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Role of Arterial Stenosis on Non-Newtonian Flow of Blood in Presence of Slip Velocity

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ABSTRACT: *An attempt has been made to study the role of arterial stenosis on blood flow in presence of slip velocity. In the present analysis Herschel-Bulkley fluid represents the non-Newtonian character of blood. The hemodynamic behaviour of blood flow is influenced by the presence of arterial stenosis. The expressions for velocity profile, pressure drop and shear stress have been investigated here. The results are shown in graphical form.*

Keywords: *Herschel-Bulkley fluid, Shear Stress, Yield Stress, Stenosis, Resistance to flow.*

I. INTRODUCTION

Many cardiovascular diseases, such as stroke, atherosclerosis, which are responsible for major cause of death, are closely related to the nature of blood flow and the dynamic behaviour of blood vessels. It is well known from medical point of view that many cardiovascular diseases directly depend on various types of arterial diseases. Among the arterial diseases the most important one is stenosis. The medical term stenosis means narrowing of body passage, including blood vessels, heart valves, vertebral canal, G. I. tract. It is believed that stenosis is formed by the accumulation of fatty substances like cholesterol or fats in the inner wall of the artery and proliferation of connecting cells. If stenosis is present in an artery, the blood flow changes to its usual state, because resistance to flow is increased and as a result supply of oxygen to each cell of the body and removal of carbon dioxide is disturbed abruptly. In other words nutrient supplement is insufficient to reach each cell of the body. Many bio-medical researchers (Shukla et. al. [1], Young and Tsai [2], Verma and Parihar [3]) feel that the hemodynamic factors may be helpful in the diagnosis, treatment and understanding of many disorders. Misra and Shit [4] have developed a mathematical model to study the non-Newtonian aspect of blood flow by using Herschel-bulkley fluid model in presence of arterial stenosis and they have shown that skin friction and resistance to flow is maximum at the throat of the stenosis and minimum at the end. Ali et. al. [5] have analysed the effect of an axial symmetric time dependent growth along the lumen of the arterial tube by considering blood as a Newtonian fluid. Biswas and Chakraborty [6], srivastava et. al. [7] have studied non-Newtonian behaviour of blood through a stenosed artery. Shah and Siddiqui [8] have studied a mathematical model to analyze the effect of peripheral layer viscosity on blood flow through stenosed arterial tube by considering the blood as power-law type non-Newtonian fluid. They have seen that the peripheral layer viscosity of blood in diabetic patients is higher than in non-diabetic patients, resulting higher resistance to flow.So peripheral layer viscosity has significant effects on resistance to flow. Thus resistance to blood flow of diabetic patients may be reduced by reducing the plasma layer viscosity.

Many researchers (Vershney et. al. [9], Singh and Shah [10], Sankar and Hemalatha [11], Sankar [12], Sanyal and Maiti [13]) have presented various types of mathematical models to study various characteristics of blood by considering blood as a non- Newtonian fluid. Many investigators (Nuber [14], Haymen [15], Chaturani and Biswas [16], Kumar et. al. [17], Biswas and Chakraborty [18], Chaturani and Ponalagusamy [19]) have analysed theoretically the blood flow model through uniform artery in presence of slip velocity to study the flow parameters of blood.

In a recent paper Biswas and Laskar [20] have presented a mathematical model to study the effect of slip velocity on steady flow of blood through constricted artery.

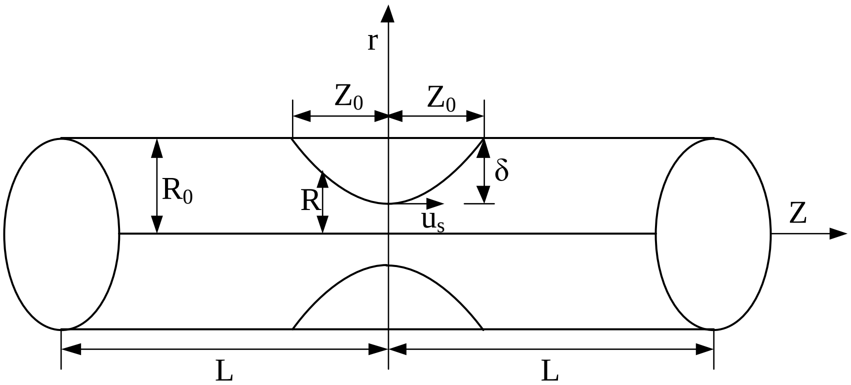

Fig. 1: Geometry of the arterial segment with stenosis.

II. MATHEMATICAL FORMULATION

Let us consider the steady one dimensional laminar flow of blood through an axially symmetric and radially non-symmetric constricted artery. The geometry of bell shaped stenosis can be taken as [4]

$$R(z) = R_0\left[1 - \frac{\delta}{R_0} exp\left(\frac{-m^2\varepsilon^2 z^2}{R_0^2}\right)\right] \tag{2.1}$$

Where $R(z)$ is the radius of the arterial tube in the stenotic region, R_0 is the normal radius of the artery, δ is the maximum height of the stenosis, m is a parametric constant and ε characterises the relative length of the constriction, defined as the ratio of the radius to half length of stenosis,

$$\frac{R(z)}{R_0} = \left[1 - a\ exp\left(-bz^2\right)\right] \tag{2.2}$$

where,

$$a = \frac{\delta}{R_0},\ b = \frac{m^2\varepsilon^2}{R_0^2}$$

The equations governing the motion are given by

$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) \tag{2.3}$$

$$0 = \frac{\partial p}{\partial r} \tag{2.4}$$

Let us consider the blood as a Herschel-Bulkley type fluid, the constitutive equation for Herschel-Bulkley fluid is given by

$$\tau = \mu\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0, \qquad \tau \geq \tau_0 \tag{2.5}$$

$$-\frac{\partial u}{\partial r} = 0, \qquad \tau < \tau_0 \tag{2.6}$$

Boundary conditions are

(i) $u = u_s$ at $r = R(z)$, (slip condition)

(ii) τ is finite at $r = 0$,(regularity condition)

III. SOLUTIONS

From (2.1),

$$\tau = \frac{Pr}{2}$$

For simplicity we take the Herschel-Bulkley fluid index as $n = \frac{1}{2}$. From (2.2) we get

$$\left(\frac{\tau - \tau_0}{\mu}\right)^2 = -\frac{\partial u}{\partial r} \tag{3.1}$$

Which can be written as

$$-\frac{\partial u}{\partial r} = \frac{1}{\mu^2}\left(\frac{Pr}{2} - \tau_0\right)^2 \tag{3.2}$$

From which we get after integration

$$u = -\frac{1}{\mu^2}\left(\frac{P^2 r^3}{12} - \frac{Pr^2 \tau_0}{2} + \tau_0^2 r\right) + A \tag{3.3}$$

Using the boundary condition (i) we get

$$u = u_s + \frac{1}{\mu^2}\left[\frac{P^2}{12}\left(R^3 - r^3\right) - \frac{P\tau_0}{2}\left(R^2 - r^2\right) + \tau_0^2\left(R - r\right)\right] \tag{3.4}$$

In the absence of stenosis (i.e., when $\delta = 0$), the axial velocity is given by

$$u_p = u_s + \frac{1}{\mu^2}\left[\frac{P^2}{12}\left(R_0^3 - r^3\right) - \frac{P\tau_0}{2}\left(R_0^2 - r^2\right) + \tau_0^2\left(R_0 - r\right)\right] \tag{3.5}$$

where the subscript p denotes the Poiseuille flow.

$$\frac{u}{u_p} = \frac{u_s + \frac{1}{\mu^2}\left[\frac{P^2}{12}\left(R^3 - r^3\right) - \frac{P\tau_0}{2}\left(R^2 - r^2\right) + \tau_0^2\left(R - r\right)\right]}{u_s + \frac{1}{\mu^2}\left[\frac{P^2}{12}\left(R_0^3 - r^3\right) - \frac{P\tau_0}{2}\left(R_0^2 - r^2\right) + \tau_0^2\left(R_0 - r\right)\right]} \tag{3.6}$$

Volumetric flow rate i.e., the flux is given by

$$Q = \int_0^R u\, 2\pi r\, dr = \pi\left[u_s . R^2 + \frac{P^2 R^5}{20\,\mu^2} - \frac{P\tau_0 R^4}{4\mu^2} + \frac{\tau_0^2 R^3}{3\mu^2}\right] \tag{3.7}$$

From which we can write

$$\frac{Q}{\pi R^2} - u_s = \frac{P^2 R^3}{20\,\mu^2} - \frac{P\tau_0 R^2}{4\mu^2} + \frac{\tau_0^2 R}{3\mu^2} \tag{3.8}$$

$$= \frac{R}{\mu^2}\left[\frac{\tau_R^2}{5} - \frac{\tau_R \tau_0}{2} + \frac{\tau_0^2}{3}\right]$$

Since $\frac{\tau_o}{\tau_R} = 1$, replacing $\frac{1}{3}$ by $\frac{5}{16}$ we finally get

$$\frac{Q}{\pi R^2} - u_s = \frac{R}{5\mu^2}\left[\tau_R^2 - \frac{10\,\tau_R \tau_0}{4} + \frac{25\,\tau_0^2}{16}\right] \tag{3.9}$$

$$= \frac{R}{5\mu^2}\left[\tau_R - \frac{5\tau_0}{4}\right]^2$$

Thus we get from above

$$P = \frac{2}{R}\sqrt{\frac{5\mu^2}{R}\left(\frac{Q}{\pi R^2} - {}_s\right)^{1/2}} + \frac{5\tau_0}{2R} \tag{3.10}$$

$$= 2\sqrt{5}\,\mu\left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3}\right)^{1/2} + \frac{5\tau_0}{2R}$$

Pressure drop across the length of the stenosis is given by

$$\Delta p = \int_{-z_0}^{z_0} \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3} \right)^{1/2} + \frac{5\tau_0}{2R} \right] dz \tag{3.11}$$

In the absence of stenosis (when $\delta = 0$)

$$(\Delta p)_p = \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R_0^5} - \frac{u_s}{R_0^3} \right)^{1/2} + \frac{5\tau_0}{2R_0} \right] 2z_0 \tag{3.12}$$

Now we write

$$K = \frac{\Delta p}{(\Delta p)_p} = \frac{\int_{-z_0}^{z_0} \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3} \right)^{1/2} + \frac{5\tau_0}{2R} \right] dz}{\left[2\sqrt{5}\mu \left(\frac{Q}{\pi R_0^5} - \frac{u_s}{R_0^3} \right)^{1/2} + \frac{5\tau_0}{2R_0} \right] 2z_0} \tag{3.13}$$

If $2L$ is the length of the arterial segment, pressure drop along the length of the artery can be given as

$$\Delta p_1 = \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R_0^5} - \frac{u_s}{R_0^3} \right)^{1/2} + \frac{5\tau_0}{2R_0} \right] (2L - 2z_0) + \int_{-z_0}^{z_0} \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3} \right)^{1/2} + \frac{5\tau_0}{2R} \right] dz \tag{3.14}$$

In the absence of stenosis the pressure drop along the length of the artery is given by

$$(\Delta p_1)_p = \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R_0^5} - \frac{u_s}{R_0^3} \right)^{1/2} + \frac{5\tau_0}{2R_0} \right] 2L \tag{3.15}$$

Now

$$K_1 = \frac{\Delta p_1}{(\Delta p_1)_p} = 1 - \frac{z_0}{L} + \frac{\int_{-z_0}^{z_0} \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3} \right)^{1/2} + \frac{5\tau_0}{2R} \right] dz}{\left[2\sqrt{5}\mu \left(\frac{Q}{\pi R_0^5} - \frac{u_s}{R_0^3} \right)^{1/2} + \frac{5\tau_0}{2R_0} \right] 2L} \tag{3.16}$$

The shear stress on the surface of the stenosis is given by

$$\tau_R = \frac{1}{2} PR = \frac{1}{2} R \left[2\sqrt{5}\mu \left(\frac{Q}{\pi R^5} - \frac{u_s}{R^3} \right)^{1/2} + \frac{5\tau_0}{2R} \right] \tag{3.17}$$

In the absence of stenosis the shear stress is given by

$$\tau_{RP} = \sqrt{5}\mu \left(\frac{Q}{\pi R_0^3} - \frac{u_s}{R_0} \right)^{1/2} + \frac{5\tau_0}{4} \tag{3.18}$$

$$\tau = \frac{\tau_R}{\tau_{RP}} = \frac{\sqrt{5}\mu \left(\frac{Q}{\pi R^3} - \frac{u_s}{R} \right)^{1/2} + \frac{5\tau_0}{4}}{\sqrt{5}\mu \left(\frac{Q}{\pi R_0^3} - \frac{u_s}{R_0} \right)^{1/2} + \frac{5\tau_0}{4}} \tag{3.19}$$

IV. NUMERICAL DISCUSSIONS

To illustrate the flow analysis, the results are shown graphically with the help of MATLAB-7.6. The effects of various parameters on pressure drop, axial velocity and wall shear stress are calculated here.

Figure 2 describes the effect of stenosis on axial velocity with respect to the axial distance z. It is observed from the figure that axial velocity decreases with the increase of stenosis size.

Figures 3-6 show the variation of pressure drop K for different values of yield stress and slip velocity with respect to stenosis size a and b. It is found that the pressure drop decreases with the increase of yield stress for the increase of a and b. But it increases as slip velocity increases with the increase of stenosis size. Similar behaviour of the pressure drop K_1 across stenosis size a and b can be seen in Figures 7-9. It is also observed that pressure drop K_1 increases with the increase of stenosis length.

Figures 10-11 show the effects of slip velocity on shear stress with respect to stenosis size. It is clear from the figures that shear stress increases with the increase of slip velocity.

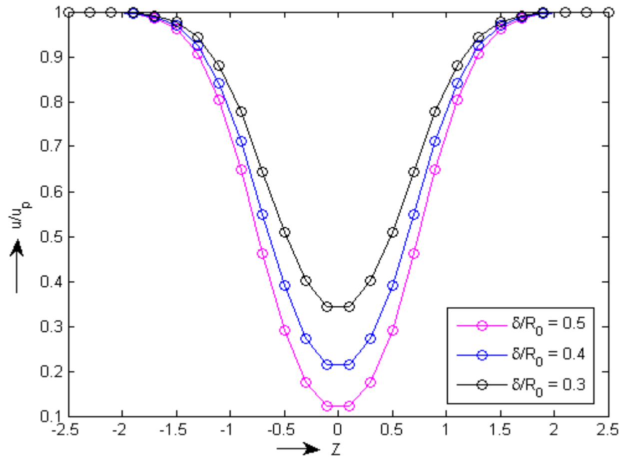

Fig. 2: Variation of axial velocity for different values of stenosis size.erent values of stenosis size.

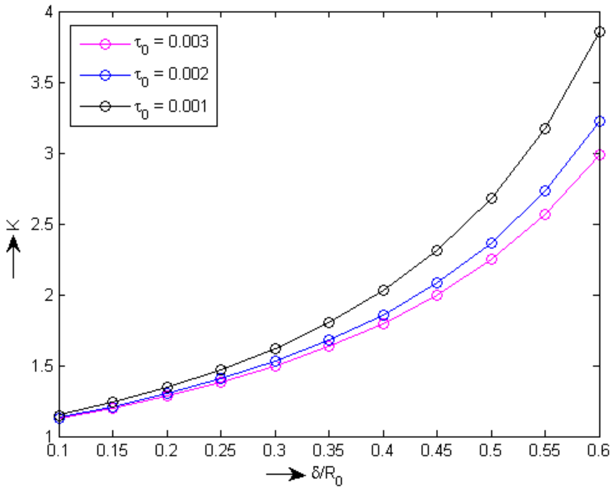

Fig. 3: Variation of Pressure drop K across the stenosis size for different values of yield stress.

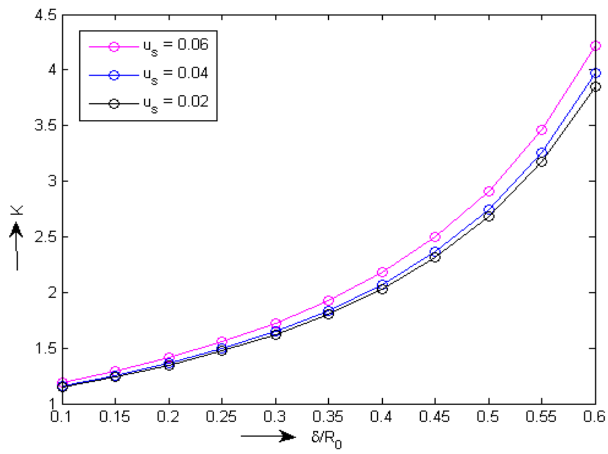

Fig. 4: Variation of Pressure drop K across the stenosis size for different values of slip velocity.

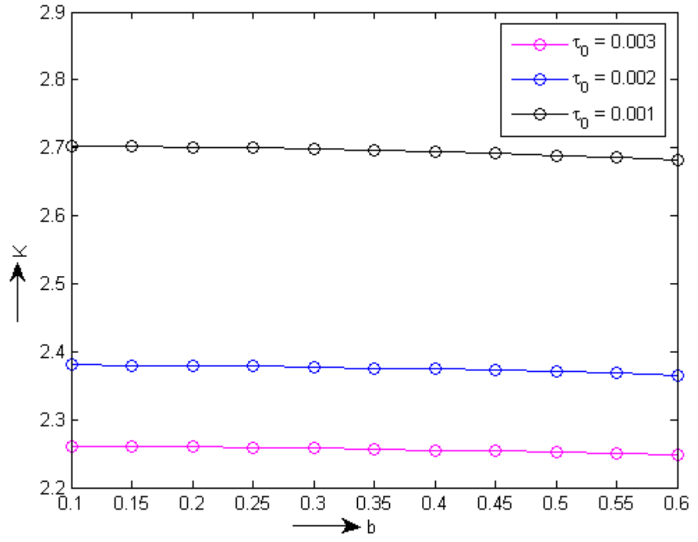

Fig. 5: Variation of Pressure drop K with respect to b for different values of yield stress.

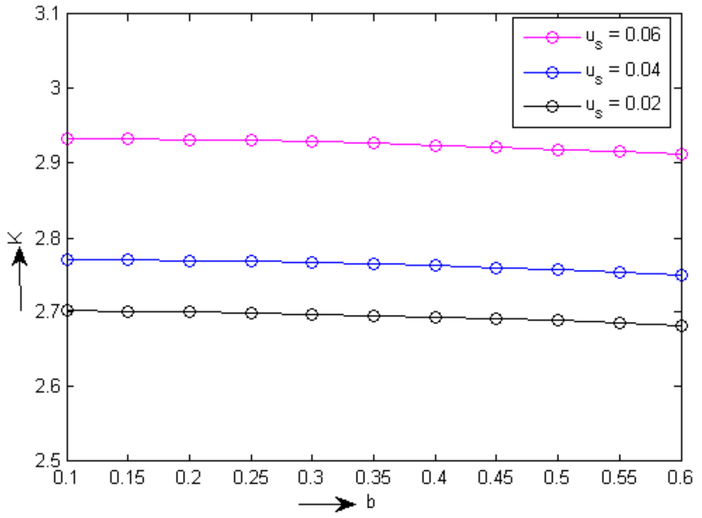

Fig. 6: Variation of Pressure drop K with respect to b for different values of slip velocity.

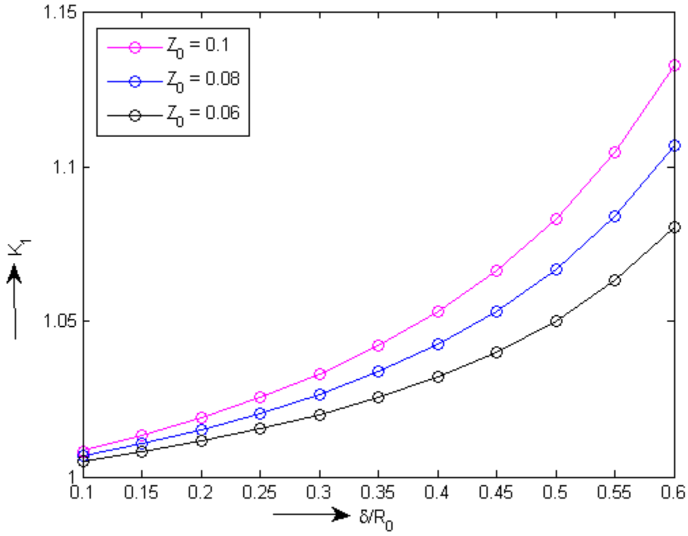

Fig. 7: Variation of Pressure drop K_1 with respect to stenosis size for the stenosis length.

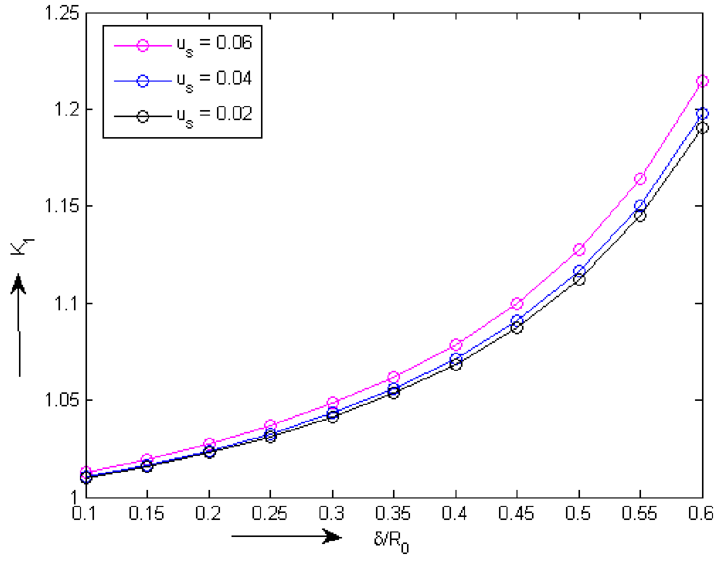

Fig. 8: Variation of Pressure drop K_1 with respect to stenosis size for the variation of slip velocity.

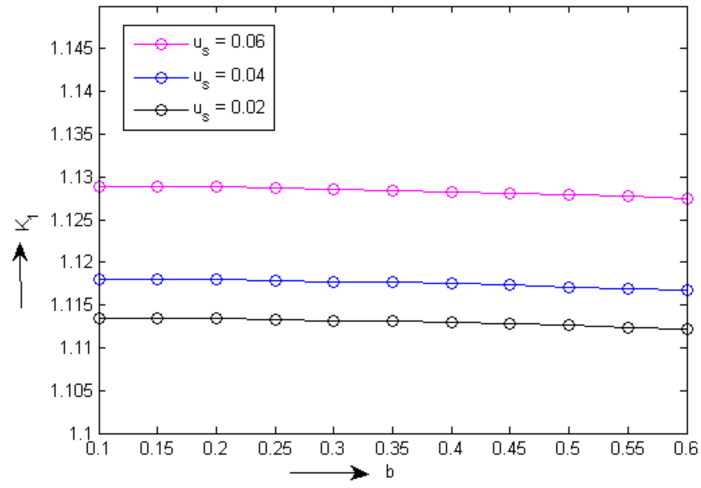

Fig. 9: Variation of Pressure drop K_1 with respect to b for the variation of slip velocity.

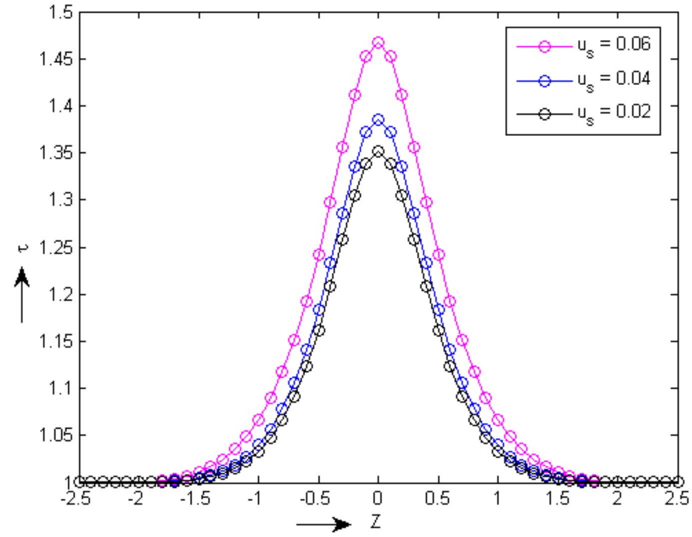

Fig. 10: Variation of shear stress with the variation slip velocity.

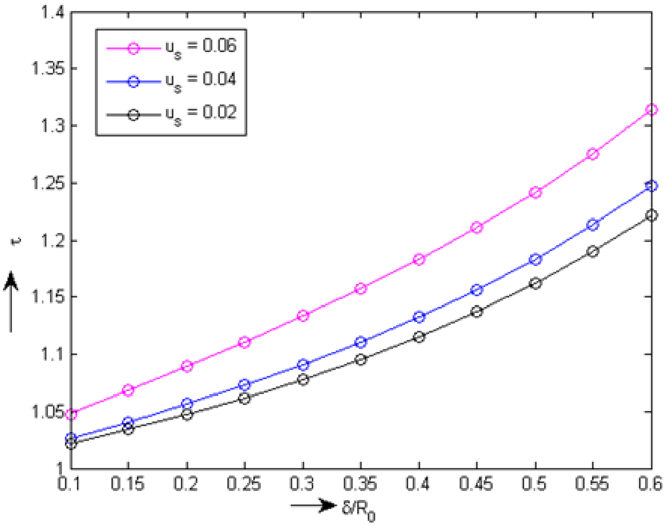

Fig. 11: Variation of shear stress across the stenosis size with the variation slip velocity.

V. CONCLUSIONS

The velocity profile, pressure drop and wall shear stress have been studied here in presence of slip velocity at the arterial wall. Many cardiovascular diseases such as stroke, hypertensions persist due to the arterial stenosis. In such diseased condition, blood behaves as a non-Newtonian fluid. It is seen that the axial velocity decreases with increase of stenosis size in presence of slip velocity, but pressure drop increases. So the present analysis may be useful for further study relating to blood flow characteristics in various types of cardiovascular diseases.

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