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Mathematical Model of Biological Oxygen Demand in Facultative Wastewater Stabilization Pond Based on Two-Dimensional Advection-Dispersion Model

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ABSTRACT: *I n this paper, we use the two dimensional advection-dispersion model for biological oxygen demand (BOD) assessment in facultative wastewater stabilization pond. This two dimensional model refers to the width and the length of the pond. We solve the model numerically by using finite difference method. We validate the model in Sewon, Yogyakarta, Indonesia facultative wastewater stabilization pond. Based on the error of the model and the sampling data, the model gives the good result in the assessment.*

Keywords**:** *Two dimensional advection-dispersion model, biological oxygen demand,, Facultative wastewater stabilization pond, finite difference method.*

I. INTRODUCTION

Quality of Groundwater and water in the river could be decreased that can be caused by direct disposal of wastewater. Wastewater treatment process is an important thing to decrease the pollutant concentration in the wastewater before it is disposed to the river or ground. Wastewater contains several parameters which are physic, chemist and biologic that can influence the oxygen dissolved in the water [1]. To reduce the pollutant concentration in the wastewater, some efforts can be applied such us using biological process that can be conducted to decrease the concentration of the organic material by relying in natural process that utilizes the existence of bacteria and algae contained in the wastewater [1,2,3,4]. This process can be done by collecting the wastewater in Wastewater Treatment Process Installation (WWTP). A WWTP contains a wastewater stabilization ponds for wastewater treatment and assessment.

A mathematical model can be used to approach the dynamic of the water flow such as flood prediction [5,6]or the dynamic of the pollutant concentration in the water such as groundwater assessment [7] and wastewater assessment by modeling the evolution of the pollutant concentration using some dynamic equation or methods such as Navier-stokes equation [8], multivariatechemometric techniques [9], dynamic equation of the organic matter [10], hydraulic-geometric dynamic equation [11] and dynamic model of stabilization pond [12]. Another mathematical equation that can be used for pollutant transport representation is advection-dispersion model. This model is adopting the advection-dispersion process to approach the evolution of the pollutant transport. The solution of this model can be determined numerically by using finite difference method [13] where the simulation of the model can be done numerically by using MATLAB [14].

In this paper, we approaching the evolution of the pollutant concentration in the domestic wastewater facultative stabilization pond by using two dimensional advection-dispersion dynamic equation.The pollutant concentration will be represented by the biological oxygen demand (BOD). The two-dimensional in the model refers to the surface of the pond i.e. the width and the length of the pond. This pollutant concentration evolution analysis can be used for pollutant assessment.

II. RESEARCHMETHODOLOGY

The two dimensional advection-dispersion model that we used to assess the BOD concentration will be validated using data sample in domestic wastewater facultative stabilization pond II at Wastewater Treatment Process Installation (InstalasiPengolahan Air Limbah/IPAL)Sewon,Bantul,Yogyakarta, Indonesia. This WWTP has 4facultative ponds and they use biological process in the wastewater treatment in the facultative ponds. The map of IPAL Sewon can be seen in **Fig. 1.**

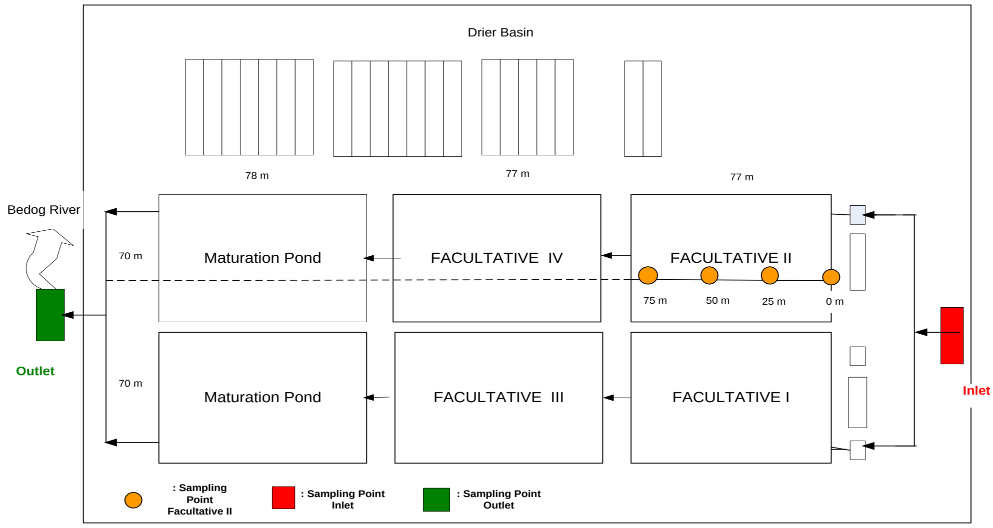

Fig. 1. Map of IPAL Sewon [1]

The dimension of the facultative pond IIis (77 x 70 x 4) m^3. We construct a grid or mesh points of the pond for data sampling of the distribution of the BOD concentration. The grid of sampling point is given by Fig. 2.

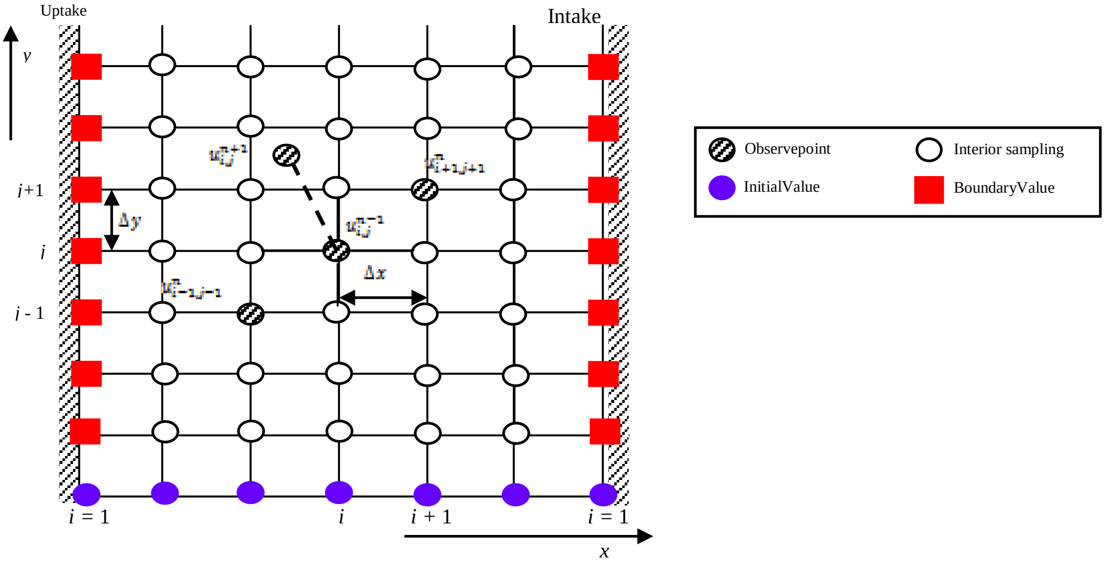

Figure2. Grid of Sampling Point

The sample at each point is taken at the surface. The two dimensional in the model refers to the width (x-direction) and the length (y-direction) of the pond and assumes that the distribution of the BOD in the z-direction (depth) equals the surface.

III. MATHEMATICAL MODEL

Let *L* denotes the length of the pond and *W* denotes the width of the pond. Following the advection-dispersion process of the BOD in the pond illustrated by Figure 3, the mathematical model of the BOD concentration is written as follows

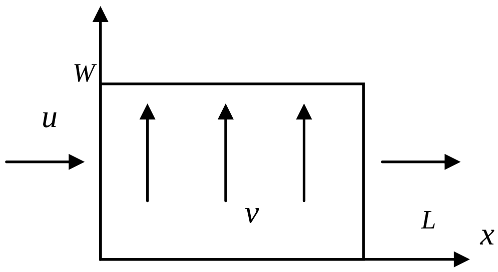

Fig. 3. The surface of the pond

$$\frac{\partial c}{\partial t} + u\frac{\partial (c)}{\partial x} + v\frac{\partial (c)}{\partial y} = D_{mx}\frac{\partial^2 c}{\partial x^2} + D_{my}\frac{\partial^2 c}{\partial y^2} \quad \text{... (1)}$$

where u is the velocity of the flow in the *x*-direction (LT^{-1}), v is the velocity of the flow in the *y*-direction (LT^{-1}) D_{mx} is dispersion constant in the *x*-direction(L^2T^{-1}), D_{my} is the dispersion constant in the *y*-direction (L^2T^{-1}) and C is the concentration of the BOD (ML^{-3}). To solve the model numerically, we use the finite difference method by substituting approximations $\frac{\partial c}{\partial t} = \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t}$, $\frac{\partial (c)}{\partial x} = \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n}}{2\Delta x}$, $\frac{\partial (c)}{\partial y} = \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n}}{2\Delta y}$, $\frac{\partial^2 c}{\partial x^2} = \frac{C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}}{(\Delta x)^2}$, and $\frac{\partial^2 c}{\partial y^2} = \frac{C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}}{(\Delta y)^2}$ into the model i.e.

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} + u\left[\frac{C_{i+1,j}^{n+1} - C_{i-1,j}^{n}}{2\Delta x}\right] + v\left[\frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n}}{2\Delta y}\right] = D_{mx}\left[\frac{C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}}{(\Delta x)^2}\right] + D_{my}\left[\frac{C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n}}{(\Delta y)^2}\right] \text{... (2)}$$

hence we have the following solution

$$\frac{C_{i,j}^{n=1}}{\Delta t} = \left(-\frac{2D_{mx}}{(\Delta x)^2} - \frac{2D_{my}}{(\Delta y)^2} + \frac{1}{\Delta t}\right)C_{i,j}^{n} + \frac{D_{mx}}{(\Delta x)^2}C_{i+1,j}^{n} + \frac{D_{mx}}{(\Delta x)^2}C_{i-1,j}^{n} + \frac{D_{my}}{(\Delta y)^2}C_{i,j+1}^{n} + \frac{D_{my}}{(\Delta y)^2}C_{i,j-1}^{n} - \frac{u}{2\Delta x}C_{i+1,j}^{n+1} + \frac{u}{2\Delta x}C_{i-1,j}^{n} - \frac{v}{2\Delta y}C_{i,j+1}^{n+1} + \frac{v}{2\Delta y}C_{i,j-1}^{n}$$

... (3)

where Δt, Δx, and Δy are the step size for time *t*, *x* and *y* respectively. The initial value of the model is obtained by data sampling given by **Tabel 1.**

Tabel 1. Initial Value of BOD concentration (mg/L) (Based on Sampling Point)

y (m)	**70**	98.20	81.32	88.54	67.58	55.32	56.84	49.52
	57.5	93.72						67.20
	46	64.20		51.32	49.52			55.54
	34.5	63.22		48.32	82.30	56.42		48.32
	23	58.54			67.32			52.22
	11.5	71.32						41.89
	0	42.59						42.32
		0	**12.5**	**25**	**37.5**	**50**	**62.5**	**77**
	x (m)							

We do the five degrees polynomial curve fitting to determine the initial value of each point on the grid. The obtained polynomial is

$f_1(x,y) = 39.2 + 2.177x + 3.524y + 0.1835x^2 - 0.323xy - 0.07977y^2 - 0.01245x^3 + 0.01188x^2y + 0.003769xy^2 - 0.002805y^3 + 0.0002236x^4 - 0.0001442x^3e - 5x^2y^2 - 3.96e - 5xy^3 + 0.00010094y^4$ -1.268e-06x^5+4.265e-07$x^4y + 9.697e - 07x^3y^2 - 3.789e - 07x^2y^3 + 4.019e - 07xy^4 - 8.66e - 07y^5$ illustrated by **Figure 5**.

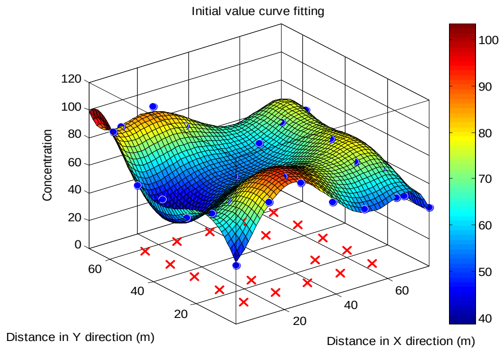

Figure 5. Initial value curve fitting

IV. RESULT AND DISCUSSION

We solve the model numerically with $\Delta x = 0.077$, $\Delta y = 0.07$, $\Delta t = 1, u = 2.10^{-4}, v = 2.10^{-4}$, $D_{mx} = 10^{-5}$ and $D_{my} = 10^{-5}$.The BOD concentration at time *t*=9 and time *t*=17 are shown by **Figure 6**. The initial value given by Figure 5 is used to solve the model. We simulate the model in Windows 8 AMD A6 2.7GHz of processor and 2GB of Memory by using software MATLAB®.

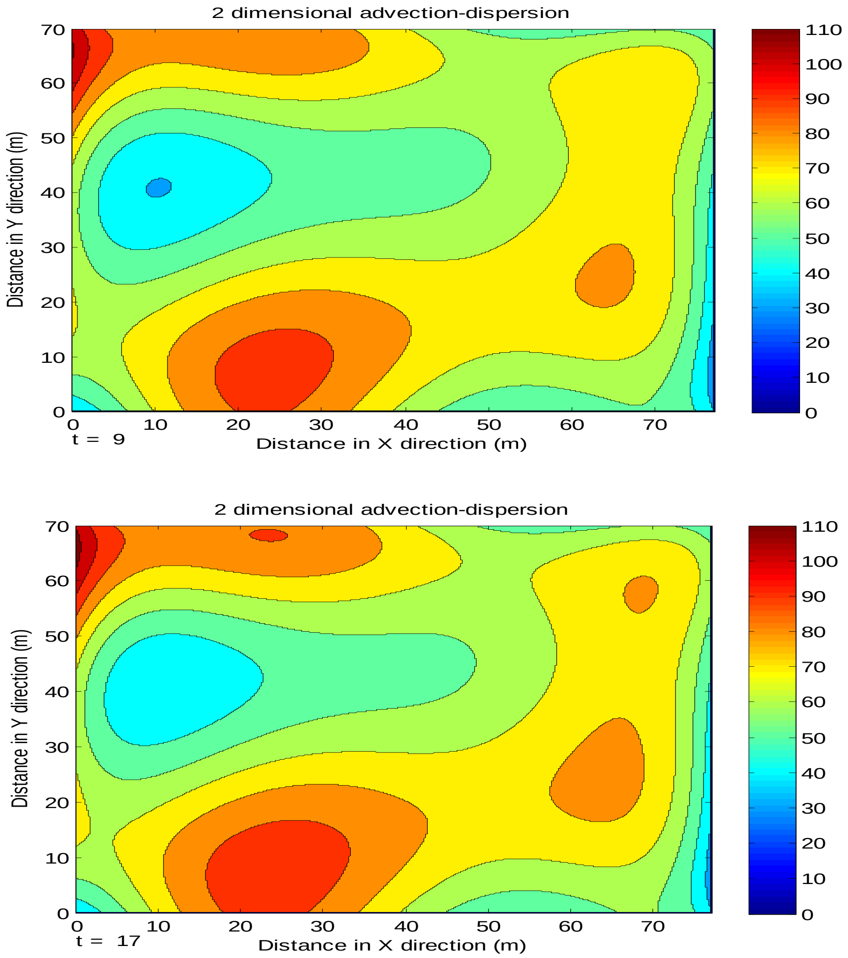

Figure 6. The Distribution of BOD concentration along the stabilization pond

Figure 6 shows the distribution of the BOD concentration at the surface of the pond. The distribution of the BOD concentration is derived by advection-dispersion phenomenon. The advection process generates the BOD concentration from one point to another point based on the advection constant. The dispersion process generates the BOD mass flux and gradient of the BOD concentration along time domain based on the dispersion constant.

V. CONCLUSION AND FUTURE WORK

In this paper, two dimensional advection-dispersion model was used to approach the distribution of the BOD concentration in a wastewater facultative stabilization pond. The model was validated using data sample in WWTP Sewon, Yogyakarta, Indonesia. This model can be used to assess the concentration of the BOD in the stabilization pond.In the future work, we will develop the assessment by using three-dimensional advection-dispersion model by generating 3D mesh points including depth of the pond. In the other work, will develop the method to optimize the biological process in the wastewater stabilization pond.

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