American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-10, pp-83-89
www.ajer.org
Research Paper Open Access

Computer Aided Analysis of Four Bar Mechanism

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ABSTRACT: *Analysis of four bar mechanism was done and set of equations were derived and solved and the solutions were converted to Fotran computer programming. The results obtained are precise and accurate when compared with graphical and analytical solutions. Furthermore the approach saves time when compared to the time spent on obtaining solution with graphical and analytical methods. It also eliminates the possibilities of errors if the simple rules are known and strictly adhered to; it reduces the high skill requirement by the other methods. Finally, it guarantees the accurate result of velocity and acceleration in the mechanism. This study is limited to four bar crank mechanism. It is recommended that similar computer aided analysis should be carried out on other mechanisms.*

Keywords: *Mechanism, Velocity, Acceleration, Analysis, Fotran.*

I. INTRODUCTION

Over the years, the problem of mechanism has received an appreciable attention by many Mechanical Engineers. Owing to this, vigorous work has been done and some methods have been adopted to solve the problem. These include graphical method (for a single position of the crank) (Robert A. Becker, 1954), instantaneous centre method, and relative velocity method (for velocity in mechanisms only), (Ham C.W., 1958). Analytical, Kliein's construction, Rennet's construction and Rither Lans construction methods can also be used, (Timoshenko P., 1972). Most of these methods have one short coming or the other and some have not given accurate result but rather generate approximated results. Examples of these are analytical method and construction method, (Khumi R.S. and Cupta J.K., 2003). Some are time consuming in use e.g. Graphical method and Analytical method, (Shigley, J., 1980). Even other may prove cumbersome in use.

Sequel to short comings in the previous methods and approaches, this study used computer aided analysis approach. Also, computer program is generated using Fotran programming language.

Objective of the Study

This study analyzes the linear and angular velocity and acceleration of four bar mechanism taking all the associated parameter into consideration. Based on this, the overall objective of the study is to use computer aided approach to solve four bar mechanism problem both in mechanical system and robotic engineering. The specific objectives are:

i. To provide the use of computer application to determine the velocity and acceleration of the mechanism, in order to produce a desired output motion for given input motion.
ii. To provide an approach which is so simple, less skilled, time and labour saving in a way that if the simple rules are strictly adhered to a workman with most elementary knowledge of the computer will be able to get the desired result.
iii. To eliminate the possibilities of errors if the simple rules are known and strictly adhered to.
iv. To reduce the time consumption which are required by the other methods.
v. To reduce the high skill requirement by the other methods.
vi. To guarantee the accurate result of velocity and acceleration in the links of mechanism.

II. MATERIALS AND METHOD

In this study, linear and angular velocity and acceleration of four bar mechanism are analyzed and the associated equations were generated after which the equations are developed into Qbasic computer program. The computer program was tested and was found satisfactory for solving four bar mechanism problem.

III. RESULTS AND DISCUSSIONS

In developing a computer program for the mechanism, the following are put into consideration:

i. Four bar mechanism was analyzed

ii. The vector diagram is considered to develop the required velocity and acceleration equations.

iii. Qbasic is used for the programming of the results obtained from the analysis.

Considering a four bar mechanism ABCD, as shown in **Fig.1**

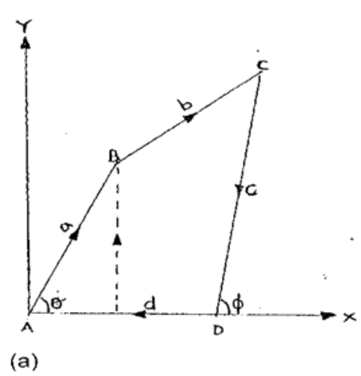


Fig. 1 Four bar mechanism

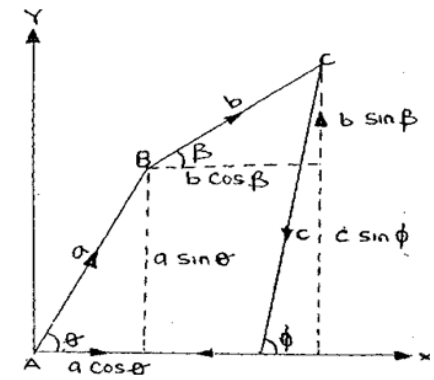


Fig. 2 Component along X-axis

AB = a, BC =b, CD= c and DA=d. The link AD is fixed and lies along X-axis. Let the link AB, BC and DC make angles θ, β and ϕ respectively along the X-axis or link AD

For equilibrium of the mechanism, the sum of the component along X-axis must be equal to zero. First of all, taking the sum of the component along X-axis as shown in **Fig. 2** , we have

$a\cos\theta + b\cos\beta - c\cos\phi - d = 0$ **(i)**

$b\cos\beta = c\cos\phi + d - a\cos\theta$

Squaring both sides

$b^2\cos^2\beta = (c\cos\phi + d - a\cos\theta)^2$

$b^2\cos^2\beta = c^2\cos^2\phi + d^2 + a^2\cos^2\theta + 2cd\cos\phi - 2ad\cos\theta - 2ac\cos\phi\cos\theta$ **(ii)**

Now taking the sum of the component along Y-axis, we have

$a\sin\theta + b\sin\beta - c\sin\phi = 0$ **(iii)**

$b\sin\beta = c\sin\phi - a\sin\theta$

Squaring both sides

$b^2\sin^2\beta = (c\sin\theta - a\sin\phi)^2 = c^2\sin^2\phi + a^2\sin^2\theta - 2ac\sin\phi\sin\theta$ **(iv)**

Adding equation (ii) and (iv)

$b^2\cos^2\beta + b^2\sin^2\beta = c^2(\cos^2\phi + \sin^2\phi) + d^2 + 2cd\cos\phi + a^2(\cos^2\theta + \sin^2\theta) - 2ac(\cos\phi\cos\theta + \sin\phi\sin\theta) - 2ad\cos\theta$

Or

$b^2 = c^2 + d^2 + 2cd\cos\phi + a^2 - 2ac(\cos\phi\cos\theta + \sin\phi\sin\theta) - 2ad\cos\theta$

$2ac(\cos\phi\cos\theta + \sin\phi\sin\theta) = a^2 - b^2 + c^2 + d^2 + 2cd\cos\phi - 2ad\cos\theta$

$$\cos\phi\cos\theta + \sin\phi\sin\theta = \frac{1}{2ac}(a^2 - b^2 + c^2 + d^2) + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta \quad \textbf{(v)}$$

Let $d/a = k_1$, $d/c = k_2$ and $(a^2 - b^2 + c^2 + d^2)/2ac = k_3$

Equation **(v)** may be written as :-

$\cos\phi\cos\theta + \sin\phi\sin\theta = k_1\cos\phi - k_2\cos\theta + k_3$ **(vi(a))**

$\cos(\phi - \theta) = k_1\cos\phi - k_2\cos\theta + k_3$ **(vi(b))**

Since it is very difficult to determine the value of ϕ for the given value of θ from equation **(vi(a))** and **(vi(b))**, therefore, it is necessary to simplify this equation.

From trigonometrically ratios, we know that

$$\sin\phi = \frac{2\tan(\phi/2)}{1 + \tan^2(\phi/2)} \quad \text{and} \quad \cos\phi = \frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)}$$

Substituting the values of sinϕ and cosϕ in equation **(vi(a))**

$$\left(\frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)}\right)\cos\theta + \left(\frac{2\tan(\phi/2)}{1 + \tan^2(\phi/2)}\right)\sin\theta = k_1\left(\frac{1 - \tan^2(\phi/2)}{1 + \tan^2(\phi/2)}\right) - k_2\cos\theta + k_3$$

$$\cos\theta\,[1 - \tan^2(\phi/2)] + \sin\theta\,[2\tan(\phi/2)] = k_1[1 - \tan^2(\phi/2)] - k_2\cos\theta\,[1 + \tan^2(\phi/2)] + k_3\,[1 + \tan^2(\phi/2)]$$

$$\cos\theta\,[1 - \tan^2(\phi/2)] + 2\sin\theta\tan(\phi/2) = k_1 - k_1\tan^2(\phi/2) - k_2\cos\theta + k_2\cos\theta\tan^2(\phi/2) + k_3 + k_3\tan^2(\phi/2)$$

Rearranging this equation

$$\cos\theta\,[1 - \tan^2(\phi/2)] + k_1\tan^2(\phi/2) - k_2\cos\theta\tan^2(\phi/2) - k_3\tan^2(\phi/2) + 2\sin\theta\tan(\phi/2) = -\cos\theta + k_1 - k_2\cos\theta + k_3$$

$$\cos\theta - \cos\theta\tan^2(\phi/2) + k_1\tan^2(\phi/2) - k_2\cos\theta\tan^2(\phi/2) - k_3\tan^2(\phi/2) + 2\sin\theta\tan(\phi/2) = -\cos\theta + k_1 - k_2\cos\theta + k_3$$

$$\tan^2(\phi/2)\,[-\cos\theta + k_1 - k_2\cos\theta - k_3] + 2\sin\theta\tan(\phi/2) + 2\cos\theta + k_1 - k_2\cos\theta + k_3 = 0$$

$$-[(1+k_2)\cos\theta + k_3 - k_1]\tan^2(\phi/2) + (2\sin\theta)\tan(\phi/2) + [k_1+k_3 - (2 + k_2)\cos\theta] \qquad \textbf{(vii)}$$

Or

$$A\tan^2(\phi/2) + B\tan(\phi/2) + C = 0$$

Where

$$\left.\begin{array}{lll} A & = & -[(1+k_2)\cos\theta + k_3 - k_1] \\ B & = & (2\sin\theta) \\ C & = & [k_1 + k_3 - (2+k_2)]\cos\theta \end{array}\right\} \qquad \textbf{(viii)}$$

The equation **(vii)** is quadratic equation, its two roots are (ϕ/2)

$$\tan(\phi/2) = \frac{-B \pm \sqrt{(B^2 - 4AC)}}{2A}$$

$$\phi = 2\tan^{-1} - B \pm \left(\frac{\sqrt{(B^2 - 4AC)}}{2A}\right) \qquad \textbf{(ix)}$$

From equation (ix) we can find the position of output link CD
(that is angle ϕ). If the length of the links (a, b, c and d) and the position of the input link AB (that is angle θ) are known.

If the relation between the position of input link AB (that is angle θ) and the position of coupler link BC (that is angle β) is required, then angle ϕ from equations (i) and (iii) should be determined.
Then equation (i) may be written as:

$$c\cos\phi = a\cos\theta + b\cos\beta - d \qquad \textbf{(x)}$$

Squaring both sides

$$c^2\cos^2\phi = a^2\cos^2\theta + b^2\cos^2\beta + d^2 + 2ab\cos\theta\cos\beta - 2ad\cos\theta - 2bd\cos\beta \qquad \textbf{(xi)}$$

Also, $$\sin\phi = a\sin\theta + b\sin\beta \qquad \textbf{(xii)}$$

Squaring both sides,

$$c^2\sin^2\phi = a^2\sin^2\theta + b^2\sin^2\beta + 2ab\sin\theta\sin\beta \qquad \textbf{(xiii)}$$

Adding equations **(xi)** and **(xiii)**

$$c^2(\cos^2\phi + \sin^2\phi) = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\beta + \sin^2\beta) + 2ab\,(\cos\theta\cos\beta + \sin\theta\sin\beta) + d^2 - 2ad\cos\theta - 2db\cos\beta$$

Or

$$2ab\,(\cos\theta\cos\beta + \sin\theta\sin\beta) = c^2 - a^2 - b^2 - d^2 + 2ad\cos\theta + 2bd\cos\beta$$

$$\cos\theta\cos\beta + \sin\theta\sin\beta = \frac{c^2 - a^2 - b^2 - d^2}{2ab} + \frac{d}{b}\cos\theta + \frac{d}{a}\cos\beta \qquad \textbf{(xiv)}$$

Let $(d/a) = k_1$, $(d/b) = k_4$ and $(c^2 - a^2 - b^2 - d^2) = k_5$

Equation **(xiv)** may be written as:-

$$\cos\theta \cos\beta + \sin\theta \sin\beta = k_1 \cos\beta + k_4 \cos\theta + k_5 \qquad \textbf{(xv)}$$

From trigonometric ratio

$$\sin\beta = \frac{2 \tan(\beta/2)}{1 + \tan^2(\beta/2)}$$

$$\cos\beta = \frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}$$

Substituting these value of sinβ and cosβ in equation **(xv)**

$$\cos\theta\left(\frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}\right) + \sin\theta\left(\frac{2\tan(\beta/2)}{1 + \tan^2(\beta/2)}\right) = k_1\left(\frac{1 - \tan^2(\beta/2)}{1 + \tan^2(\beta/2)}\right) + k_4 \cos\theta + k_5 \qquad \textbf{(xvi)}$$

Opening the bracket,

$$\cos\theta\,[1 - \tan^2(\beta/2)] + 2\sin\theta \tan(\beta/2) = k_1\,[1 - \tan^2(\beta/2)] + k_4 \cos\theta\,[1 + \tan^2(\beta/2)] + k_5\,[1 + \tan^2(\beta/2)]$$

$$\cos\theta - \cos\theta \tan^2(\beta/2) + 2\sin\theta \tan(\beta/2) = k_1 - k_1 \tan^2(\beta/2) + k_4 \cos\theta \tan^2(\beta/2) + k_5 + k_5 \tan^2(\beta/2)$$

$$-\cos\theta \tan^2(\beta/2) + k_1 \tan^2(\beta/2) - k_4 \cos\theta \tan^2(\beta/2) - k_5 \tan^2(\beta/2) + 2\sin\theta \tan(\beta/2) - k_1 - k_4 \cos\theta - k_5 + \cos\theta = 0$$

$$-\tan^2(\beta/2)\,[(k_4 + 1)\cos\theta + k_5 - k_1] + 2\sin\theta \tan(\beta/2) - [(k_4 - 1)\cos\theta - k_5 + k_1] = 0$$

Or

$$[(k_4 + 1)\cos\theta + k_5 - k_1] \tan^2(\beta/2) + (-2\sin\theta)\tan^2(\beta/2) + [(k_4 - 1)\cos\theta - k_5 + k_1] = 0 \qquad \textbf{(xvii)}$$

Or

$$D \tan^2(\beta/2) + E \tan(\beta/2) + F = 0$$

$$D = (k_4 + 1)\cos\theta + k_5 - k_1$$

$$E = -2\sin\theta$$

$$A = (k_4 - 1)\cos\theta - k_5 + k_1 \qquad \textbf{(xviii)}$$

Equation (xvii) is a quadratic equation in tan (β/2)

Its two roofs are $\tan(\beta/2) = \dfrac{-E \pm \sqrt{(E^2 - 4DF)}}{2D}$

$$\beta = 2\tan^{-1}\left(\frac{-E \pm \sqrt{(E^2 - 4DF)}}{2D}\right) \qquad \textbf{(xix)}$$

From equation **(xix)**, we can find the position of coupler link BC (i.e angle β)

Analyzing the mechanism to get the velocity,

Let

W_1 = Angular velocity of the link AB = $\dfrac{d\theta}{dt}$

W_2 = angular velocity of the link BC = $\dfrac{d\beta}{dt}$

W_3 = Angular velocity of the link CD = $\dfrac{d\phi}{dt}$

Differentiating equation (i) and other equations with respect to time, it can be shown that

$$-aw_1 \sin\theta - bw_2 \sin\beta + cw_3 \sin\phi = 0 \qquad \textbf{(xx)}$$

$$w_3 = \frac{-aw_1 \sin(\theta - \beta)}{c \sin(\phi - \beta)} \qquad \textbf{(xxi)}$$

$$w_2 = \frac{-aw_1 \sin(\theta - \phi)}{b \sin(\beta - \phi)} \qquad \textbf{(xxii)}$$

From equation **(xxi)** and **(xxii)**, we can find w_3 and w_2, if a, b, c, θ, ϕ, β and w_1 are known

In analyzing the mechanism to get the acceleration,

Let

$\propto_1$ = Angular acceleration of the link AB = $\frac{dw_1}{dt}$

$\propto_2$ = Angular acceleration of the link BC = $\frac{dw_2}{dt}$

$\propto_3$ = Angular acceleration of the link CD = $\frac{dw_3}{dt}$

Differentiating equation (xxi) and other equations with respect to time

$$- a\left(w_1 \cos\theta \frac{d\theta}{dt} + \sin\theta \frac{dw_1}{dt}\right) - b\left(w_2 \cos\beta \frac{d\beta}{dt} + \sin\beta \frac{dw_2}{dt}\right) + c\left(w_3 \cos\phi \frac{d\phi}{dt} + \sin\phi \frac{dw_3}{dt}\right) = 0$$

or

$$- aw_1^2 \cos\theta - a\propto_1 \sin\theta \propto_1 - bw_2^2 \cos\beta - b\propto_2 \sin\beta + cw_3^2 \cos\phi + c\propto_3 \sin\phi = 0 \quad \textbf{(xxiii)}$$

$$\propto_2 = aw_1^2 \cos(\theta - \phi) - a\propto_1 \sin(\theta - \phi) + bw_2^2 \cos(\beta + \phi) - \frac{cw_3^2 (\cos\phi \cos\phi - \cos\phi \sin\phi) + c\propto_3 (\sin\phi \cos\phi - \sin\phi \sin\phi)}{b \sin(\beta - \phi)} \quad \textbf{(xxiv)}$$

$$\alpha_3 = + aw_1^2 \cos(\theta - \beta) + a\alpha_1 \sin(\theta - \beta) + bw_2^2 - \frac{cw_3^2 (\cos\phi \cos\beta - \cos\phi \sin\beta)}{c(\sin\phi \cos\beta - \sin\phi \sin\beta)} \quad \textbf{(xxv)}$$

In designing the computer program for the generated equations, the detailed specification of the solution to the problem are represented by the detailed logical stages to the solution in form of a descriptive language called Pseudo code or a diagrammatic representation in form of a flow chart. As such, seven stages will be taken into consideration in developing the program. These are:

1) Logarithm preparation
2) Algorithm preparation
3) Flow chart design
4) Coding into a high level language
5) Input preparation
6) Compilation
7) Correction (De bugging)
8) Testing process.

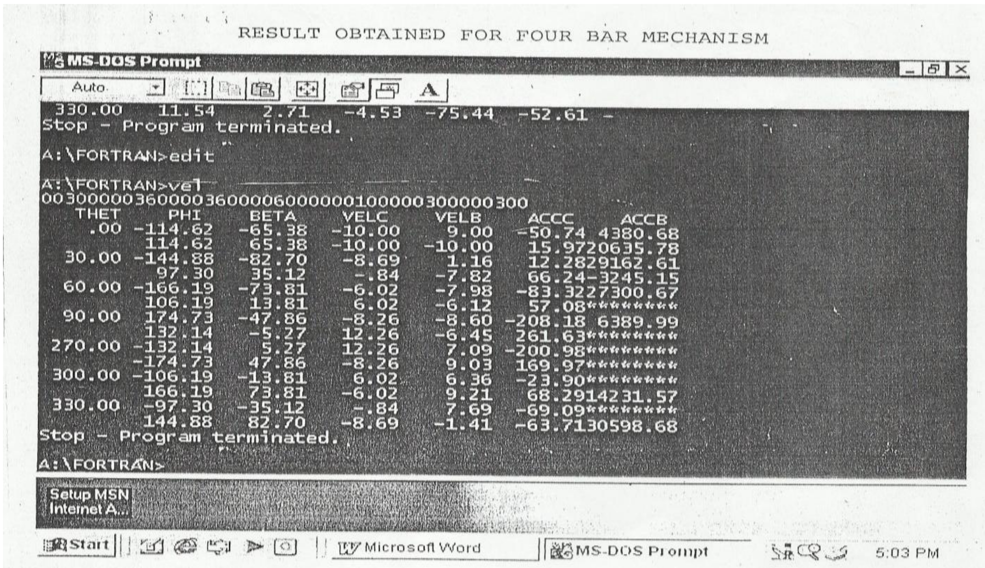
The computer program developed is shown in **index I**

INDEX I

```
C       PROGRAM TO FIND THE VELOCITY AND ACCELERATION IN A   FOUR-BAR MECHANISM
        DIMENSION PH (2), PHI (2), PP (2), BET (2), BT (2), VELC (2), VELB (2)
        DIMENSION
        ACCC (2), ACCB (2), C1 (2), C2 (2), C3 (2), C4 (2), B1 (2), B2 (2)
        DIMENSION B3 (2), B4 (2)
C       READ (*, *) A, B, C, D, VELA, ACCA, THETA
        READ (*, 2) A, B, C, D, VELA, ACCA, THETA
        2 FORMAT (7F6.1)
        PI= 4.0 * ATAN (1.0)
        THET= 0
        IHT= 180/THETA
        DTHET= PI/IHT
        DO 10   J=1, 2 * ITH
        THET = (J-1) * DTHET
        AK= (A*A- B*B + C*C + D*D) * 0.5)
        TH= THET * 180/PI
```

```
          AA= AK-A* (D-C) * COS (THET)- (C*D)
          BB= -2.0 *A*C*SIN (THET)
          CC= AK-A* (D+C) * COS (THET) + (C*D)
          AB=BB**2-4*AA*CC
          IF (AB.LT.0) GOTO 10
          PHH= SQRT (AB)
          PH (1)= -BB+PHH
          PH (2)= -BB –PHH
          DO 9     I=1,2
          PH I (I)= ATAN (PH (I) * 0.5/AA)*2
          PP (I)I= PHI (I) *180/PI
          BET (I)=ASIN ( C*SIN (PHI (I))- A*SIN (THET))/B)
          BT (I)= BET (I) *180/PI
          VELC (I)= A *VELA*SIN (BET (I)-THET)/ (C*SIN (BET (I)-PHI (1)))
          VELB (I)= (A*VELA*SIN (PHI (I)- THET))/ (B*SIN (BET (I)- (I)))
          C1 (I)= A* ACCA*SIN (BET (I)- THET)
          C2 (I)=A*VEL**2*COS (BET (I)- THET) + B*VELB (I)**2
          C3 (I)= C*VELC (I) **2*COS (PHI (I)-BET (I))
          C4 (I)=C* SIN (BET (I)- PHI (I))
          ACCC (I)= (C1 (I)-C2 (I) + C3 (I))/ C4 (I)
          BI (I)= A* ACCA*SIN (PHI (I)- THET)
          B2(I)= A*VELA**2*COS (PHI (I)- THET)
          B3 (I)= B*VELB (I) **2* COS (PHI (I)- BET (I))- C* VELC (I)**2
          B4 (I)=B* (SIN (BET (I)-PHI (I) ))
9         ACCB (I)= B1 (I)- B2 (I)-B3 (I)/ B4 (I)
          IF (J.NE.1) GOTO 8
          WRITE (*, 7)
7         FORMAT (4X, ‘THET’, 4X, ‘PHT’ 4X, ‘BETA’, 4X, ‘VELC’ 4X, ‘VELB’ 4X, ‘ACCC’
          1, 4X, ‘ACCB’)
8         WRITE (*, 6) TH, PP (1), BT (1), VELC (1), VELB (1), ACCC (1), ACCB (1)
          6 FORMAT (8F8.2)
          WRITE (*,5) PP (2), BT (2), VELC (2), VELB (2), ACCC (2), ACCB (2)
5         FORMAT (8X, 8F8.2)
10        CONTINUE
          STOP
          END
```

Plate1 shows the result of the computer program when it was tested. **VEL** is used to open the computer program.



IV. CONCLUSION

In the attempt to solve four bar mechanism problem, sets of equations were generated and Fotran computer programming language was developed from the equations to solve the mechanism problem. When the program was tested, the following advantages were discovered:

1) It provides an approach which is so simple and labour saving in a way that if the simple rules are strictly adhered to a workman with most elementary knowledge of the computer will be able to get the desired result.
2) It eliminates the possibilities of errors if the simple rules are known and strictly adhered to.
3) It reduces the time consumption which are required by the other methods
4) It reduces the high skill requirement by the other methods.
5) It guarantees the accurate result of velocity and acceleration in the mechanism.

REFERENCES

[1]. Ham C.W. (1958), Mechanics of Machine, Graw-Hill Book Company.
[2]. Khumi, R.S and Gupta J.K. (2003), Theory of Machine, Eurasia Publishing House Ltd, 7361, Ram Nagar, New Delhi.
[3]. Robert A. Becker (1954), Introduction to Theoretical Mechanism, MC Graw-Hill Book Company New-York, London, Toronto.
[4]. Shigley, J. (1980), Theory of Machines Mechanisms, MC GRAW-Hill Book New York, Toronto, London.
[5]. Timoshenko, P. (1972), Mechanics of Materials, Sternly Thornes (Publishers) Limited.