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# Weakly GeneralizedHomeomorphism in Intuitionistic Fuzzy Topology

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**ABSTRACT:** In this paper the concepts of weakly  $\pi$  generalized homeomorphism on intuitionistic fuzzy topological space is introduced with numerical examples. Some of their properties are investigated. **Keywords:** Intuitionistic fuzzy (IF) topology, IF weakly  $\pi$  generalized closed set and open set, IF completely weakly  $\pi$  generalized continuous mappings and closed mapping, IF weakly  $\pi$  generalized homeomorphism,  $IF_{w\pi}T_{1/2}$  space and  $IF_{w\pi g}T_q$  space.

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## I. INTRODUCTION

In 1965, the concept of Fuzzy set (FS) theory and its applications are first proposed by Zadeh [20]. It provides a framework to encounter uncertainty, vagueness and partial truth. It is represented by introducing degree of membership for each member of the universe of discourse to a subset of it. In 1968, the concept of fuzzy topology has been introduced by Chang [3]. They are referred as a concept and its context. The concept of FS has generalized into intuitionistic fuzzy (IF) by Atanassov [1] in 1986. After that many research articles have been published in the study of examining and exploring, how far the basic concepts and theorems, defined in crisp sets and in fuzzy sets remain true in IF sets. In 1997, Coker [4] has initiated the concept of generalization of fuzzy topology into IF topology. In his article, the apprehension of semi closed,  $\alpha$  closed, semi pre-closed, weakly closed are introduced. Further its properties are derived.

In this paper, the concept of IF weakly  $\pi$  generalized homeomorphism in IF topological space is introduced and suitable examples are given. Some of its characteristics are obtained. The inter-relationships among the various existed classes, which form IF homeomorphisms are established. Numerical illustrations are also given to substantiate the derived results.

This paper is organized into four sections. In the first section, historical development of the concepts is briefed. The basic definitions and results, needed for this work are listed in the second section. Section three discusses the IF weakly  $\pi$  generalized homeomorphism and suitable examples are given. Section four contains the Conclusion remarks.

#### **II. PRELIMINARIES**

**Definition 2.1**: [1] Let *X* be a non-empty crisp set. An intuitionistic fuzzy (IF) set *A*, in *X* is defined as an object of the form

 $A = \{ < x, \mu_A(x), \nu_A(x) > | x \in X \},\$ 

where the functions  $\mu_A(x) : X \to [0, 1]$  and  $\nu_A(x) : X \to [0, 1]$  denote respectively the degree of membership (briefly  $\mu_A$ ) and the degree of non-membership (briefly  $\nu_A$ ) of each element  $x \in X$  to the set A, for each  $x \in X$  $0 \le \mu_A(x) + \nu_A(x) \le 1$ . The collection of all IF sub-sets in X, is denoted by IFS(X).

**Definition 2.2**: [1] Let *A* and *B* be two different IFSs defined by,  $A = \{ < x, \mu_A(x), \nu_A(x) > | x \in X \}$ ,  $B = \{ < x, \mu_B(x), \nu_B(x) > | x \in X \}$ . The brief notation of  $A = \{ < x, \mu_A(x), \nu_A(x) > | x \in X \}$  and  $B = \{ < x, \mu_B(x), \nu_B(x) > | x \in X \}$  are  $A = < x, \mu_A, \nu_A >$  and  $B = < x, \mu_B, \nu_B >$  respectively.

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The operations  $\land$  and  $\lor$  are defined on  $\mu_A$ ,  $\mu_B$ ,  $\nu_A$ , and  $\nu_B$  as follows:

i)  $\mu_A \vee \mu_B = \max \{ \mu_A, \mu_B \},$ ii)  $\mu_A \wedge \mu_B = \min \{ \mu_A, \mu_B \},$ iii)  $\nu_A \vee \nu_B = \max \{ \nu_A, \nu_B \},$  and iv)  $\nu_A \wedge \nu_B = \min \{ \nu_A, \nu_B \}.$ 

Then,

i)  $A \subseteq B$ , if and only if,  $\mu_A \leq \mu_B$  and  $\nu_A \geq \nu_B$  for all  $x \in X$ , similarly  $A \supseteq B$  can be defined. ii) A = B, if and only if, both  $A \subseteq B$  and  $B \subseteq A$  are valid. iii)  $A^{C} = \{ \langle x, \mu_A', \nu_A' \rangle | x \in X \}$ , where  $\mu_A' = \nu_A$  and  $\nu_A' = \mu_A$ . iv)  $A \cap B = \{ \langle x, \mu_A \land \mu_B, \nu_A \lor \nu_B \rangle | x \in X \}$ , and v)  $A \cup B = \{ \langle x, \mu_A \lor \mu_B, \nu_A \land \nu_B \rangle | x \in X \}$ .

The intuitionistic fuzzy sets  $0_{\sim}$  and  $1_{\sim}$  are defined respectively as,  $0_{\sim} = \{ < x, 0, 1 > | x \in X \}$  and  $1_{\sim} = \{ < x, 1, 0 > | x \in X \}$ . The sets  $0_{\sim}$  and  $1_{\sim}$  are known as the empty IF set and the whole IF set of *X* respectively.

**Definition 2.3**: [4] An intuitionistic fuzzy topology (IFT) is a family  $\tau$  of IFS defined on X, satisfying the following axioms :

i)  $0_{\sim}, 1_{\sim} \in \tau$ ,

ii)  $G_1 \cap G_2 \in \tau$ , whenever  $G_1, G_2 \in \tau$ ,

iii)  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i | i \in J\} \subseteq \tau$ .

Then the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS) and any IFS in  $\tau$  are known as an intuitionistic fuzzy open set (IFOS) in X.

If, A is an IFOS, in an IFTS  $(X, \tau)$ , then its complement A<sup>C</sup> is called an intuitionistic fuzzy closed set (IFCS) in X.

**Definition 2.4**: [4] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in X. Then the intuitionistic fuzzy closure and an intuitionistic fuzzy interior are defined by,

 $cl(A) = \cap \{ G \mid G \text{ is an IFCS in } X \text{ and } A \subseteq G \}$ , and int  $(A) = \cup \{ K \mid K \text{ is an IFOS in } X \text{ and } K \subseteq A \}$ .

Note that for any IFS, A in X,  $cl(A^{C}) = (int(A))^{c}$  and  $int(A^{c}) = (cl(A))^{c}$ .

**Definition 2.5:** [4] An IFS,  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS( $X, \tau$ ) is said to be an, i) intuitionistic fuzzy closed set (IFCS) in  $X \Leftrightarrow cl(A) = A$ , and ii) intuitionistic fuzzy open set (IFOS) in  $X \Leftrightarrow int(A) = A$ .

**Definition 2.6**: [14] A subset A of a space  $(X, \tau)$  is called, i) regular open, if, A = int(cl(A)), and ii)  $\pi$  open, if, A is the union of regular open sets, symbolically A is an IF $\pi$ OS in X.

**Definition 2.7:** [5] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS $(X, \tau)$  is said to be an, i) intuitionistic fuzzy semi-closed set (IFSCS) if  $int(cl(A)) \subseteq A$ , and ii) intuitionistic fuzzy semi-open set (IFSOS) if  $A \subseteq cl(int(A))$ .

**Definition 2.8**: [18] Let A be an IFS of an IFTS  $(X, \tau)$ . Then the semi-closure of A (simply scl(A)) and semi- interior of A (simply sint(A)) are defined as,

i)  $scl(A) = \cap \{ G \mid G \text{ is an IFSCS in } X \text{ and } A \subseteq G \}$ , ii)  $sint(A) = \cup \{ K \mid K \text{ is an IFSOS in } X \text{ and } K \subseteq A \}$ .

**Result 2.1:** [16] Let A be an IFS in  $(X, \tau)$ , then

i)  $scl(A) = A \cup int(cl(A))$ , and

ii)  $sint(A) = A \cap cl(int(A)).$ 

**Definition 2.9:** [5] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS( $X, \tau$ ) is said to be an,

i) intuitionistic fuzzy  $\alpha$  closed set (IF $\alpha$ CS), if,  $cl(int(cl(A))) \subseteq A$ , and

ii) intuitionistic fuzzy  $\alpha$  open set (IF $\alpha$ OS), if,  $A \subseteq int(cl(int(A)))$ .

**Definition 2.10**: [11] Let  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS of an IFTS( $X, \tau$ ). Then, the  $\alpha$  closure of A ( $\alpha$  cl(A)) and  $\alpha$  interior of  $A(\alpha int(A))$  are defined as,

 $\alpha cl(A) = \cap \{ G \mid G \text{ is an } IF\alpha CS \text{ in } X \text{ and } A \subseteq G \}$ , and  $\alpha$  int (A) =  $\cup \{K \mid K \text{ is an } IF\alpha OS \text{ in } X \text{ and } K \subseteq A\}.$ 

**Result 2.2:** [12] Let A be an IFS in  $(X, \tau)$ , then,

i)  $\alpha cl(A) = A \cup cl(int(cl(A)))$ , and ii)  $\alpha$  int $(A) = A \cap int(cl(int(A)))$ .

**Definition 2.11**: An IFS,  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS $(X, \tau)$  is said to be an

i) intuitionistic fuzzy pre-closed set [5] (IFPCS) if,  $cl(int(A)) \subseteq A$ ,

ii) intuitionistic fuzzy regular closed set [5] (IFRCS) if, cl(int(A)) = A,

iii)intuitionistic fuzzy generalized closed set [17] (IFGCS) if,  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is an IFOS in X,

iv)intuitionistic fuzzy generalized semi closed set [13] (IFGSCS) if,  $scl(A) \subseteq U$  whenever  $A \subseteq U, U$  is an IFOS in X.

v)intuitionistic fuzzy  $\alpha$  generalized closed set [12] (IF $\alpha$ GCS) if,  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is an IFOS in Χ.

**Definition 2.12**: [7] An IFS, A is said to be an intuitionistic fuzzy weakly  $\pi$  generalized closed set (IFW $\pi$ GCS) in  $(X, \tau)$  if,  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and U is an IF $\pi$ OS in X.

The family of all IFW $\pi$ GCS of an IFTS (*X*,  $\tau$ ) is denoted by IFW $\pi$ GCS(*X*).

**Result 2.3:** Every IFCS, IFaCS, IFGCS, IFRCS, IFACS, IFaCS are IFW $\pi$ GCS [7] but the converse need not be true.

**Definition 2.13:** [7] An IFS, A is said to be an intuitionistic fuzzy weakly  $\pi$  generalized open set (IFW $\pi$ GOS) in  $(X, \tau)$  if, the complement  $A^c$  is an IFW $\pi$ GOS in X.

The family of all IFW $\pi$ GOS of an IFTS (X,  $\tau$ ) is denoted by IFW $\pi$ GOS (X).

**Definition 2.14**: [5] Let f be a mapping defined on an IFTS  $(X, \tau)$  into IFTS  $(Y, \sigma)$ . Then f is said to be intuitionistic fuzzy continuous (IF cts) if,  $f^{-1}(B) \in IFOS(X)$  for every  $B \in \sigma$ .

**Definition 2.15**: Let f be a mapping from an IFTS  $(X, \tau)$  into IFTS $(Y, \sigma)$ . Then f is said to be

- intuitionistic fuzzy weakly  $\pi$  generalized continuous mapping [8] (IFW $\pi$ G cts) if,  $f^{-1}(B)$  is an IFW $\pi$ GCS i) in  $(X, \tau)$  for every IFCS, B of  $(Y, \sigma)$ ,
- ii) intuitionistic fuzzy semi continuous mapping [19] (IFS cts) if,  $f^{-1}(B) \in \text{IFSO}(X)$ , for every  $B \in \sigma$ ,
- iii) intuitionistic fuzzy  $\alpha$  continuous mapping [19] (If  $\alpha$  cts) if,  $f^{-1}(B) \in IF\alpha O(X)$ , for every  $B \in \sigma$ , iv) intuitionistic fuzzy pre-continuous mapping [19] (IFP cts) if,  $f^{-1}(B) \in IFPO(X)$ , for every  $B \in \sigma$ , v) intuitionistic fuzzy completely continuous mapping [6] if,  $f^{-1}(B) \in IFPO(X)$ , for every  $B \in \sigma$ ,
- vi) intuitionistic fuzzy generalized continuous mapping [13] (IFG cts) if,  $f^{-1}(B) \in IFGO(X)$ , for every  $B \in$ σ,

- vii) intuitionistic fuzzy generalized semi continuous mapping [12] (IFGS cts) if,  $f^{-1}(B) \in \text{IFGSO}(X)$ , for every  $B \in \sigma$ , and
- viii) intuitionistic fuzzy  $\alpha$  generalized continuous mapping [12](IF $\alpha$ G cts) if,  $f^{-1}(B) \in IF\alpha$ GO (X), for every  $B \in \sigma$ .

**Definition 2.16**: [9] Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS. A mapping  $f: (X, \tau) \to (Y, \sigma)$  is called an intuitionistic fuzzy weakly  $\pi$  generalized closed mapping (IFW $\pi$ GCM) if, f(A) is an IFW $\pi$ GCS in Y, for every IFCS, A in X. In other words, every IFCS in X are mapped into IFW $\pi$ GCS in Y.

**Definition 2.17:** [10] Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS. A mapping  $f: (X, \tau) \to (Y, \sigma)$  is called an intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mapping (IF completely W $\pi$ G cts) if,  $f^{-1}(B)$  is an IFRCS in  $(X, \tau)$  for every IFW $\pi$ GCS, B of  $(Y, \tau)$ .

**Definition 2.18:** [7] An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $_{w\pi}T_{1/2}$  (IF  $_{w\pi}T_{1/2}$ ) space if, every IFW $\pi$ GCS in X is an IFCS in X.

**Definition 2.19:** [7] An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $_{w\pi g}T_q$  (IF  $_{w\pi g}T_q$ ) space, (0 < q < 1) if, every IFW $\pi$ GCS in X is an IFPCS in X.

## III. INTUITIONISTIC FUZZY WEAKLY GENERALIZEDHOMEOMORPHISM

The main objective of this section is to study the weakly  $\pi$  generalized homeomorphism defined on a topological spaces, in this connection some of its properties are obtained.

**Definition 3.1**: Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS. A bijection mapping  $f: (X, \tau) \to (Y, \sigma)$  is called an intuitionistic fuzzy weakly  $\pi$  generalized (IFW $\pi$ G) homeomorphism if, both the functions, f and  $f^{-1}$  are IFW $\pi$ G continuous mappings.

**Example 3.1**: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$ ,  $G_1 = \langle x, (0.2, 0.3, 0.4), (0.8, 0.6, 0.6) \rangle$  for  $x \in X$  and  $G_2 = \langle y, (0.8, 0.6, 0.6), (0.2, 0.3, 0.4) \rangle$  for  $y \in Y$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Define a bijection mapping  $f: (X, \tau) \to (Y, \sigma)$  by f(a) = u, f(b) = v and f(c) = w. Then, f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. Therefore f is an IFW $\pi$ G homeomorphism.

**Proposition 3.1**: Every IF homeomorphism is an IFW $\pi$ G homeomorphism.

**Proof:** Let  $f: (X, \tau) \to (Y, \sigma)$  be an IF homeomorphism. Then f and  $f^{-1}$  are IF continuous mappings. This implies f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. Therefore f is an IFW $\pi$ G homeomorphism.

Example 3.2: (Converse of Proposition 3.1 need not be true)

Let  $X = \{a, b, c\}, Y = \{u, v, w\}, G_1 = \langle x, (0.3, 0.4, 0.5), (0.7, 0.6, 0.5) \rangle$  for  $x \in X$  and

 $G_2 = \langle y, (0.7, 0.7, 0.6), (0.3, 0.3, 0.4) \rangle$  for  $y \in Y$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Consider a bijective mapping  $f: (X, \tau) \to (Y, \sigma)$  defined by f(a) = u, f(b) = v and f(c) = w. Then, f is an IFW $\pi$ G homeomorphism but not an IF homeomorphism, since f and  $f^{-1}$  are not IF continuous mappings.

**Proposition 3.2**: Every IF $\alpha$  homeomorphism is an IFW $\pi$ G homeomorphism.

**Proof:** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS. Suppose,  $f: (X, \tau) \to (Y, \sigma)$  is an IF $\alpha$  homeomorphism. Then both f and  $f^{-1}$  are IF $\alpha$  continuous mappings. This implies f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. Therefore f is an IFW $\pi$ G homeomorphism.

**Example 3.3**: (Converse of Proposition 3.2 need not be true)

Let  $X = \{a, b, c\}, Y = \{u, v, w\}, G_1 = \langle x, (0.4, 0.3, 0.2), (0.6, 0.7, 0.8) \rangle$  for  $x \in X$ , and

 $G_2 = \langle y, (0.7, 0.8, 0.8), (0.3, 0.2, 0.2) \rangle$  for  $y \in Y$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Define a bijective mapping  $f: (X, \tau) \to (Y, \sigma)$  defined by f(a) = u, f(b) = v and f(c) = w. Then, f is an IFW $\pi$ G homeomorphism but not an IF $\alpha$  homeomorphism, because f and  $f^{-1}$  are not IF $\alpha$  continuous mappings.

**Proposition 3.3**: Every IFG homeomorphism is an IFW $\pi$ G homeomorphism.

**Proof:** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS and  $f: (X, \tau) \to (Y, \sigma)$  be an IFG homeomorphism. Then f and  $f^{-1}$  are IFG continuous mappings. This implies f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. So, f is an IFW $\pi$ G homeomorphism.

## Example 3.4: (Converse of Proposition 3.3 need not be true)

Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$   $G_1 = \langle x, (0.2, 0.3, 0.4), (0.8, 0.7, 0.6) \rangle$  for  $x \in X$ , and  $G_2 = \langle y, (0.8, 0.8, 0.7), (0.2, 0.2, 0.2) \rangle$  for  $\in Y$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Consider a bijective mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  defined by f(a) = u, f(b) = v and f(c) = w. Then, f is an IFW $\pi$ G homeomorphism but not an IFG homeomorphism, since both f and  $f^{-1}$  are not IFG continuous mappings.

**Proposition 3.4**: Every IF $\alpha$ G homeomorphism is an IFW $\pi$ G homeomorphism.

**Proof:** Let  $f: (X, \tau) \to (Y, \sigma)$  be an IF $\alpha$ G homeomorphism. Then f and  $f^{-1}$  are IF $\alpha$ G continuous mappings. This implies that both f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. Therefore f is an IFW $\pi$ G homeomorphism.

**Example 3.5**: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $G_1 = \langle x, (0.5, 0.6, 0.6), (0.5, 0.4, 0.4) \rangle$ ,  $G_2 = \langle y, (0.6, 0.5, 0.5), (0.3, 0.5, 0.5) \rangle$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Consider a bijective mapping  $f: (X, \tau) \to (Y, \sigma)$ , given by f(a) = u, f(b) = v and f(c) = w. Then, f is an IFW $\pi$ G homeomorphism but not an IF $\alpha$ G homeomorphism, since f and  $f^{-1}$  are not IF $\alpha$ G continuous mappings.

**Proposition 3.5**: Let  $f: (X, \tau) \to (Y, \sigma)$  be a bijective mapping from an IFTS  $(X, \tau)$  onto an IFTS  $(Y, \sigma)$ . Then the following statements are equivalent:

- a) f is an IFW $\pi$ G open mapping,
- b) f is an IFW $\pi$ G closed mapping,
- c)  $f^{-1}: (Y, \sigma) \to (X, \tau)$  is an IFW $\pi$ G continuous mapping.

#### **Proof:**

(i) (a)  $\Rightarrow$  (b): Let A be an IFCS in X, then  $A^c$  is an IFOS in X. By Definition,  $f(A^c) = (f(A))^c$  is an IFW $\pi$ G open set in Y. Therefore f(A) is an IFW $\pi$ G closed set in Y. Thus f is an IFW $\pi$ G closed mapping.

(ii) (b)  $\Rightarrow$ (c): Let A be an IFCS in X. Since f is an IFW $\pi$ G closed mapping,  $f(A) = (f^{-1})^{-1}(A)$  is an IFW $\pi$ G closed set in Y. So,  $f^{-1}$  is an IFW $\pi$ G continuous mapping.

(iii)(c)  $\Rightarrow$ (a): Let A be an IF open set in X. By Definition  $(f^{-1})^{-1}(A) = f(A)$  is an IFW $\pi$ G open set in Y. Therefore f is an IFW $\pi$ G open mapping.

Hence from (i) to (iii), all the statements (a) to (c) in the proposition (3.5) are equivalent.

**Proposition 3.6**: Let  $f: (X, \tau) \to (Y, \sigma)$  be a bijective mapping from an IFTS  $(X, \tau)$  onto an IFTS  $(Y, \sigma)$ . If f is an IFW $\pi$ G continuous mapping, then the following statements are equivalent:

- a) f is an IFW $\pi$ G closed mapping,
- b) f is an IFW $\pi$ G open mapping,
- c) f is an IFW $\pi$ G homeomorphism.

**Note(3.1)**: The proof of the above proposition (3.6) is similar to the proof of the proposition (3.5).

**Remark 3.1**: The composition of two IFW $\pi$ G homeomorphism need not be an IFW $\pi$ G homeomorphism in general. It is illustrated by means of the following example(3.6).

**Example 3.6**: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $Z = \{p, q, r\}$ . Let  $G_1 = \langle x, (0.7, 0.5, 0.5), (0.3, 0.5, 0.5) \rangle$  for  $x \in X$ ,  $G_2 = \langle y, (0.7, 0.8, 0.8), (0.3, 0.2, 0.2) \rangle$ 

for  $y \in Y$  and  $G_3 = \langle z, (0.8, 0.8, 0.7), (0.2, 0.2, 0.3) \rangle$  for  $z \in Z$ . Then,  $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$  and

 $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$  and  $\pounds = \{0_{\sim}, G_3, 1_{\sim} \}$  are IFTs on X, Y and Z respectively. Define a bijection mapping  $f: (X, \tau) \to (Y, \sigma)$  by f(a) = u, f(b) = v and f(c) = w and  $g: (Y, \sigma) \to (Z, \pounds)$  by g(u) = p, g(v) = q and g(w) = r. Then, both the functions f and  $f^{-1}$  are IFW $\pi$ G continuous mappings. Also g and  $g^{-1}$  are IFW $\pi$ G continuous mappings. Therefore f and g are IFW $\pi$ G homeomorphism. But the composition  $g \circ f: (X, \tau) \to (Z, \pounds)$  is not an IFW $\pi$ G homeomorphism, since  $g \circ f$  is not an IFW $\pi$ G continuous mapping.

**Proposition 3.7**: Let  $f:(X,\tau) \to (Y,\sigma)$  be an IFW $\pi$ G homeomorphism from an IFTS  $(X,\tau)$  into an IFTS  $(Y,\sigma)$ . Then *f* is an IF homeomorphism if  $(X, \tau)$  and  $(Y, \sigma)$  are IF<sub>w</sub> $_{\pi}T_{1/2}$  spaces.

**Proof**: Let *B* be an IFCS in *Y*. By definition,  $f^{-1}(B)$  is an IFW $\pi$ GCS in *X*. Since  $(X, \tau)$  is an IF  $_{w\pi}T_{1/2}$  space,  $f^{-1}(B)$  is an IFCS in *X*. So, *f* is an IF continuous mapping. Also by definition,  $f^{-1}: (Y, \sigma) \rightarrow$  $(X, \tau)$  is an IFW $\pi$ G continuous mapping. Let *A* be an IFCS in *X*. Then  $(f^{-1})^{-1}(A) = f(A)$  is an IFW $\pi$ GCS in Y, by definition. Since  $(Y, \sigma)$  is an

IF  $_{w\pi}T_{1/2}$  space, f(A) is an IFCS in Y. Therefore  $f^{-1}$  is an IF continuous mapping. Thus f is an IF homeomorphism.

**Proposition 3.8**: Let  $(X, \tau)$  and  $(Y, \sigma)$  be two IFTS. Suppose,  $f: (X, \tau) \to (Y, \sigma)$  and

 $g: (Y, \sigma) \to (Z, \pounds)$  be two IFW $\pi$ G homeomorphisms and  $(Y, \sigma)$  is an IF $_{w\pi}T_{1/2}$  space. Then  $g \circ f$  is an IFW $\pi$ G homeomorphism.

**Proof:** Let A be an IFCS in Z. Since  $g: (Y, \sigma) \to (Z, E)$  is an IFW $\pi$ G continuous mapping,  $g^{-1}(A)$  is an IFW $\pi$ GCS in Y. Then  $g^{-1}(A)$  is an IFCS in Y as  $(Y, \sigma)$  is an IF $_{w\pi}T_{1/2}$  space. Also since  $f: (X, \tau) \to (Y, \sigma)$  is an IFW $\pi$ G continuous mapping,  $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$  is an IFW $\pi$ GCS in X. Therefore  $g \circ f$  is an IFW $\pi$ G continuous mapping.

#### **IV. CONCLUSION**

In this paper, a particular type of intuitionistic fuzzy homeomorphism, namely intuitionistic fuzzy weakly  $\pi$  generalized homeomorphism is defined. The relationships among the various existed classes, which form IF homeomorphisms are established. By means of suitable numerical examples, it is established that the converse of the propositions describing the properties, need not be true.

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