	3

2.1.9. Recommended Grading System

Table 1b. Recommended grading system.

Class performance (Atten., particip., assignments)	10%
Labs (3)	15%
Quizzes/ Tests (3-5)	10%
First Exam I	10%
Second exam II	10%
Project; Written/Oral; (Report + Presentation)	15%
Final	30%
Total	100%

2.1.10. Pre-course

A special *pre-course* recommended to be offered in the week before the course begins. This pre-course gives a concise introduction to four main topics: linear algebra, ordinary differential equations, complex theory and dynamical systems**.**

III. TEACHING PLAN AND TOPICS EXPLAINED

Table 1c. Teaching plan and explained topics of the course.

Topics to be covered
1) Introduction to control theory and related concepts. (T1:1, 1, 2): First Week, 1 Lecture, 2 Hours. *a)* Course overview: first day materials; describe course structure, objectives, administration,. *b)* Definition of main control concepts and terminologies; Control, control system, Controller, control algorithm, control system components; Sensor, Actuator, Plant, Process. Input, output, disturbance, test input signals, response and plots (transient & steady state), performance, steady state error, performance evaluation, Control low, Design, Analysis, Control history. *c)* Advantages of control systems and application examples. *d)* Definition and classifications/types of each of: Control (automatic and manual), Processes (SISO, MIMO), Control systems (Discrete (ON/Off), Multistep, Continuous (P-, PI, PD, PID, Lead, Lag…)), control loops (A single variable control loop (Feedback, Feedforward), Multivariable Control loop (Feedback plus Feedforward, cascade control, ratio control)). *e)* Introduce (proposed) steps for control system selection and design Figure 2a, b, c. *f)* Introduce role of control systems analysis and design software (e.g. MATLAB, Labview) to aid in the analysis, design and simulation of control systems.
2) Mathematical foundation: Review of related mathematical theories; differential equations, Complex-variable theory and Laplace transform. (T2:1, 1, 2): First week, 1 lecture, 2 hours. *a)* Ordinary differential equations (First and Second order HODE, solving/ plotting solution, relating differential equation terminologies with control system terminologies (e.g. solution/response, particular integrate/transient response, forced function/steady state response, characteristic equation …). *b)* Complex variables (complex plane, complex conjugate, phase, magnitude, Complex Arithmetic. *c)* Laplace transform and elements of the Laplace transform (Laplace table)
3) System representation-mathematical modelling: represent physical systems (electrical, mechanical, hydraulic and pneumatic) using differential equations, Block diagrams, transfer function, state space equation, and signal flow graphs. (T3:2-4, 5, 10): II by IV Week, 5 Lectures, 10 Hours. *a)* Introducing I & II order systems, why in control engineering, we most interested in study of such systems?. *b)* Modeling basics and definition of concepts. Forms of mathematical models;(differential equations, state space equations, transfer function, block diagrams...). Developing mathematical model in the form of differential equations for mechanical systems (translational, rotational, and combination, mechanical elements; Spring, Mass, Damper), Electric systems (circuits & elements; resistor, capacitor, inductor RC, RLC circuits), electromechanical (DC motor), hydraulic and pneumatic systems, including; First order systems (car spring-damper suspension system, car cruise control, tank level control, pressure control, RC circuit), and Second order systems (Two tank system, Spring-mass-damper…), higher order system (e.g. DC motor, Two-degrees-of freedom system), analogies. *c)* Linearization of nonlinear systems. *d)* Representing system using state equations. Concepts, definition and equations development. *e)* Representing system using Block diagram and Block diagram algebra; block diagrams reduction techniques (Table 2), Representing system using signal flow graphs, Mason's gain formula. *f)* Representing system using transfer function: forward, open loop, closed loop, overall transfer function, Poles, Zeros, Pole-Zero map. (Most of proposed is recommended to be taught in parallel, e.g. derive mathematical model of a given system in the form of differential equation, (and/or write state equations), apply Laplace transform, develop transfer function, find Poles, Zeros, and plot pole-zero map.
4) Analysis and evaluation of (basic) system (plant) performance; Stability and (transient and steady state) response analysis of first and second order systems, dominant poles of high order systems. (T4:4-7, 6, 12): IV by VII Week, 6 Lectures, 12 Hours. *a)* Introducing design steps depicted in Figure 2 *b)* Introduce the three predominant objectives of systems' performance analysis and design with corresponding concepts and definitions; *a)* Stability analysis: Ensure stability and the degree or extent of system stability, *b)* Transient performance analysis; calculate transient performance specification (measures): T, 5T, T_R, T_P, T_S, M_P, OS%..., *c)* Steady-state performance analysis; (Performance accuracy) Calculate steady state error, test input signals. *c)* Stability analysis (absolute and relative) in time and state space domains. Routh-Hurwitz stability criterion: relative stability analysis: parameters change, to determine system parameters (and ranges) to yield stability, critical stability and instability), design using Routh-Hurwitz stability criterion. *d)* Test input signals, definition and application.; pulse, impulse, step, ramp, parabolic, sinusoidal. *e)* Response analysis of I order system: concepts, definition and classification (transient response and steady state response). Response specifications/Measures, Introducing Table 3a, *and for/or* the next below topics: *f)* Response analysis of I order systems (Introducing Table 3a): Time constant is the only parameter needed to evaluate I order system performance. General form of I order systems without zeros, in terms of differential equation, transfer function, and both in terms of time constant. Forms of I order system response (natural growth/decay). I order system response measures (T, T_r, T_s, DC gain, Ess). The effect of changing (increasing/decreasing) time constant (pole location on complex plane) upon system response curve (speeds-up/slows response….). Application examples of I order systems (Car spring-damper suspension system, Car cruise control, level control, pressure control, RC circuit....). *g)* Response analysis of II order system (Introducing Table 3b, c): The two parameters needed to evaluate II order system performance (damping ratio and undamped natural frequency). General form of II order systems without zeros, in terms of differential equation, transfer function, and in terms of these two parameters. (Table 3b, c) The four forms of stable II order system responses (Undamped, underdamped, critically damped, overdamped). The relations between damping ratio (ξ), Poles (roots) nature, location and response form, properties and time solution for performance/error *prediction*. II order system response measures in term of damping ratio and undamped natural frequency (T, ζ, ω_n, ω_d, 5T, T_R, T_P, T_S, M_P,%OS,..). The effect of changing (increasing/decreasing) time constant or damping ratio and/or undamped natural frequency upon system response curve (speed up/slow response, increase/decrease overshoot…..). Calculating damping ratio and undamped natural frequency from plant's parameters, and from poles location on complex plane (phase and magnitude of complex pole). Damping ratio line and overshoot,

magnitude of complex pole-circle and undamped natural frequency. Application examples of II order systems (Spring-mass-damper, two tank system, RLC circuit…).

h) Response analysis of higher than second order systems: Dominant poles and systems approximation, finding dominant pole(s), DC motor as application example (speed and position control), dominant poles in terms of damping ratio and undamped natural frequency.

i) Writing/Finding transfers function and/or plant parameters from response curve or from transfer function. Convert between differential equation/transfer function / state-space models. Finding time response from the state-space representation.

j) The effects of nonlinearities on the system time response. I and II order systems with zeros.

k) Response analysis; Steady state error (introducing Table 4): measure of system accuracy, relation between input signal *R(s)*, system type and steady-state error for performance/error prediction. Steady-state error in terms of open loop and closed loop transfer functions. Static error constants. Finding the steady-state error for a unity and non unity feedback system. Finding the steady-state error for systems represented in state-space.

l) Selective design: (e.g. achieving desired response without adding control system), to select (I or II order) system's parameters to result in desired response specification, done by reverse solving system's parameters in terms of desired response specification.

m) Analysis and evaluation of system performance using analysis and design software (e.g. MATLAB, Labview) to aid in the analysis, design and simulation of control systems (MATLAB built-in functions for analysis and plotting of systems' response subjected to an input signal. Introducing control system toolbox *sisotool* and *rltool*).

Lab (1): Using MATLAB/Simulink: Test input signal. Response analysis and evaluation of I and II order systems' (Spring-Mass-Damper, Car system, RLC circuit). Effect of changing systems' parameters upon response. Response measures.
(T4-L1:7, 1, 3): VII Week, 1 Session, 3 Hours.

Review: (T1-7, 0.5, 1): VIII Week, 0.5 Lecture, 1 Hour.
First exam: (1 hour): Covering all previous (up) topics
(T1:5:7, 0.5, 1): VIII week, 0.5 Lecture, 1 Hour.

5) Selection and design of control system in time domain, to meet and maintain desired overall system performance.
(T5:7-10, 5, 10): VIII by X Week, 5 Lectures, 10 Hours.

a) Introducing Table 5: Review: Definition and classifications/types of control systems; (Discrete (ON/Off), Multistep, Continuous (P-, PI, PD, PID, Lead, lag…)), control loops (A single variable control loop (Feedback, Feedforward), Multivariable Control loop (Feedback plus Feedforward, cascade control, ratio control)), with emphasize on Continuous feedback (closed loop) control systems.

b) Types (modes), mathematical model, and transfer function of control systems/compensators; P-, PI, PD, PID, Lead, lag and leadlag. Controllers' gain definition. Effect of each controller mode on system's response (transient and steady state). Equivalency of each controller mode in terms of pole/zero addition to plant's transfer function. Controller selection Criteria. Implementing controllers using passive components (amplifier-resistor-capacitor) and concept of gains.

c) Introductory review to control system selection and design: (Calculating damping ratio and undamped natural frequency from poles location on complex plane (phase and magnitude of complex pole. Damping ratio line and overshoot, magnitude of complex pole-circle and undamped natural frequency) and corresponding response measures). The design *region* in the complex plane where a pair of second-order poles must be *located* to satisfy specification.

d) Controller design in time domain: Selective design: Review (achieving desired response without adding control system). Comparison for design: comparing a given form of systems closed-loop transfer function (e.g. with P-controller) with the corresponding "standard" form of I or II -order systems, and by comparison to calculate the controller's gain(s).

e) Control system design via root locus; definition, power of root locus (provide solutions for stability and response analysis and design for systems of order higher than second order, analysis and design for stability; ranges for stability, instability and break into *oscillation*). Sketching rules. Control systems/compensators (P-, PI, PD, PID, Lead, lag and leadlag) design via root locus; applying controller pole/zero addition, angle criterion, magnitude criterion, design verification.

f) MATLAB builtin function for control system design and analysis e.g. *tf()*, *rlocus*. Applying control system toolbox (*sisotool* and *rltool*) to select and design a control system (P-, PI, PD, PID, Lead, lag and leadlag) to achieve desired performance.

Project assignment, (Report + oral presentation):
Given and explained in: week No. X.
So as students can apply, gained abilities and knowledge in solving control system/algorithm selection and design as a stage of Mechatronics system design, the course, as many other, Mechatronics courses, is project-based and include a project on the selection and design of control system/algorithm to control a given Mechatronic device based on/to meet given objectives and design specifications. *Recommended*: Given a dynamic system e.g. DC motor based motion control application (robot arm, mobile robot, suntracker, Automated conveyer system, electric car) with defined parameters, and desired performance specification (or student can select desired actual system response specifications). Each student is to apply all selection, design and analysis steps to design a control system to meet desired performance, and using MATLAB), and verify design.

Second exam: (T1-4:10, 1, 0.5): X week, 0.5 lecture, 1 hours.
Covering selection and design of control system in time domain, to meet and maintain a desired performance.

Lab (2): (T4L2:11, 1, 3): XI Week, 1 Session, 3 Hours
Identify closed loop systems components and their role. System representation. Data collecting, response plotting and interpreting: Analysis of open and closed loop response, and/or observing effects of plant's parameters (and/or T, ζ, ω_n) change on resulting step/ramp/parabolic response of I and II order systems, and/or observing effects of PID-modes and parameters-gains change on system response. Selection and design of control system to meet desired performance: e.g. DC motor based motion control (speed/position); Robot arm, Ball and beam system.

6) Controller design in frequency domain:
(T6:11-12, 4, 8): XI by XII week, 4 lectures, 8 hours.

a) Introducing Table 6.

b) Definition of main concepts; History, Frequency transfer function, advantages and applications of the frequency response techniques.

c) The three graphical representations tools of the frequency response representation (Bode diagrams, Nyquist diagrams, and Nichols charts).

d) Performance and stability analysis in frequency domain; gain margin (GM), phase margin (PM), Nyquist stability criterion, Bode stability criterion, frequency-domain performance specifications;(M_r, ω_r, ω_p, ω_g, ω_b, BW, E_{ss}). Using Bode and Nyquist plots to determine closed loop stability and performance.

e) Controller selection and design in frequency domain: Design Lead/Lag compensators to achieve closed loop bandwidth and

stability margins
7) Introduction to analysis and design in state-space: (T7:13, 2, 4): XIII week, 2 lectures, 4 hours. *a)* Introducing Table 7, *b)* The general state-space representation, *c)* Stability analysis in state space, *d)* Steady-state error for systems in state space, *e)* Controller design in state space (pole placement)
Lab (3): frequency response (T4L3:14, 1, 3): XIV Week, 1 session, 3 Hours.
Course Project defense: (Report + Oral) (T1-7:14, 1, 2): XIV Week, 1 Lecture, 2 Hours.
Course Review of; objectives / gained abilities and knowledge, Selection and design examples, relation to other subjects. (T1-7:14, 1, 2): XIV week, 1 lecture, 2 hours.
TOTAL: 7 Topics, 15 weeks, 28 lectures, 2 lectures for Midterms +quizzes, 60 hours, 3 Lab sessions 9 hours.

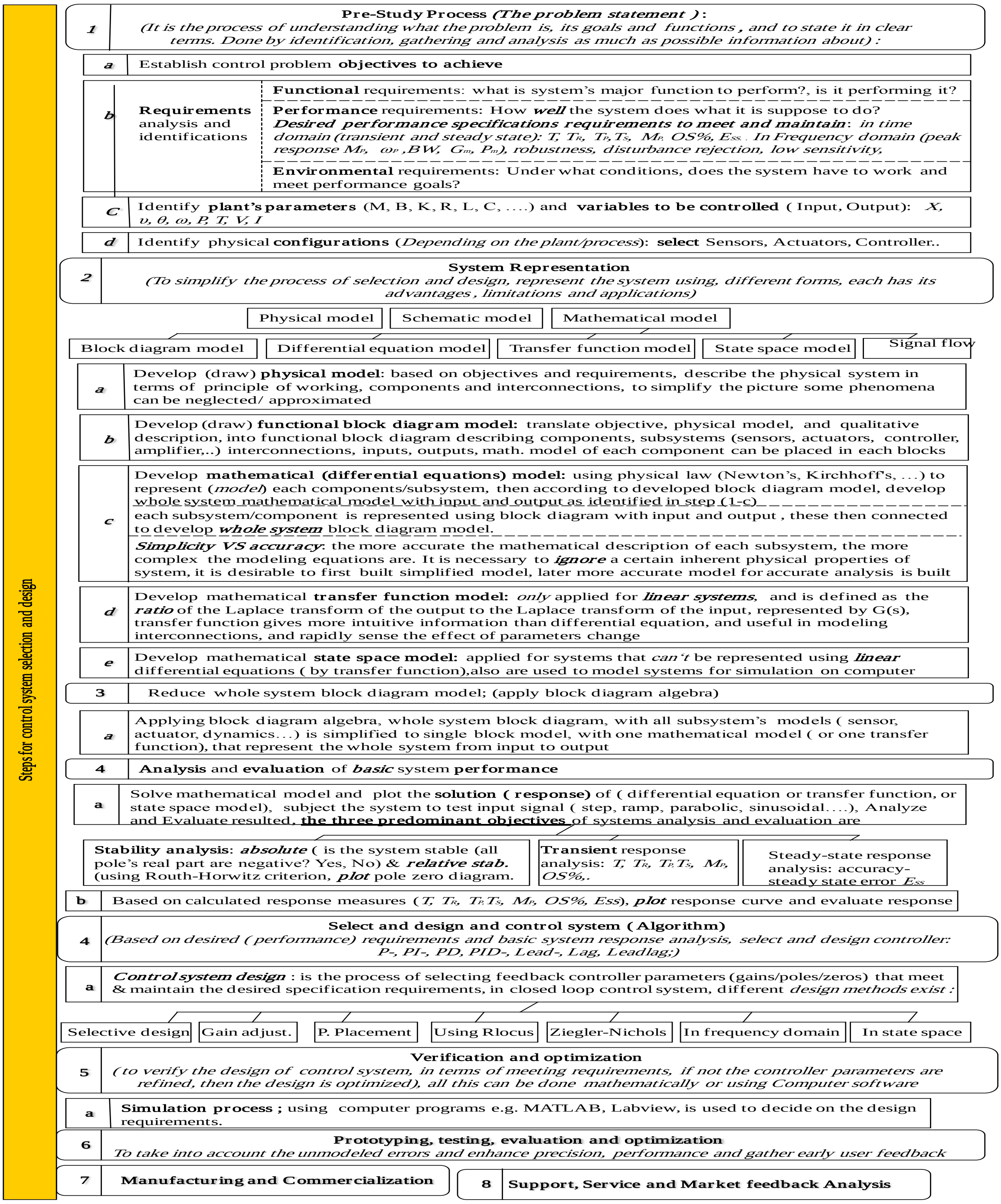


Figure 2a. Course proposed design steps, for selection and design of control systems.

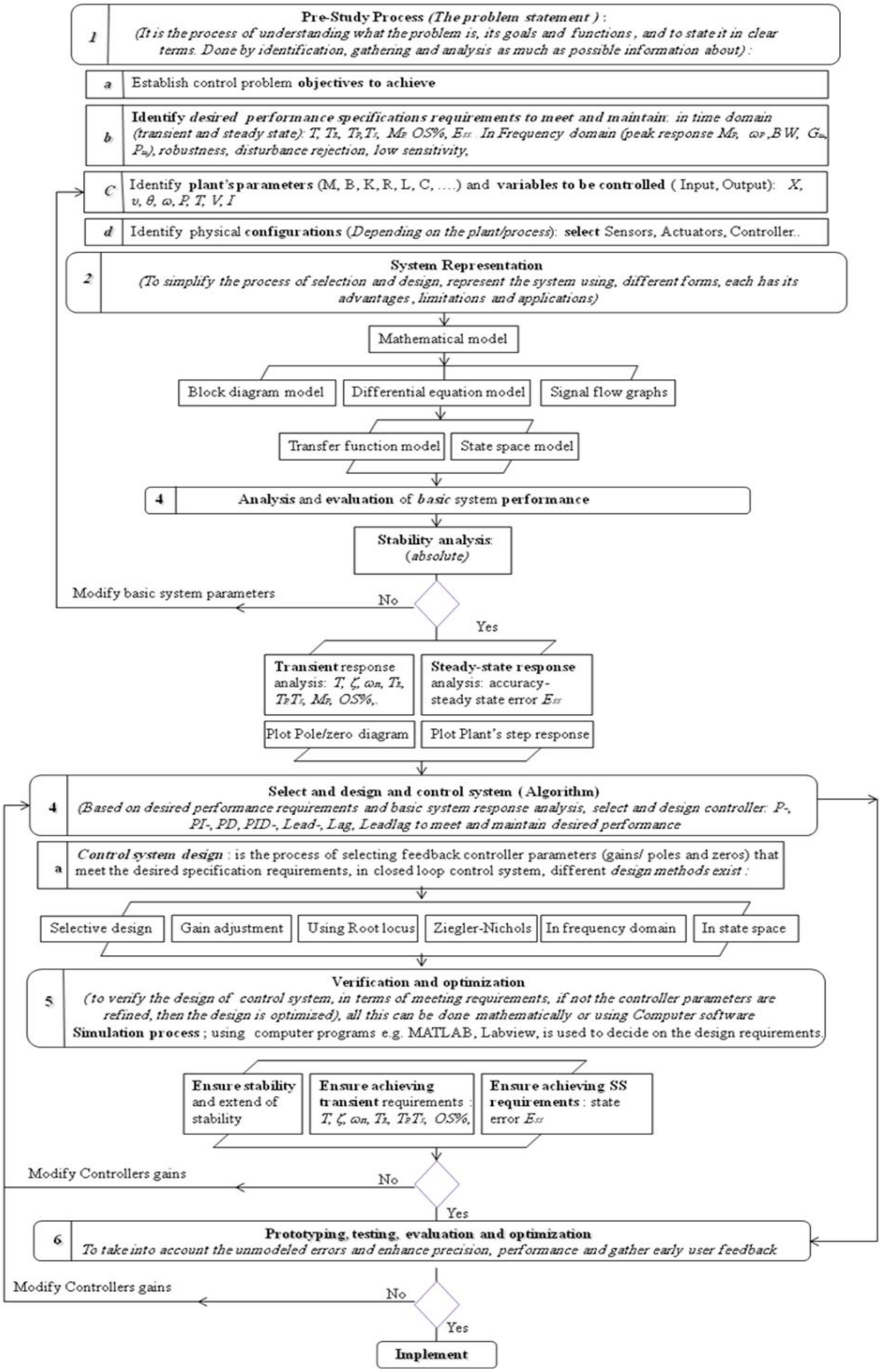


Figure 2b. Proposed design steps, for selection and design of control systems.

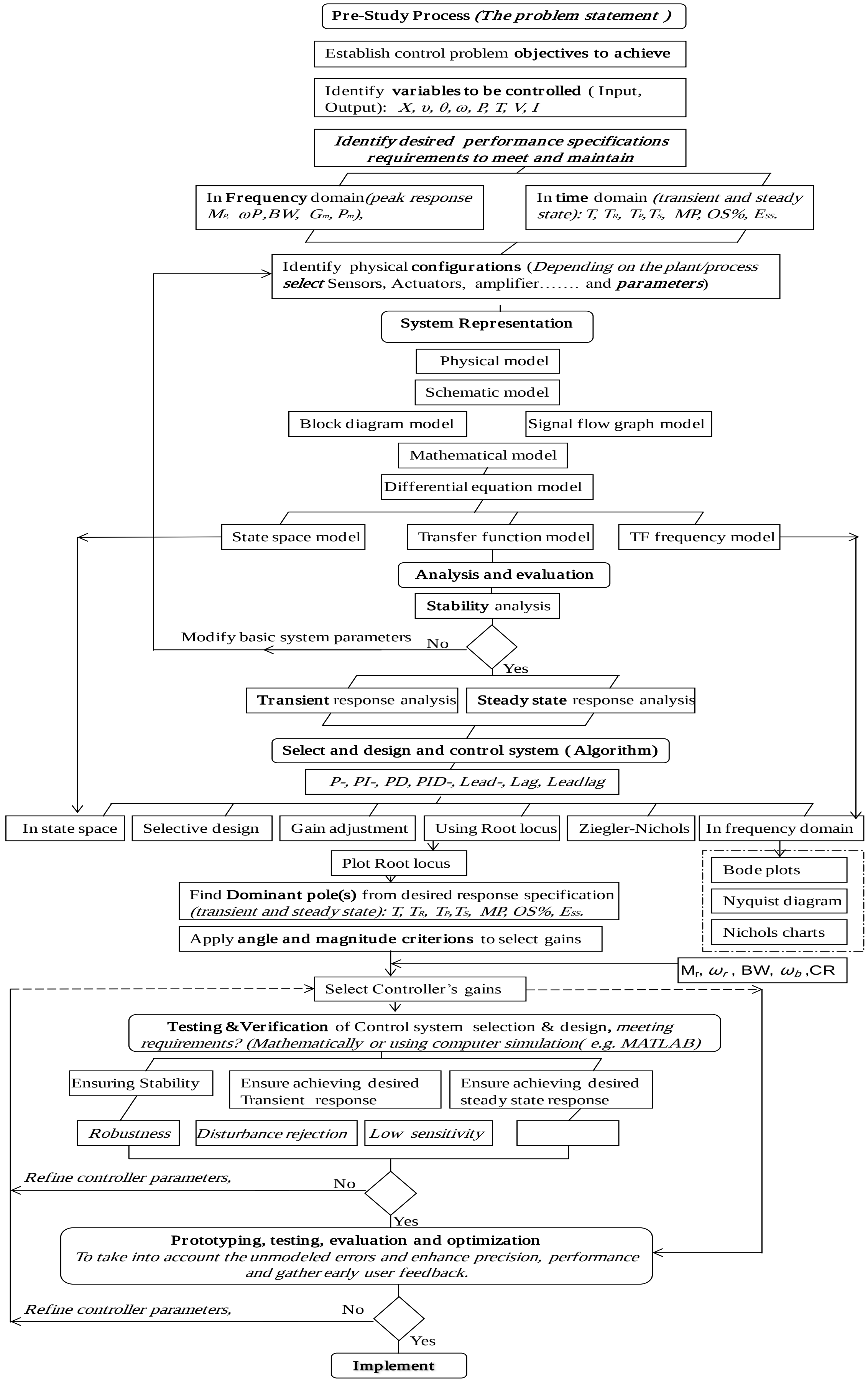


Figure 2c. Flowchart; proposed design steps, for selection and design of control systems.

Table 2. Some basic rules of block diagram algebra [3].

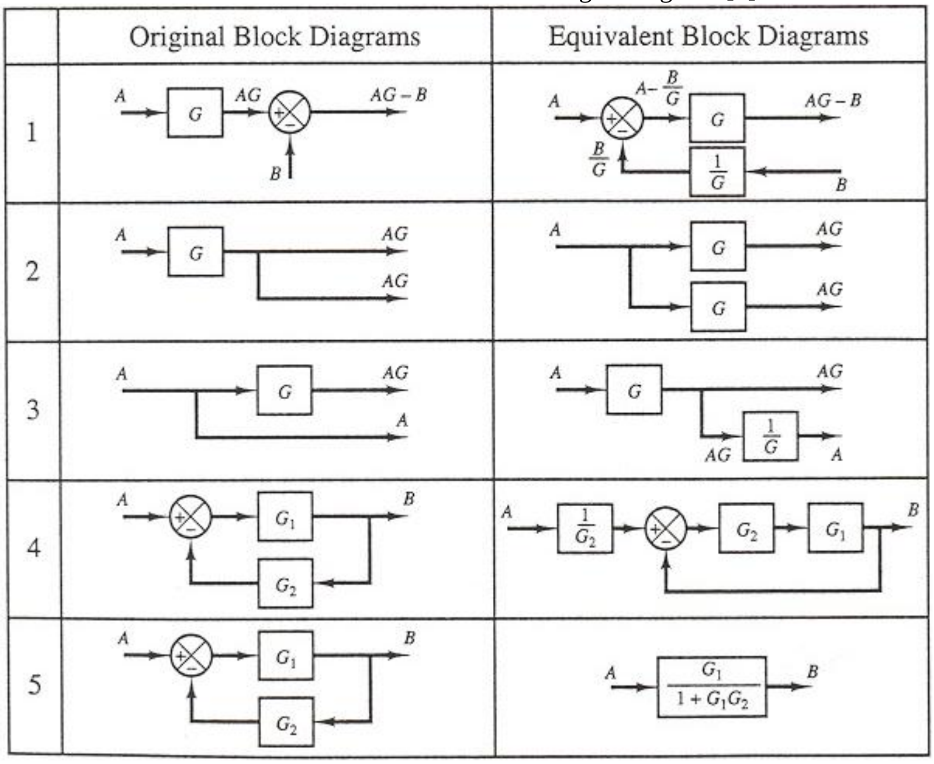
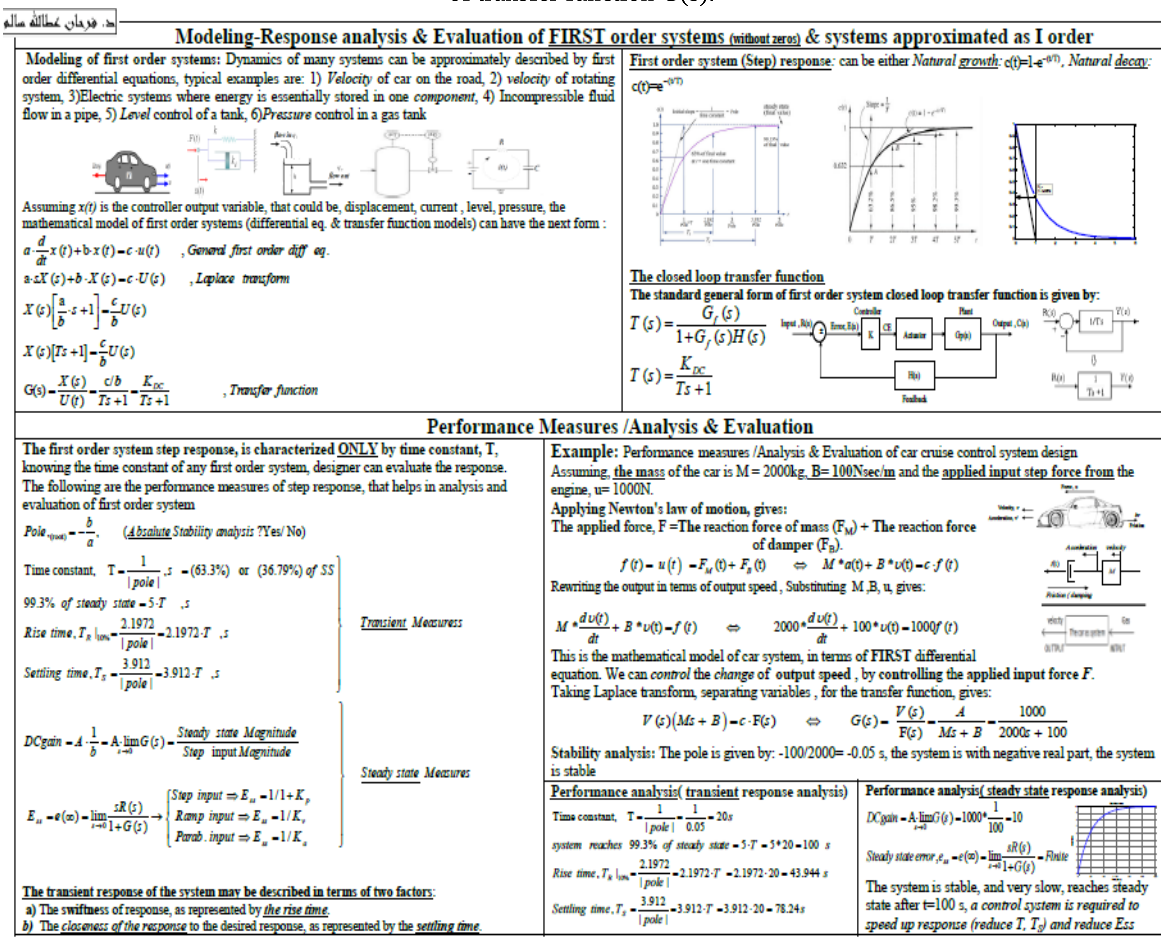


Table 3a. I order systems modelling, response analysis, Performance measures, control loop, and general forms of transfer function G(s).


د. فرحان عطاالله سالم

Modeling-Response analysis & Evaluation of FIRST order systems (without zeros) & systems approximated as I order

Modeling of first order systems: Dynamics of many systems can be approximately described by first order differential equations, typical examples are: 1) *Velocity* of car on the road, 2) *velocity* of rotating system, 3)Electric systems where energy is essentially stored in one *component*, 4) Incompressible fluid flow in a pipe, 5) *Level* control of a tank, 6)*Pressure* control in a gas tank

Assuming *x(t)* is the controller output variable, that could be, displacement, current , level, pressure, the mathematical model of first order systems (differential eq. & transfer function models) can have the next form :

$a\cdot\frac{d}{dt}x(t)+b\cdot x(t)=c\cdot u(t)$, *General first order diff eq.*

$a\cdot sX(s)+b\cdot X(s)=c\cdot U(s)$, *Laplace transform*

$X(s)\left[\frac{a}{b}\cdot s+1\right]=\frac{c}{b}U(s)$

$X(s)[Ts+1]=\frac{c}{b}U(s)$

$G(s)=\frac{X(s)}{U(t)}=\frac{c/b}{Ts+1}=\frac{K_{DC}}{Ts+1}$, *Transfer function*

First order system (Step) response: can be either *Natural growth*: $c(t)=1-e^{-(t/T)}$, *Natural decay*: $c(t)=e^{-(t/T)}$

The closed loop transfer function

The standard general form of first order system closed loop transfer function is given by:

$T(s)=\frac{G_f(s)}{1+G_f(s)H(s)}$

$T(s)=\frac{K_{DC}}{Ts+1}$

Performance Measures /Analysis & Evaluation

The first order system step response, is characterized ONLY by time constant, T, knowing the time constant of any first order system, designer can evaluate the response. The following are the performance measures of step response, that helps in analysis and evaluation of first order system

$Pole_{(root)}=-\frac{b}{a}$, (*Absalute Stability analysis* ?Yes/ No)

Time constant, $T=\frac{1}{|pole|}$, s $=(63.3\%)$ or (36.79%) *of SS*

99.3% *of steady state* $=5\cdot T$, s

Rise time, $T_R|_{10\%}=\frac{2.1972}{|pole|}=2.1972\cdot T$, s

Settling time, $T_S=\frac{3.912}{|pole|}=3.912\cdot T$, s

Transient Measuress

$DCgain=A\cdot\frac{1}{b}=A\cdot\lim_{s\to 0}G(s)=\frac{Steady\ state\ Magnitude}{Step\ input\ Magnitude}$

$E_{ss}=e(\infty)=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}\to\begin{cases}Step\ input\Rightarrow E_{ss}=1/1+K_p\\ Ramp\ input\Rightarrow E_{ss}=1/K_v\\ Parab.\ input\Rightarrow E_{ss}=1/K_a\end{cases}$

Steady state Measures

The transient response of the system may be described in terms of two factors:

a) The swiftness of response, as represented by *the rise time*.

b) The *closeness of the response* to the desired response, as represented by the *settling time*.

Example: Performance measures /Analysis & Evaluation of car cruise control system design

Assuming, the mass of the car is M = 2000kg, B= 100Nsec/m and the applied input step force from the engine, u= 1000N.

Applying Newton's law of motion, gives:

The applied force, F =The reaction force of mass (F_M) + The reaction force of damper (F_B).

$f(t)=u(t)=F_M(t)+F_B(t)\quad\Leftrightarrow\quad M*a(t)+B*\upsilon(t)=c\cdot f(t)$

Rewriting the output in terms of output speed , Substituting M ,B, u, gives:

$M*\frac{d\upsilon(t)}{dt}+B*\upsilon(t)=f(t)\quad\Leftrightarrow\quad 2000*\frac{d\upsilon(t)}{dt}+100*\upsilon(t)=1000f(t)$

This is the mathematical model of car system, in terms of FIRST differential equation. We can *control* the *change* of output speed , by controlling the applied input force *F*.

Taking Laplace transform, separating variables , for the transfer function, gives:

$V(s)(Ms+B)=c\cdot F(s)\quad\Leftrightarrow\quad G(s)=\frac{V(s)}{F(s)}=\frac{A}{Ms+B}=\frac{1000}{2000s+100}$

Stability analysis: The pole is given by: -100/2000= -0.05 s, the system is with negative real part, the system is stable

Performance analysis(transient response analysis)

Time constant, $T=\frac{1}{|pole|}=\frac{1}{0.05}=20s$

system reaches 99.3% *of steady state* $=5\cdot T=5*20=100$ s

Rise time, $T_R|_{10\%}=\frac{2.1972}{|pole|}=2.1972\cdot T=2.1972\cdot 20=43.944$ s

Settling time, $T_S=\frac{3.912}{|pole|}=3.912\cdot T=3.912\cdot 20=78.24s$

Performance analysis(steady state response analysis)

$DCgain=A\cdot\lim_{s\to 0}G(s)=1000*\frac{1}{100}=10$

Steady state error, $e_{ss}=e(\infty)=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}=$ *Finite*

The system is stable, and very slow, reaches steady state after t=100 s, *a control system is required to speed up response (reduce T, T_S) and reduce Ess*

Table 3b. II order systems modelling, response analysis, Performance measures and general forms of transfer function G(s).

د. فرحان عطاالله سالم

Modeling-Response analysis & Evaluation of SECOND order systems (without zeros) & systems approximated as II order

Modeling of second order systems: Dynamics of many systems can be approximately described by second order differential equations, typical examples are: 1) *Position* of car on the road, 2) *Motion* control systems, 3) Stabilization of satellites, 4) Electric systems where energy is stored in two elements, 5) Levels in two connected tanks, 6) Pressure in two connected vessels.

$$a\cdot\frac{d^2}{dt^2}x(t)+b\cdot\frac{d}{dt}x(t)+c\cdot x(t)=d\cdot u(t)\quad \text{, General second order diff eq}$$

$$a\cdot s^2X(s)+b\cdot sX(s)+c\cdot X(s)=d\cdot U(s)\quad \text{, Laplace transform}$$

$$X(s)\left[as^2+bs+c\right]=d\cdot U(s)$$

$$G(s)=\frac{X(s)}{U(s)}=\frac{d}{as^2+bs+c}=\frac{(d/a)}{s^2+(b/a)s+(c/a)}$$

Performance Measures /Analysis & Evaluation

The second order system step response, is characterized by damping ratio and undamped natural frequency. knowing both, designer can analyze and evaluate the response of any II order system ***without*** zeros or approximated as such. (the performance measures of ***step*** response of second order system are given next):

Absolute Stability analysis ?Yes/ No? : $Poles_{(roots)}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\xi\omega_n\pm j\,\omega_n\sqrt{1-\xi^2}$

For all systems & WITH Zeros, From system parameters, $\Rightarrow$ **We find:** $\begin{cases}\xi=\frac{b}{2\cdot\sqrt{a\cdot c}}\\ \omega_n=\sqrt{\frac{c}{a}}\ Rad/sec\end{cases}$, $\omega_d=\omega_n\sqrt{1-\xi^2}$

Performance measures for Underdamped response $(0<\xi<1)$*, ONLY for II order systems WITHOUT zeros, :*

Transient measures:

- Time constant, $T=\frac{1}{\xi\omega_n}$, s
- 99.3% *of steady state* $=5\cdot T$, s
- *Rise time*, $T_R|_{90\%}=\frac{2.16\xi+0.6}{\omega_n}$ s, $(0<\xi<0.8)$
- Peak *time*, $T_P=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}=\frac{\pi}{\omega_d}$ s, $(\zeta<0.7)$

Settling time, $T_S|_{2\%}=\frac{4}{\xi\omega_n}=4T$, s

Max. overshoot, $M_P=c_{max}(t)-c_{ss}(\infty)=e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$

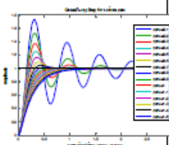
Percent overshoot, $OS\%=MP\cdot 100=\frac{c_{max}(t)-c_{ss}(\infty)}{c_{ss}(\infty)}\cdot 100$

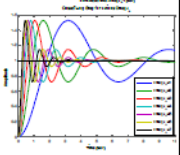
Finding ξ *from* given $OS\%$, $\xi=\frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$

Minima Time, $T_M=\frac{2\pi}{\omega_n\sqrt{1-\xi^2}}=\frac{2\pi}{\omega_d}$

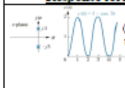
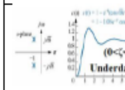
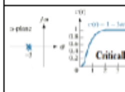
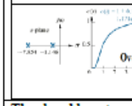
The decay ratio $D_R=e^{-2\xi\pi/\sqrt{1-\xi^2}}$,

$$E_{ss}=e(\infty)=\lim_{s\to 0}\frac{sR(s)}{1+G(s)}\begin{cases}Step\ input\Rightarrow E_{ss}=1/1+K_p\\ Ramp\ input\Rightarrow E_{ss}=1/K_v\\ Parab.\ input\Rightarrow E_{ss}=1/K_a\end{cases}$$





Second order system (Step) response *of stable system can have any of the* ***FOUR*** *response forms, shown in table below.* This table shows **The relations between Damping ratio** (ξ), **Poles (roots) location** and **response Form, and Properties.**

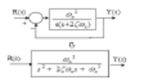
Response form	Example	Type (nature) of roots	Properties of response
 (ζ=0) Undamped	G(s): $R(s)=\frac{1}{s}$ → $\frac{9}{s^2+9}$ → C(s); Undamped; $y''(t)+4y(t)=1$; $s^2+4s=1$	**Complex-conjugate imaginary poles**, *with zero real part, (lies on the imaginary axis)*, given by: $P_{1,2}=\pm j\,\omega_n$	The transient response is ***simple harmonic undamped-sustained oscillations motion***, *theoretically never damps out, (it is a sine wave of constant amplitude).*
 (0<ζ<1) **Underdamped**	G(s): $R(s)=\frac{1}{s}$ → $\frac{9}{s^2+2s+9}$ → C(s); Underdamped; $y''(t)+2y'(t)+4y(t)=1$; $s^2+2s+4=1$	***Complex-conjugate*** poles, *with negative real part*, given by: $P_{1,2}=-\xi\omega_n\pm j\,\omega_n\sqrt{1-\xi^2}$	The transient response ***is a decaying exponential combined with an oscillatory****, damped sinusoid* (**with overshoot that finally damps-out**)
 (ζ=1) Critically damped	G(s): $R(s)=\frac{1}{s}$ → $\frac{9}{s^2+6s+9}$ → C(s); Critically damped; $y''(t)+4y'(t)+4y(t)=1$; $s^2+6s+9=9$	***Real repeated*** *(multiple)* ***negative*** poles, given by: $P_{1,2}=-\xi\,\omega_n$	The transient response is ***without overshoot***, it is related ***to critical points*** **boundary** of *underdamped* and *overdamped* responses
 (ξ>1) Overdamped	G(s): $R(s)=\frac{1}{s}$ → $\frac{9}{s^2+9s+9}$ → C(s); Overdamped; $y''(t)+6y'(t)+4y(t)=1$; $s^2+6s+9=9$	**Real *distinct negative*** roots and given by: $s_{1,2}=-\xi\omega_n\pm j\,\omega_n\sqrt{1-\xi^2}$	The transient response is ***Sluggish*** (بطيء) ***response***; it is a ***decaying exponential without overshoot*** **and** ***without oscillation.***

The closed loop transfer function

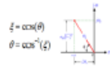
The standard general form of second order system closed loop transfer function is given below Using this closed loop transfer function (WITHOUT zeros), we can Find *closed loop* ζ **and** ω_n *, also design control systems to meet* ***desired*** specifications

$$T(s)=\frac{G_f(s)}{1+G_f(s)H(s)}$$

$$T(s)=\frac{\omega_n^2}{s^2+(2\xi\omega_n)s+\omega_n^2}$$



System approximation: Done by locating pole(s) nearest to *imaginary* axis ***or*** from desired *specification (OS%, T, Ts) find desired* ζ and ω_n or T *and substitute in standard closed* transfer function



Second order systems **Dominant poles: are given by** $P_{1,2}=-\xi\omega_n\pm j\,\omega_n\sqrt{1-\xi^2}$

First order systems Dominant pole is given by: $P=1/T$

***Ess*, *input R(s)* & System type:**

Step Ess	$Ess\|_{step}=\frac{1}{1+K_p}$	$K_p=1+\lim_{s\to 0}G(s)$
Ramp Ess	$Ess\|_{ramp}=\frac{1}{K_v}$	$K_v=\lim_{s\to 0}sG(s)$
Parabolic Ess	$Ess\|_{parabolic}=\frac{1}{K_a}$	$K_a=\lim_{s\to 0}s^2G(s)$

R(t) / Type	Step	Ramp	Para.
0	F	∞	∞
1	0	F	∞
2	0	0	F

Processes with time delay (e^{-Ls})**, FOPDT/SOPDT :**

$$G(s)=\frac{K_{DC}}{Ts+1}e^{-Ls}$$

$$G(s)=\frac{\omega_n^2}{s^2+(2\xi\omega_n)s+\omega_n^2}e^{-Ls}$$

Table 3c. The nature of second order system poles (roots) and the effect of changing damping and undamped natural frequency on systems response.

Table : The ***natures of poles (roots)*** **of the characteristic equation, their Discriminant, (*conditions*), general solution and corresponding response**

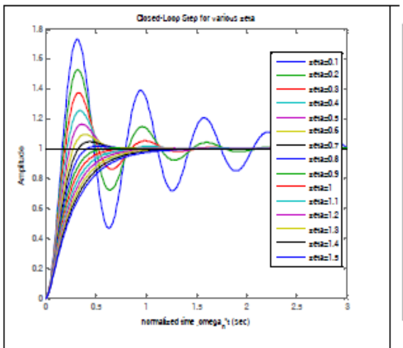
The *characteristic equation* of II order system has the general form given by Eq.(1). Poles *(Roots)* can be found by *factoring* or by the use of the *quadratic formula* that is given by Eq.(2): Where :*a, b,* and *c* are the plant's parameters (e.g. M, K, B, R, L, C, ..) . The part (b^2- 4*ac*) is called the *discriminant*, Δ= b^2- 4ac

$$A(s)=as^2+bs+c \qquad (1) \qquad\qquad P_1,P_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{\Delta}}{2a} \qquad (2)$$

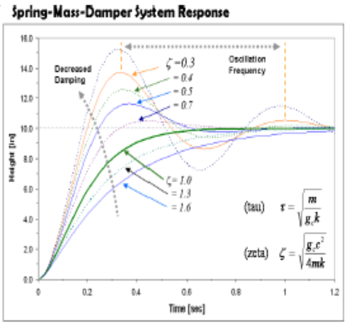
Nature of poles (roots)	Discriminant	General Solution	Response types
Real and distinct roots, P_1, P_2	$b^2-4ac>0$	$y=Ae^{P_1t}+Be^{P_2t}$	1 < ξ , Overdamped
Real and *equal* roots, $P_1=P_2$	$b^2-4ac=0$	$y=e^{Pt}(A+Bt)$	ξ=1, Critically damped
Complex *conjugate* roots $P_{1,2}=\alpha\pm j\omega$	$b^2-4ac<0$	$y=e^{\alpha t}(A\cos\omega t+B\sin\omega t)$	0 < ξ < 1 Underdamped

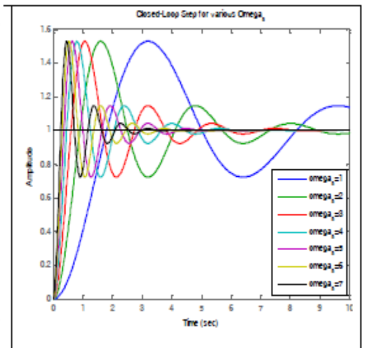
Table : Effect of increasing/Decreasing damping ratio and undamped natural frequency upon response

1) By **increasing** the damping ratio, *from zero toward one*: The response become *more decaying* exponential (*less* overshoot), combined *with an oscillatory*.
2) By ***more*** **increasing** the damping ratio, *toward one (and more than one)* The response become with less overshoot up to no overshoot



Step responses for *various* 0≤ ζ ≤1.5 with const. ω_n=10.





Step response for *various* ω_n with constant ζ =0.2.

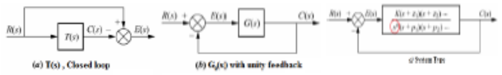
Table 4. The steady state error dependence on input signal and system type.

The relations between steady state error E_{ss}, Reference input signal $R(s)$ and system Type *Dr. Farhan A. Salem*

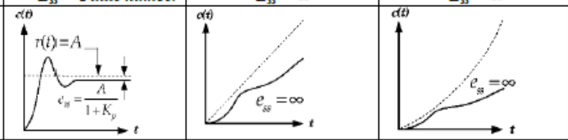
The standard measure of *performance accuracy* is *steady state error E_{SS}*, it can only have *three* possible values: *a) Zero*, *b)* non-zero *(finite number)* , *c)* infinity.

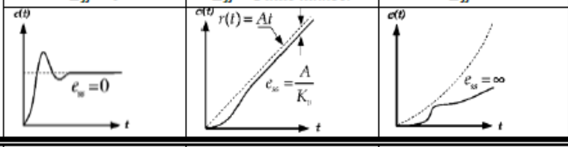
- ✓ Steady-state error can be found in terms of *open loop* transfer function *Go(s)* or *closed loop* transfer function *T(s)*. We find more insight for analysis and design by expressing the steady-state error in terms of *open loop* transfer function *Go(s) with unity feedback*, rather than *closed loop* transfer function *T(s)*.
- ✓ The steady state error is found at time at which system reached the steady-state e.g. greater that *5T* , but for more accurate results, we can assume that time become very large, that is infinity*(t =∞)*, applying final value theorem, we can obtain the value of the steady state error.
- ✓ **The steady state error depends on :** *a)* The type of input signal *R(s)* and , *b)* Type of the system (*this is shown in below table*)
- ✓ In terms of *open loop* transfer function *Go(s)* unity feedback, E_{ss} is given by Eq.(1), while in terms of *closed loop* transfer function *T(s)* , E_{ss} is given by Eq.(2).

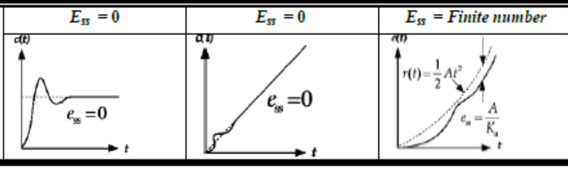
$$e(\infty) = E(0) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \quad (1) \qquad e(\infty) = E(0) = \lim_{s\to 0} sR(s)[1-T(s)] \quad (2)$$



Sys. Type	Examples	Step (*position*) input	Ramp (*Velocity*) input	Parabolic (*Accel.*) input
Type *zero* system	$G(s)=\frac{K}{(s+2)(1.2s+1)}$ $G(s)=\frac{18}{(s+3)}$	E_{ss} = *Finite number*	$E_{ss} = \infty$	$E_{ss} = \infty$
Type *One* system	$G(s)=\frac{K}{s(s+2)(s+1)(s^2+2s+1)}$ $G(s)=\frac{18}{s(s+2)}$	$E_{ss} = 0$	E_{ss} = *Finite number*	$E_{ss} = \infty$
Type *two* system	$G(s)=\frac{18(s+6)}{s^4+7s^3+12s^2}$ $G(s)=\frac{K(1+2s)}{s^3}$	$E_{ss} = 0$	$E_{ss} = 0$	E_{ss} = *Finite number*







Ess , input R(s) & System type:

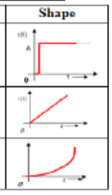
R(s) / Type	Step	Ramp	Para.
0	F	∞	∞
1	0	F	∞
2	0	0	F

Static error constants.

Step Ess	$Ess\|_{step} = \frac{1}{1+K_p}$	$K_p = 1+\lim_{s\to 0} G(s)$
Ramp Ess	$Ess\|_{ramp} = \frac{1}{K_v}$	$K_v = \lim_{s\to 0} sG(s)$
Parabolic Ess	$Ess\|_{parabolic} = \frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2G(s)$

Test input reference signals, R(s)

Test Input signal	Shape
Step (*Position*) input *r(t) = A* *R(s)=A/s*	
Ramp (*Velocity*) input *r(t) = At* $R(s)=A/s^2$	
Parabolic (*Accel.*) input $r(t) = A\,t^2/2$ $R(s)= A/s^3$	



steady-state error

Table 5a. Control systems/algorithms transfer function, actions, selection criteria.

Dr. Farhan A. Salem

Controller's mathematical Models and transfer functions: (Control algorithms)

Proportional *Control* : $u(t)=K_pe(t) \Rightarrow U(s)= E(s)K_p \Rightarrow G_p(s)=\frac{U(s)}{E(s)}=K_p$

Derivative *Control* : $u(t)=K_D\frac{de(t)}{dt} \Rightarrow U(s)= sE(s)K_D \Rightarrow G_D(s)=\frac{U(s)}{E(s)}=K_Ds$

Integral *Control* : $u(t)=K_I\int e(t) \Rightarrow U(s)= K_I\frac{1}{s}E(s) \Rightarrow G_I(s)=\frac{U(s)}{E(s)}=\frac{K_I}{s}$

PD–*Control*, $G_{PD}(s)=K_p+K_Ds=K_D\left(s+\frac{K_p}{K_D}\right)=K_D(s+Z_{PD})=K_p(1+T_Ds)\approx K_p(1+\frac{T_Ds}{1+T_Ds})=K_p(1+\frac{T_Ds}{1+T_Ds/N})$

PI–*Control*, $G_{PI}(s)=K_p+\frac{K_I}{s}=\frac{K_ps+K_I}{s}=\frac{K_p\left(s+\frac{K_I}{K_p}\right)}{s}=\frac{K_p(s+Z_{PI})}{s}=K_p\left(1+\frac{1}{T_Is}\right)=K_p\left(\frac{T_Is+1}{T_Is}\right)$

PID–*Control*, $G_{PID}(s)=K_p+\frac{K_I}{s}+K_Ds=\frac{K_Ds^2+K_ps+K_I}{s}=\frac{K_D\left[s^2+\frac{K_p}{K_D}s+\frac{K_I}{K_D}\right]}{s}=\frac{K_D(s+Z_{PI})(s+Z_{PD})}{s}$

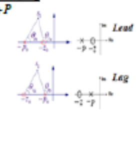
$=K_p+\frac{K_I}{s}+K_Ds=K_p\left(1+\frac{1}{T_Is}+T_Ds\right)=K_p\frac{[T_IT_Ds^2+T_Is+1]}{T_Is}$, *Where* : Integral time, $T_I=\frac{K_p}{K_I}$, Derivative time, $T_D=\frac{K_D}{K_p}$

Compensator's transfer functions :

$G_{PD}(s)\approx G_{Lead}(s)=K_p+K_D\frac{Ps}{s+P}=\frac{K_p(s+P)+K_DPs}{s+P}=\frac{(K_p+K_DP)s+K_pP}{s+P}=(K_p+K_DP)\frac{s+\left[\frac{K_pP}{K_p+K_DP}\right]}{s+P}$

Let $K_c=K_p+K_DP$ *,and* $Z=\left[\frac{K_pP}{K_p+K_DP}\right]$, $\Rightarrow G(s)=K_c\frac{s+Z}{s+P}$

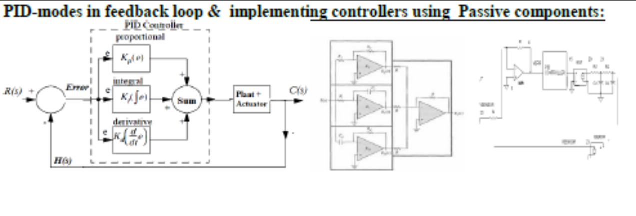
$\begin{cases} \text{If } \|Z\|<\|P\| \Rightarrow \textbf{lead } compensator, (Z \text{ closer to origin than } P), \text{Both } P\,\&\,Z \text{ close to the origin} \\ \text{If } \|Z\|>\|P\| \Rightarrow \textbf{lag } compensator, (P \text{ closer to origin than } Z), \end{cases}$



$G_{Lead/Lag}(s)=K_C\frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$, $P_2>Z_2>Z_1>P_1>0$, $Z_2Z_1=P_2P_1$

$G(s)=\alpha\frac{s+j\omega T}{s+j\omega\alpha T}\begin{cases}0<\alpha<1 & \textbf{,lead } compensator \\ \alpha>1 & \textbf{,lag } compensator\end{cases}$

PID-modes in feedback loop & implementing controllers using Passive components:



Controller (*Algorithm*) action and selection *criteria*

	T	T_s	M_p	T_S	E_{SS}	Notes/During design
P-	Reduce	Reduce	Increase	Small Change	Decrease, but never eliminate	1) Improves transient and steady state responses up to a limit, then has reveres effect up to instability. 2) Reducing T_R result in increasing OS% & vise versa, no compromise
I-	Reduce	Reduce	Increase	Increase	Eliminate	May make transient response worse
D-	Increase	Reduce	Reduce	Reduce	Small Change	Provide fast response **Increase stability**
PI-	Increase	Increase	Reduce	Increase	eliminates	May cause worse transient response or **instability**
Lag	Increase	Increase	Reduce	Increase	Reduces	Reduces steady state error but never eliminate
PD-	Reduce	Reduce	Reduce	Reduce	Reduce	Improves transient : speeds up response , Increase stability, could increase noise
Lead	Reduce	Reduce	Reduce	Reduce	Reduce	Allows to achieve desired **both** T_R & OS%

Effect of Pole-Zero addition on overall response

PI-	**One Zero** addition at $Z_{PI}=K_I/K_p$ **One Pole** at origin, $P_{PI}=0$	**Pole addition** will tend to shift root locus to the **right** (*toward* imaginary axis) of s-plane, resulting in: system becomes *more* oscillatory *decreasing* stability, (**slower** overall **response** increasing T_R , T_p , T_S)
PD-	**One Zero** at addition $Z_{PD}=K_p/K_D$	
PID-	**TWO Zeros** at $Z_{PID}=K_p/K_D$ & $Z_{PID}=K_I/K_p$ **One Pole** at origin, $P_{PID}=0$	**Zero addition** will tend to shift root locus to the **left** (*away* from imaginary axis) of s-plane, resulting in: system becomes *less* **oscillatory**, *increasing* **stability**, **faster overall response**, (Decreases T_R , T_p , T_S). Zeros addition on *right* half of s-plane: slows response & system exhibits *inverse* response
Lead/ Lag	One Pole and One Zero	
Lead- Lag	TWO Poles and TWO Zeros	

Root locus : *rules* for sketching

1) **Find** dominant pole for desired response specification
2) **Find** number of poles (*P*) and zeros (*Z*) of open loop G(s), & **plot** pole-zero diagram.
3) Identify locus Branches = number of *T(s)* poles, *or*, =P-Z
4) **Identify** locus asymptotes , **Draw** the asymptotes centered at *centroid* σ and leaving the real axis at asymptote's angles φ
5) **Compute** angles of departure from pole & angles of arrivals.
6) Braches on the real axis: exists to the *left* of an *odd* number of finite P and/or
7) **Find** the breakaway/Break-in points (if any) , differentiate (*KG(s)H(s)* =- 1) with respect to *s* and find its *optimal* values
8) **Find** Imaginary axis crossings (if any), apply Routh-Hourwits criterion, 8) **sketch** root locus plot, by connecting all these points.

Centroid, $\sigma=\frac{\sum P-\sum Z}{P-Z}$,

Asym. angle, $\varphi=\frac{(2k+1)}{(N\ of\ P)-(N\ of\ Z)}$

$\phi_{dep}=\sum\angle Z-\sum\angle P-180$

$\phi_{arr}=\sum\angle P-\sum\angle Z-180$

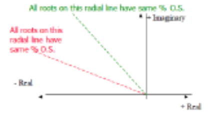
1) a *point* in the s-plane is *on the root locus* for a particular value of gain, *K*, if $\sum\angle Z-\sum\angle P=(2k+1)180^\circ$

2) Gain *K* at that point by *Magnitude criterion:* $K=\frac{\prod Poles\ lengths}{\prod Zeros\ lengths}$

Table 5b. Control algorithm design via root locus.

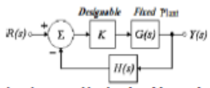
Dr. Farhan A. Salem

Root locus technique for control *system/algorithm* design



When the process (plant) to be controlled, G(s), is *fixed* (parameters can *NOT* be changed) but Controller *gains K*, are *variable* and can be **selected/designed** to meet specifications.

The *open-loop transfer* function of the system shown is given by: $G_{open}(s) = K G(s) H(s)$



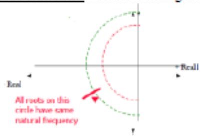
The *closed loop transfer* function is given by: $T(s) = \dfrac{KG(s)}{1+KG(s)H(s)}$

The **characteristic equation** (*c.e.*) is: $1+KG(s)H(s) = 1+K\dfrac{N(s)}{D(s)} = 0$ or $D(s)+KN(s) = 0$

Definition:-As *gain K* changes, so do locations of **closed** loop poles (i.e., zeros of *c.e.*). **Graphically**, the locus **is the *set of paths*** in the *complex plane* traced by the ***closed-loop poles* as *gain, K*, is *varied from* zero to infinity.**

Controller design:- ***By selecting a point along* the root locus that *coincides* with a *desired damping ratio* ζ and *natural frequency ωn, a gain, K*, can be *calculated and implemented* in the controller.** Rules for sketching the location of these poles for K=0→∞ (i.e., K≥0), are given in table below.

Real power of Root locus lies in its ability to provide solutions for systems of order higher than second order, where:



1) **For Analysis and design for stability** :The root locus gives a graphical representation of a system's stability, where we can see:
   - a) ***Ranges of stability***.
   - b) ***Ranges of instability***, *and*
   - c) The **conditions** that cause a system to break into ***oscillation***.
2) **For Analysis and design for transient response specifications**: locus can be used to describe qualitatively the performance of a system as *various parameters are changed*.
   - a) The effect of varying *gain* upon percent overs**hoot PO%, settling time Ts, and peak time Tp, can be clearly displayed**. The qualitative description can then be verified with quantitative analysis.
   - b) The root locus can be used to design for *the damping ratio ζ and natural frequency ω_n*, of a feedback system, where as shown on Figure:
     - 1) **Lines** of *constant damping ratio ζ*, can be drawn *radially* from the *origin*.
       - ✓ ***All roots on the line drawn a long the damping ratio ANGLE line have the same overshoot*** : $\zeta = \cos\theta$, $\theta = \cos^{-1}\zeta$
     - 2) **Lines** of *constant natural frequency ω_n* can be drawn as *arcs* whose center points *coincide with the origin*.
       - ✓ ***The undamped natural frequency ω_n is the magnitude*** of the complex roost, which is the *distance* of the root from the origin in the s-plane. (Or, you can interpret that as the length of the vector from the origin to the *pole*.
       - ✓ All roots on the circle shown in Figure (on the right), **have the same undamped natural frequency**
   - c) ***By selecting a point along* the root locus that *coincides* with a *desired damping ratio* ζ and *undamped natural frequency ω_n*, *a gain, K*, can be *calculated and implemented* in the controller.**

Rules (*steps*) for sketching Root locus:

1) Find dominant poles from *desired* response specification.

2) Find open and closed loop *T(s)* ,transfer functions.

3) Find number of poles (*P*) and zeros (*Z*) of **open loop** K*G(s)H(s).
- ✓ There are as many *closed loop poles* as there are *open loop poles* .
- ✓ If *poles > zeros* , then the number of are **zeros at infinity** = *Poles - Zeros*
- ✓ The root locus **starts** at *the* ***open*** *loop* **pole** (**when K=0**) and **ends** at *the* ***open*** *loop* **zero** *(***when K→∞***)*.

4) Plot the Pole-Zero Diagram; Place the *open* loop poles and zeros on the complex plane

5) Identify locus **Branches**:
Number of Branches = number of *closed* loop *T(s)* poles, (*one Branch for each closed loop pole*), *or*,
Number of the branches = the *order of the characteristic equation.*

6) Braches on the real axis(*axis crossings*): root locus exists to the *left* of an *odd number* of finite P and Z

7) Starting and Ending Points: locus starts (K=0) at *open* **loop poles**, and **ends** (K→∞) at *open* **loop zeros**.

8) Identify locus asymptotes: (*The asymptotes tell us how we get to zeros at infinity*):
Draw the asymptotes centered **at** ***centroid*** **σ** at real axis and leaving the real axis **at asymptote's angles φ**:

$$\text{Centroid}, \sigma = \frac{\sum P - \sum Z}{P-Z} \quad , \quad \text{Asym. angle}, \varphi = \frac{(2k+1)}{(N\ of\ P)-(N\ of\ Z)} \ ,or, \quad \varphi = \pm k\frac{180}{P-Z} \quad where: k = 1, 3, 5\ldots$$

- ✓ *The zeros at infinity are at the ends of the asymptotes.*
- ✓ *The asymptotes* show ***which direction*** *the roots move* as the gain moves from zero (angles of departure, at the forward-loop poles) to infinity.

9) On the complex plane ,(on the already dawn *the pole/zero diagram*), **Draw the asymptotes centered at the centroid σ, and leaving the real axis at angles asymptote angle, θa**

10) Compute angles of **departure** from **complex Pole** & angles of **arrivals at complex Zero**

$$\phi_{dep} = \sum\angle Z - \sum\angle P - 180 \quad , \quad \phi_{ar} = \sum\angle P - \sum\angle Z - 180$$

11) Find the **breakaway/Break-in** points (*if any*) : **Each *break point*** is a point where a **double** or **higher** order (***real and equal* repeated *poles***) **root *exists*** for some value of *K (critically damped response)*
- ✓ **Break-in points exist** between **two zeros** (***locus breaks in** where the gain is minimum*)
- ✓ **Breakaway points exist** between **two poles** (***locus breaks away** where the gain is maximum*)
- ✓ Differentiate (*KG(s)H(s)* =- 1) with respect to *s* and find its *optimal* values:
  N(s)D'(s)-N'(s)D(s)=0.

12) Find Imaginary axis crossings (if any): Apply Routh-Hourwits criterion,
- a) Using the closed loop *T(s)*: ***Construct Routh table, Find gain values that result in row of zeros, Form the auxiliary equation***, Find its ***roots, these are imaginary axis crossing.***
- b) The value of *ω* at the imaginary axis crossing yields the ***frequency of oscillation,***
- c) ***The gain*** at the ***imaginary** axis* crossing , *jω*, yields **the *maximum positive gain* for system *stability***

13) Sketch root locus plot, by connecting all calculated above points.

14) A *point* in the s-plane is *on the root locus* for a particular value of gain, *K*, if *angle criterion is met:*

$$\sum\angle Z - \sum\angle P = (2k+1)180^{\circ}$$

15) Finding the gain *K* at that point by *Magnitude criterion:*

$$K = \frac{1}{|G(s)H(s)|} = \frac{\text{The product of POLE lengths}}{\text{The product of ZERO lengths}} = \frac{\prod \text{POLE lengths}}{\prod \text{ZERO lengths}}$$

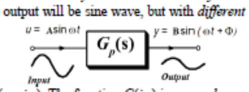
- *a)* If the desired Dominant pole location is *on the root locus*, then a simple *gain adjustment* is all that is required in order to meet the response specification.
- *b)* If the desired Dominant pole location is *NOT* on the root locus, we cannot design the system by simply adjusting the gain, K.
- *c)* The solution is: Reshaping root locus by addition of poles and/or zeros (that is controller) to plant's transfer function such that the system has a root locus that goes through the desired Dominant pole location.

Table 6a. Analysis and design in Frequency domain.

Introduction to Analysis and design in Frequency domain

Dr. Farhan A. Salem

Frequency response methods developed by *Nyquist and Bode* in the 1930, are older than/ and an alternative to the root locus for analyzing and designing feedback control systems. It is the ***steady state response*** characteristics of the system when *subject to sinusoidal inputs*. Where the input sine wave is varied over a given range of frequencies (0 and ∞), the output will be sine wave, but with *different amplitude A, and phase Φ*. By measuring the *output amplitude and phase* of a system over the range, a particular version of the dynamic response is built:

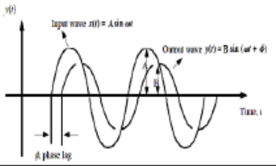


$$\text{Sinusoidal input of the form}: \ x(t) = A\sin(\omega t) \Rightarrow \text{Sinusoidal output (if the system is stable)}: \ y(t) = B\sin(\omega t + \Phi)$$

$$\text{where}: B = |G(j\omega)| \quad \text{and} \quad \Phi = \angle G(j\omega)$$

The frequency transfer: The frequency-domain analysis of the closed-loop system can be conducted from the frequency-domain plots of *G(s)* by setting (*s* =*jω*). The function *G(jω)* is a *complex* and can be written as below, where |*G(jω)*| is the magnitude of complex G(*jω*), and ∠G(jω) is the phase angle. *function* of the frequency *ω*, and is a *complex number, with real and imaginary parts*

$$G_{open(s)} = G(j\omega)H(j\omega) \quad , \quad T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)} \quad , \quad G(j\omega) = |G(j\omega)|\angle G(j\omega)$$

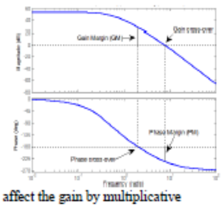


Graphical representations and analysis tools (Plotting the frequency response) : there are ***three graphical tools***: Bode plots , Nyquist diagram and Nichols charts, All methods display the same information, the difference lies in the way the information is presented. If G(s) is the ***open loop*** transfer function of a system and *ω* is the frequency vector, *we then plot* ***G(jω)*** *versus ω* : ***a)*** Create a vector of frequencies varying between zero and infinity and, b) Compute the value of the plant transfer function *magnitude* (|G(jω)|)and *phase* ∠ G(jω) at those frequencies. c) Select the graphical representations tool and plot it (e.g. Bode, Nyquist and Nichols). d) Since the frequency transfer function *G(jω)* is a complex number, we can plot both its magnitude |G(jω)| and phase∠(G(jω) against frequency (Bode plots) or its *position* in the complex plane (the Nyquist plot), as well as Nichols diagram

Bode plots: Two separate graphs of open loop transfer function:
- ***a)*** First plot of ***open loop*** magnitude, log |*G(jω)*| versus log *ω* frequency.
- ***b)*** The other is plot of ***open loop*** phase (∠G(jω) versus log ω frequency.

Bode's stability criterion: The closed-loop system is stable, if the frequency response of the ***open loop*** system has a *gain* ***less than unity*** *(0 dB) when the phase is -180°.*
The system is stable if magnitude *G(jω) <0 dB*, when the phase *G(jω)=±180°*
- ✓ **The magnitude** response in decibels dB, is given by: $K_{dB} = 20\log_{10}|G(j\omega)|$,
- ✓ **A decibel or dB is 1/10 of a bel**
- ✓ **Bode magnitude plot** starts at *the low-frequency gain* of *Open loop G(s)H(s)*, that is found by setting *s* to *zero*. **The phase plot begins** at 0°
- ✓ The low frequency and high frequency asymptotes intersect at Break frequency (corner frequency)

Magnitude plot: The value *ω* = *0* is infinitely far to the left of the bode plot. The value of the slope of the line at *ω* = *0* is 0dB/Decade.



Curves that turn up or down: a) Curves that turn up or down are drawn for *each corner (break) frequency* , associated with the locations of every poles and every zeros ***b)*** From ***each pole's*** break point, the *slope* of the graph ***decreases*** by 20dB/Decade (graph to *turn up*). ***c)*** From ***each zero*** break point the *slope* of the ***graph increases*** by 20dB/Decade (graph to *turn down*).***d)*** squared (*second-order*) terms would have a slope of +/- 40dB/decade , so as, Double, triple, or higher amounts of repeat poles and zeros affect the gain by multiplicative amounts.

Phase plot: ***a)*** for positive system gain; straight line with zero slope at -180 degrees, ***b)*** For every zero, curve ***slopes up*** slopes at 45 degrees per decade when *ω= Z/10*, that is 1 decade before the break frequency, *c)* For every pole, the curve ***slopes down*** at 45 degrees per decade when *ω= P/10* that is 1 decade before the break frequency

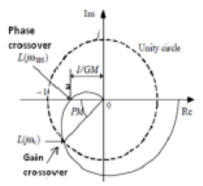
Nyquist plot: is a semigraphical plot of ***open loop*** G(s), ***magnitude*** and ***phase positions*** in the complex plane when frequency *ω* is varied (The real part of the *transfer function* is plotted on the *X* axis and the imaginary part is plotted on the *Y* axis). Nyquist plot is used to determine the stability of a ***close loop*** system by investigating the properties of the ***open loop*** frequency-domain plot

The Nyquist's Stability Criterion says: The ***closed loop*** system is ***marginally*** stable, ***if*** the Nyquist curve of the ***open loop*** goes through (intersects) the critical point of (−1, 0), Or for stability, a zero encirclements of (−1, 0). For stability, system needs both PM > 0, GM > 0

Phase and gain stability margins and gain and phase-crossovers from the Nyquist diagram:
- ✓ **Gain margin:** *In order for the gain margin to exist, the Nyquist diagram must cross (intersect) real axis.*

The gain margin; is the increase in the system gain when phase = —180° that will result in a marginally stable system with intersection of the — 1 + *j0* point. How much designer can change the gain before the system become unstable? Change means Multiplication, that is what multiplier with transfer function gain, will result in intersection of the Nyquist plot with point (-1, 0). This multiplier is the gain margin. Find the intercept point *(a)* of Nyquist diagram with real axis , then gain margin (GM) is given by:

$$GM = 1/|a| \quad , \quad -a^*K = -1 \Rightarrow K = \frac{1}{a}$$



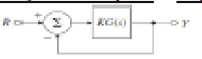
- ✓ **Phase margin:** *In order for phase margin to exist, the Nyquist diagram must cross* ***unity circle***. At point *(-1,0)*, where the gain is *unity or zero decibel* ,(curve intersects unity circle) if a phase shift of *a* degrees occurs, the system becomes unstable.
- ✓ **Phase-crossover** is a *point* at which the plot intersects **the negative real** axis *(where* $\angle G(j\omega) = 180^{\circ}$).
- ✓ **Gain-crossover** is a *point* at which **the magnitude of** | *G(jω)*|=1
- ✓ The Nyquist criterion can tell us how many closed loop poles are in the right half-plane, but does not give the exact location of the characteristic equation roots.
- ✓ If the Nyquist plot ***d't*** intersect ***both*** negative real *axis*, and unity circle this means: PM=0 and GM=∞dB.
- ✓ If the Nyquist plot *intersect real axis*, at point (-1,0), this means PM=1 and GM = 0 dB:(critically stable)
- ✓ If the Nyquist plot encloses point (-1,0), this means: PM>1 and GM < 0 dB:(un stable)
- ✓ The use of Nyquist plots is more common in *multiloop* or *multivariable* analyses

Table 6b. Analysis and design in Frequency domain.

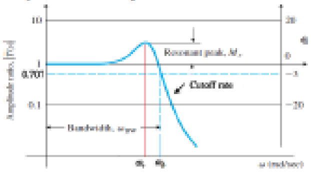
Frequency-Domain Analysis; Performance Specifications: Correlation between time and frequency domain specifications(1a)

The usefulness of frequency response specifications and their relation to the *actual* transient performance depend upon the approximation of the system by ***a second-order pair of complex poles*** For linear, time-invariant, higher-order systems having a dominant pair of complex conjugate closed-loop poles, the below relationships exist between the step transient response and frequency response:

$$T(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}=\frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2}=\frac{1}{1+j2\xi(\omega/\omega_n)-(\omega/\omega_n)^2}$$



Margin *is defined* as how much space you have to a boundary. In control design, the boundary is instability How much designer can *change the gain and phase* before the system become *unstable*. **Two quantities that measure the *stability* of a system are the Gain and Phase margins.**



Gain margin (GM) : Is the amount of Additional *gain expressed in* ***decibels*** *(dB),* that can be allowed (required at -180° of phase shift),to increase in the ***forward loop*** system $KG(j\omega)$, before ***closed loop*** system *reaches instability.(Change means Multiplication, that is what multiplier with transfer function gain, multiplier is the gain margin)*

Phase margin (PM): the change in ***open-loop*** phase shift required at unity gain *(0 dB)* to make the ***closed-loop*** system unstable (The change by which the phase of ***forward loop*** system $KG(j\omega)$ exceeds -180°)

- ✓ The PM and ζ are related by given, on the right, equation.
- ✓ *The* ***higher phase margin****, the larger the* ***damping ratio****, which implies the* ***smaller the overshoot****.*
- ✓ For $0\le\zeta\le0.6$: *PM* and ζ are related approximately by a straight line as given by equation on the right:
- ✓ *System stability is improved by increasing phase and gain margins.* Lower useful gain and phase margins ; GM > 2.5 , PM > 30

$$PM=\tan^{-1}\left(\frac{2\xi}{\sqrt{\sqrt{1+4\xi^4}-2\xi^2}}\right)\qquad \zeta=\frac{PM}{100}$$

Gain-Crossover Point: is the point at which the gain curve crosses the *0 dB line ,(unity* **gain** , $|G(j\omega)|=1$)

Gain-crossover frequency ω_g : The frequency where the ***amplitude*** curve *crosses* the *0dB* (The frequency at the gain-crossover point)

- ✓ ***Increasing*** the gain crossover will *increase the bandwidth* , resulting in *faster transient response* . (As the bandwidth *increases*, the *rise time* of the step response of the system will *decrease).*

$$\omega_g=\omega_n\sqrt{\sqrt{1+4\xi^4}-2\xi^2}$$

Phase-Crossover Point: is the point at which the phase curves crosses the *-180° line ,(* ∠G(jω)=-180°)

Phase-crossover frequency ω_p : The frequency where ***phase*** curve ***crosses*** ***-180° point*** (The frequency at the phase-crossover point) .

- ✓ For many control problems, there is **only a single** crossover frequencies, ω_g , ω_p. But multiple values can occur

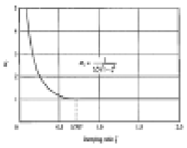
$$\omega_p=-90^\circ-\tan^{-1}\left(\frac{\omega_g}{2\xi\omega_n}\right)$$

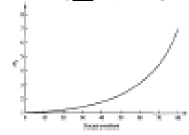
Resonant Peak M_r *(or,* M_p*)*: also called the **peak amplitude ratio**, it is the maximum value of |M(jω)|, with respect to frequency ω, it occurs at the ***Resonant Frequency*** ω_r, and is given by below Eq.

- ✓ It is a relative stability criterion, (the higher M_r the poorer the relative stability)
- ✓ Satisfactory transient performance is usually obtained if the value of M_r should be within the range $1.0 < Mr < 1.4$ that is (0 dB < M_r < *3* dB) which corresponds to an effective damping ratio of (0.4 < δ < 0.7). *(M_p=1.3* is often used as compromise between speed of response and relative stability)
- ✓ If the system is subjected to noise signals whose frequencies are near the resonant frequency ω_r, the noise will be amplified in the output and will present serious problems.
- ✓ Resonant Peak M_r is a function of the *damping ratio* ζ only; Where:
  - a) The given equation for ζ <= 0.707
  - b) For ζ > 0.707, $M_r = 1$, $\omega_r = 0$, *and there is no resonant peak* ,
  - c) When ζ =0, M_r is *infinite* ,
  - d) As ζ increases, Overshoot decreases and M_r, decreases, (a large M_r corresponds to a *large* maximum overshoot) , *as shown in graphs on the right* **down** *(M_r versus OS%)* **up***(M_r versus ζ)*
  - e) When ζ is *negative*, the system is *unstable*, and the value of M_r ceases to have any meaning,
  - f) BW occurs For ζ < 0.707, *e)* When $G(j\omega)$ = - 1 , |M(jω)| is infinite, and the closed-loop system is marginally stable.

$$M_r=\frac{1}{2\zeta\sqrt{1-\zeta^2}}\quad ,\zeta\le0.7$$

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}\quad ,\zeta\le0.7$$





Resonant Frequency ω_r *(or,* ω_p*)*: is the frequency at which the peak resonance, M_r occurs, and is given by up Equation:

- ✓ Indicative *(criterion)* of the speed of the transient response. (The larger the value of ω_r, the faster the time response is).
- ✓ Resonant Frequency is a function of both ζ and ω_n. For ζ <= 0.707, where:
  - g) For ζ > 0.707, Resonant Frequency ω_r = 0 there is no resonant peak
  - h) When the *damping ratio* ζ approaches zero, the resonant frequency ω_r approaches *undamped natural frequency* ω_n
- ✓ Since the values of M_r and ω_r can be easily measured in a physical system, they are quite useful for checking agreement between theoretical and experimental analyses.
- ✓ In terms of the open-loop frequency response, the damped natural frequency in the transient response is somewhere between the gain crossover frequency and phase crossover frequency.

Table 7a. The modern State space (variable) approach.

د. فرحان عطالله سالم — ***The modern State space (variable) approach*** — ***Dr. Farhan A. Salem***

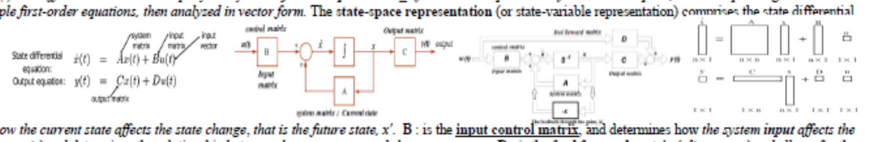
The state of a system is a set of *variables* whose values, together *with the input signals* and the *equations describing the dynamics*, will *provide the future state and output* of the system.

Advantages: *1)* Represent *nonlinear* systems, *2)* Handle, systems with *non zero initial conditions, 3) Time varying systems,4) Multiple inputs and multiple outputs systems* (MIMO), *5)* model and manipulate systems that cannot be adequately described using the Laplace Transform.

In state space, the response of a system is described by ***a set of first-order*** differential equations written ***in terms of the state variables*** ($x_1, x_2 \dots x_n$) and the ***inputs*** *($u_1, u_2, \dots, u_m$)* such that the initial values $x_i(t_0)$ of this set and the system inputs $u_j(t)$ are *sufficient* to describe uniquely the system's ***future response*** of $t \ge t_0$. *That is to represent a system in state space, we decompose a given second-order or higher, differential equation into multiple first-order equations, then analyzed in vector form.* The state-space representation (or state-variable representation) comprises the state differential equation and the output equation, given by:

$$\frac{dx}{dt}=Ax(t)+Bu(t)\quad ,\ State\ diff.\ eq.$$

$$y(t)=Cx(t)+Du(t)\quad ,\ Output\ eq.$$



Where: A: is the **system matrix**, *and relates how the current state affects the state change, that is the future state,* x'. B : is the **input control matrix**, and determines how *the system input affects the state change*. C : is the **output matrix**, (*sensor matrix*) and determines the relationship between *the system state* and *the system output*. D : is the **feed forward matrix**,(*direct term*) and allows for the system *input to affect the system output directly.*

A proposed systematic procedure for developing state space representation

(for the case when ***no derivatives*** *with respect to the input u(t) ,are present)*

a) Represent the system in the form of differential equation.

$$\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\dots+a_1\frac{dy(t)}{dt}+a_0y(t)=u(t)$$

b) Let the ***coefficient*** of the ***highest*** order term of differential equation is **unity**.

c) Selection of the state variables: ***1) Principle***: the number of state variable chosen to represent the system should be *as small as possible.* **2)** ***The minimum*** *number of* ***state variables*** *chosen* ***equals*** *to the order of differential equation.* 3) Choose **the output, *y(t)*,** and its *(n - 1) derivatives* as the state variables.

d) Take derivatives of state variables

e) Substitute state variables and corresponding derivatives; here note that by differentiating both sides , the following is observed: $x'_1 = x_2$ and $x'_2 = x_3$ and so on ,

f) Rearrange first order equations and , substituting corresponding ***ones*** to *replace* the *state variables* and substituting *zeros* to replace the ***missing*** state variables.

g) Represent these linear *first order equations* in *vector-matrix form*

g) Substitute in the state-space representation; the state differential equation and the output equation,

Select the state variables to be: x_1, x_2, x_3..	Taking derivatives of selected variables. Notice that: $x'_1=x_2$, $x''_2=x_3$		Rearranging first order eqs. and , substituting corresponding ones and zeros:
$x_1=y(t)$ $x_2=\frac{dy(t)}{dt}$ $x_3=\frac{d^2y(t)}{dt^2}$ .... $x_n=\frac{d^{n-1}y(t)}{dt^{n-1}}$	$\frac{dx_1}{dt}=\frac{dy(t)}{dt}$ $\frac{dx_2}{dt}=\frac{d^2y(t)}{dt^2}$ $\frac{dx_3}{dt}=\frac{d^3y(t)}{dt^3}$ .... $\frac{dx_n}{dt}=\frac{d^ny(t)}{dt^n}$	$\frac{dx_1}{dt}=x_2$ $\frac{dx_2}{dt}=x_3$ $\frac{dx_3}{dt}=x_4$ .... $\frac{dx_n}{dt}=x_n$	$\frac{dx_1}{dt}=0x_1\quad 1x_2\quad 1x_3\quad 0\ \dots\dots+\ 0u$ $\frac{dx_2}{dt}=0x_1\quad 0x_2\quad 1x_3\quad 0\ \dots\dots+\ 0u$ .... $\frac{dx_{n-1}}{dt}=0x_1\quad 0x_2\quad 0x_3\quad 1x_n\ \dots\dots+\ 0u$ $\frac{dx_n}{dt}=a_0x_1-a_1x_2-a_2x_3\ \dots\dots\ a_{n-1}x_n+b_0u$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \vdots\\ \dot{x}_{n-1}\\ \dot{x}_n\end{bmatrix}=\begin{bmatrix}0&1&0&0&0&0&\cdots&0\\0&0&1&0&0&0&\cdots&0\\0&0&0&1&0&0&\cdots&0\\ \vdots\\0&0&0&0&0&0&\cdots&1\\-a_0&-a_1&-a_2&-a_3&-a_4&-a_5&\cdots&-a_{n-1}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\ \vdots\\x_{n-1}\\x_n\end{bmatrix}+\begin{bmatrix}0\\0\\0\\ \vdots\\0\\b_0\end{bmatrix}u\qquad y=\begin{bmatrix}1&0&0&\cdots&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\ \vdots\\x_{n-1}\\x_n\end{bmatrix}$$

Analysis and design in state space:

The transfer function matrix:

Relates the output vector Y(s), to the input vector U(s), and is rewritten as:

$$G(s)=\frac{Y(s)}{U(s)}=C(sI-A)^{-1}B+D=C\left[\frac{adj(sI-A)}{\det(sI-A)}\right]B+D$$

Where: *det*: matrix ***determinant***, I is an n x n identity matrix, n : the order of the system

The characteristic equation is found by:

$$B(s)=|(sI-A+B)|$$

Stability in state space (Eigenvalues)

The poles are found from the common denominator of the matrix $(sI-A)^{-1}$.

The Eigenvalues of the *system matrix A*, are equal to the poles of the system's transfer function , and are found by : $\det(sI-A)=0$

Steady state error in state space *(final value theorem)*

$$\lim_{s\to0} sE(s)=\lim_{s\to0} sR(s)[1-C(sI-A)^{-1}B]$$

Controller Design)

a) If we feed back each of the state variables to summing junction, *through* a gain, k_i, *then there would be* n gains k_i, *that* could be adjusted, to yield the required closed-loop pole values.

b) The feedback through the gains, k_i is represented in below Figure by the feedback vector -K.

c) based on this, Introducing into the closed loop system, a number n, of adjustable parameters, equals to the highest power of transfer function/differential equation, and *relate* them to the system's coefficients; all the poles of the closed-loop system can be set to any desired location.

d) The state equations for the closed-loop system (*below* figure), can be written by to be as:

$$\frac{dx}{dt}=Ax+B(-Kx+r)u=(A-BK)x+Br$$

$$y(t)=Cx$$

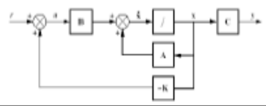
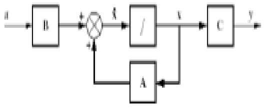


Table 7b. Controller design in state space (variable) approach.

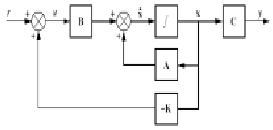
د. فرحان عطاالله سالم

Controller Design via state space *(Pole Placement)*

Shortly: the philosophy is to pick the control gains such that the eigenvalues (*poles*) of the closed loop system match desired eigenvalues: If we know the *locations* of desired closed loop poles, we can built the characteristic equation of desired system, then equate the coefficients of desired characteristic equation with that we got from state space representation, as explained below:



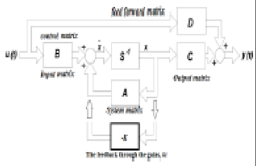
1) From desired performance specifications (OS%, T, T_S, …) find the desired closed loop *poles (locations)*.
2) If the system is third (or higher) order; for third order, select a third order pole, either to *cancel* the closed-loop zero, or near it.
3) Based on determined poles (location) of the closed loop system; write the both *desired* closed loop transfer function and characteristic equation.
4) Represent the systems (plant) in state space (write state and output equations).
5) Represent the closed loop system in state space;
   a) The state equation will have the form: $\frac{dx}{dt} = Ax + B(r - Kx) = (A-BK)x + Br$
   b) Where K is column vector with size $1xn$, meanwhile x is column vector with size $nx1$, and the resulted matrix will be of $2x2$ size.
6) Find the characteristic equation of the closed-loop system, by: $B(s) = |(sI - A + BK)|$



7) Find the gains, $k_1, k_2, k_3, ..$, by *comparing* the characteristic equation from step (6) and with desired in step (4) .
8) applying this method for systems higher that I and II order system , will be laborious, *MATLAB can be a good solution*, using built-in function `place(A,B,P)` , to return the gains, where A, B :are the system matrix, *and* input control matrix. meanwhile P: are the desired poles, defined as *row* vector

Example : a system is represented by transfer given below, ***evaluate the coefficients of the state feedback gains***, such that the closed loop system will respond with critically damped response, and time constant T= 0.5 second

$$G(s) = \frac{1}{s(s+4)}$$



From desired response, the damping ratio is $\zeta = 1$, the undamped natural frequency ω_n, is calculated from time constant as shown next:

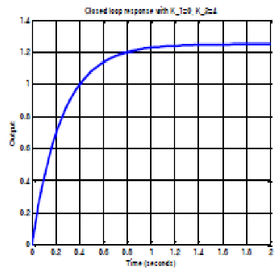
$$T = \frac{1}{\zeta\omega_n} \Rightarrow \omega_n = \frac{1}{\zeta T} = 2 \ Rad/s$$

The desired closed loop *poles (locations)* are given by:

$$P_{1,2} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2} = -2, -2$$

The *desired* closed loop transfer function and characteristic equation, are found by either from calculated poles by finding (s+2)(s+2) or from standard general *II* order system transfer function form:

$$\left.\begin{matrix}(s+2)(s+2) \\ \dfrac{\omega_n^2}{s^2 + (2\xi\omega_n)s + \omega_n^2}\end{matrix}\right\} \Rightarrow \quad B(s) = s^2 + 4s + 2$$



Representing the systems (plant) in state space, gives:

$$\begin{bmatrix}\frac{d}{dt}x_1 \\ \frac{d}{dt}x_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -4\end{bmatrix} * \begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix} * \begin{bmatrix}x_1 \\ x_2\end{bmatrix}$$

Finding the characteristic equation of the closed-loop system, by expanding the next formula:

$$B(s) = |(sI - A + BK)| = \left|\left(s\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 \\ 0 & -4\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\begin{bmatrix}K_1 & K_2\end{bmatrix}\right)\right| = s^2 + (4+K_2)s + K_1$$

Now, Finding the gains, $k_1, k_2, k_3, ..$, by *comparing* both characteristic equations:

$$s^2 + (4+K_2)s + K_1 = s^2 + 4s + 2 \Rightarrow \begin{cases}4 + K_2 = 4 \Rightarrow K_2 = 0 \\ K_1 = 2\end{cases}$$

The Simulink representation of the closed loop system and resulting response are shown in figure on the right

IV. CONCLUSIONS

In this paper, are proposed and discussed, a proper for Mechatronics education, Control systems design and analysis course detailed description, topics with specific learning objectives, prerequisites, administration, simple but effective teaching approach supported by simple and easy to memorize education oriented steps and tables, that integrate course outcomes, to solve control problems, all intended to support educators in teaching process, help students in concepts understanding, maximum knowledge and skills transfer /gaining in solving controller/algorithm selection and design problems as a stage of Mechatronics system design stages, and prepare them for other further courses applied in Mechatronics curricula including; Mechatronics fundamentals, Mechatronics systems design, Process control, Embedded systems design, Robotics, PLC, CNC and others.

REFERENCES

[1]. Farhan A. Salem, Ahmad A. Mahfouz. Mechatronics Design and Implementation Education-Oriented Methodology; A Proposed Approach, Journal of Multidisciplinary Engineering Science and Technology, Volume 1, Issue 03, October 2014.
[2]. Richard M. Murray. Principles of Feedback and Control Course Overview, California Institute of Technology, 2002.
[3]. Norman Nise. Control system engineering, sixth edition, John Wiley & *Sons*, Inc, 2011.
[4]. Katsuhiko Ogata. Modern control engineering, third edition, Prentice Hall, 1997.
[5]. Farid Golnaraghi, Benjamin C. Kuo. Automatic Control Systems, sixth edition, John Wiley & Sons, INC, 2010.
[6]. Richard C. Dorf, Robert H. Bishop. Modern Control Systems, Twelfth edition, Prentice Hall, 2011.
[7]. T. Dean and M. P. Wellman. Planning and Control. Morgan Kaufman, San Mateo, CA, 1991.
[8]. G. F. Franklin, J. D. Powell, and A. Emami-Naeini. Feedback Control of Dynamic Systems. Addison-Wesley, Reading, MA, 1994.
[9]. Jean-Jacques E. Slotine and Weiping Li. Applied Nonlinear Control. Prentice-Hall, 1991.
[10]. H. K. Khalil. Nonlinear Systems. Prentice-Hall, Englewood Cliffs, NJ, 2nd edition, 1996.
[11]. J. H. D'Azzo and C. H. Houpis. Linear Control System Analysis and Design: Conventional and Modern. McGraw-Hill, New York, 1995.
[12]. J. C Doyle, B. A. Francis, and A. R. Tannenbaum. Feedback Control Theory. Macmillan, New York, 1992.
[13]. B. Friedland. Control System Design. McGraw-Hill, New York, 1986.
[14]. http://www.abet.org/accreditation/