American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-9, pp-116-121
www.ajer.org
Research Paper Open Access

Solution of Advection-Diffusion Equation for Concentration of Pollution and Dissolved Oxygen in the River Water by Elzaki Transform

Pradip R. Bhadane[1,] Kirtiwant P. Ghadle[2],

[1]*Assistant Professor, Department of Mathematics, Arts, Commerce and Science College Ozar (Mig) Nasik-424008(MS) INDIA*

[2]*Associate Professor, Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad-431004 (MS) INDIA*

ABSTRACT : *In this article, a new integral transform called as Elzaki transform is used to solve the model consists of a couple of advection-diffusion equations. These advection-diffusion equations have much importance in chemistry, engineering and sciences. These advection-diffusion equations play a very important role to study the parameters regarding river pollution and are used to predict the level of pollution concentration and level of dissolved oxygen concentration in the river water. We have obtained the solutions for advection-diffusion equations by Elzaki transform. These results prove that the Elzaki transform is quite applicable and useful for finding solutions of the problems related to pollution and dissolved oxygen level in river water.*

Keywords: *Elzaki transform, Advection-diffusion equation, Chemical parameters, Water pollution.*

I. INTRODUCTION

There are many partial differential equations such as Laplace equation, wave equation etc. which are quite useful and applicable in Engineering, Physics, Chemistry and Sciences. Such partial differential equations are very difficult and complicated to solve to obtain its exact solution. In recent years many researchers have paid attention to find the solution of partial differential equations by using various methods. One of such methods is Elzaki transform, which is a very useful technique. Elzaki transform was proposed originally by Tarig M. Elzaki in 2011. He developed the method for solving ordinary and partial differential equations in the time domain [1] [2] [3].

These advection-diffusion equations are the partial differential equations and also known as Mathematical Model for river pollution. In Chemistry, advection is the transport of pollutants in a river by bulk water flow downstream and diffusion is the spread over of gas through water.

Different types of Mathematical Models in Engineering, Chemistry and Earth Sciences have been solved by B. Pimpunchat in 2009 [4]. Recently in 2012 Saleem Azara Husain presented a simple Mathematical model for the concentration of Pollution and dissolved oxygen in river water [5].

In this paper, Elzaki transform is used to solve advection-diffusion equations for studying the level of pollutant and dissolved oxygen concentrations in river water and the result is presented.

1.1 Elzaki Transform

Mainly Fourier, Laplace and Sumudu transforms are used as convenient mathematical tools for solving differential equations. Recently a new transform called as Elzaki transform is derived from the classical Fourier integral and some of it is fundamental properties are used to solve the differential equations.

A new transform called the Elzaki transform defined for function of exponential order, we consider functions in the set A defined by

$$A = \left\{ f(t) : \exists M, K_1, K_2 > 0, \left| f(t) \right| < M e^{\frac{|t|}{k_j}}, if, t \in (-1)^j X [0, \infty) \right\} \qquad (1)$$

For a given function in the set A, the constant M must be finite number, K_1, K_2 may be finite or infinite

The Elzaki transform denoted by the operator $E\,(.)$ defined by the integral equation.

$$E[f(t)] = T(u) = u\int_0^\infty f(t)e^{\frac{-t}{u}}dt, t \geq 0, K_1 \leq u \leq K_2, 0 \leq t \leq \infty \qquad (2)$$

The variable u in this transform is used to factor the variable t in the argument of the function; this transform has deeper connection with the Laplace transform.

Elzaki Transform of the Some Functions

We have Elzaki transform of simple functions.

1) $If, f(t) = 1, then; E(1) = u\int_0^\infty e^{\frac{-t}{u}}dt = u\left[-ue^{\frac{-t}{u}}\right] = u^2$

2) $If, f(t) = t, then; E[t] = u\int_0^\infty te^{\frac{-t}{u}}dt = u^3$

3) $If, f(t) = e^{at}, then; E(e^{at}) = u\int_9^\infty e^{at}e^{\frac{-t}{u}}dt = \frac{u^2}{1+au}$

Theorem: - If $E[f(t)] = T(u)$ then

1) $E\left[\frac{df}{dt}\right] = E\left[f'(t)\right] = \frac{T(u)}{u} - uf(0)$

2) $E\left[\frac{d^2f}{dt^2}\right] = E\left[f''(t)\right] = \frac{T(u)}{u^2} - f(0) - uf'(0)$

3) $E\left[\frac{d^nf}{dt^n}\right] = E\left[f^n(t)\right] = \frac{T(u)}{u^n} - \sum_{k=0}^{n-1} u^{2-n+k} f^k(0)$

1.2 Mathematical Model

In this model we assumed that the advection–diffusion-equation may be a good approximation to study model of river pollution. We assumed that the river is linear and it has a uniform cross-sectional area. The river cross-section portion considered under study is one with an arbitrary interior and endpoints at $x = 0$ and $x = L.$

These equations account expansion of the pollutant and the dissolved oxygen concentrations respectively are as

$$\frac{\partial(RG_s(x,t))}{\partial t} = \theta_p\frac{\partial^2(RG_s(x,t))}{\partial x^2} - \frac{\partial(wARG_s(x,t))}{\partial x} + M\int G_s(x,t)dx \qquad (3)$$

with boundary conditions $G_s(0) = \frac{r}{kR}$

$$\frac{\partial(RF_s(x,t))}{\partial t} = \theta_x\frac{\partial^2(RF_s(x,t))}{\partial x^2} - \frac{\partial(wRF_s(x,t))}{\partial x} + \gamma\int(S - F_s(x,t))dx \qquad (4)$$

with boundary conditions $F_s(0) = S + \frac{r}{kR}$

The first equation includes the mass transferred of solid to the water (river) in m^2/day, and it is the concentration of pollution. The second equation is a mass balance for dissolved oxygen, with addition through the surface at a rate proportional to the degree of saturation of dissolved oxygen (S-F),

with boundary conditions $G_s(0) = \frac{r}{kR}$

$$\frac{\partial(RF_s(x,t))}{\partial t} = \theta_x\frac{\partial^2(RF_s(x,t))}{\partial x^2} - \frac{\partial(wRF_s(x,t))}{\partial x} + \gamma(S - F_s(x,t)) \qquad (5)$$

with boundary conditions $F_s(0) = S + \frac{r}{kR}$

For these the only variation is with the distance downstream on the river and so we write $G(x,t) = G(x)$ and $F_S(x,t) = F_S(x)$.

To simplify the equations, we will consider only the steady–state solutions.
where

L	-	Polluted length of river	(15787m)
θ_x	-	dispersion coefficient of dissolved oxygen in the x direction.	(m^2/ day)
θ_p	-	dispersion coefficient of pollutant in the x direction	(m^2/day)
R	-	cross-section area of the river	(m^2)
S	-	Saturated oxygen concentration(less than 5)	(kg/m^3)
γ	-	Mass transfer of oxygen from air to water of the river	(m^2/day)
M	-	Mass transfer of solid (solute) to the water of the river	(m^2/day)
k	-	Degradation rate coefficient	(per Day)
r	-	Rate of pollutant addition along the river	(kg/m/day)
w	-	Water velocity in the x- direction.	(m^2)

II. ANALYTIC STEADY-STATE SOLUTIONS FOR SPECIAL CASE (ZERO DISPERSION)

Consider the equation (5), when dispersion $\theta_p = 0$, with boundary conditions $G_s(0) = \frac{r}{kR}$,

[that is no pollution upstream because of the absence of dispersion.]

$$-\frac{d(wRG_s(x,t))}{dx} + MG_{s(x,t)} = 0$$

$$wR\frac{d(G_s(x,t))}{dx} - MG_{s(x,t)} = 0$$

$$\frac{d(G_s(x,t))}{dx} - \frac{M}{wR}G_{s(x,t)} = 0$$

Appling Elzaki transform on both sides, we get

$$E\left[\frac{d(G_s(x,t))}{dx}\right] - \frac{M}{wR}E[G_{s(x,t)}] = 0$$

$$\frac{T(u)}{u} - u.G_s(o) - \frac{M}{wR}T(u) = 0 \text{ , by initial condition } G_s(0) = \frac{r}{kR}$$

$$\frac{T(u)}{u} - u.\frac{r}{kR} = \frac{M}{wR}T(u)$$

$$T(u) = \frac{1}{k}\left[\frac{u^2 wr}{(wR - uM)}\right]$$

Now, by appling inverse Elzaki transform on both sides, we get

$$E^{-1}[T(u)] = \frac{1}{k}E^{-1}\left[\frac{u^2 wr}{(wR - uM)}\right]$$

$$E^{-1}[T(u)] = \frac{wr}{wkR}E^{-1}\left[\frac{u^2}{\left[1 - \left(\frac{M}{wR}\right)u\right]}\right]$$

$$G_s(x) = \frac{r}{kR}.e^{\left(\frac{M}{wR}\right)x}$$

$$G_s(x) = C.e^{\left(\frac{M}{wR}\right)x}, Where \quad C = \frac{r}{kR}$$

This shows that the pollutant concentration downstream of the river.

If $x = 0$, then $G_s(x) = G_s(0) = \frac{r}{kR}$ and

If $x = \infty$, then $G_s(x) = G_s(\infty) = \frac{r}{kR}e^{\infty}$ (6)

The solution of the equations result reveals that the amount of pollutants are found to be increased at $x = L$ than $x = 0$.

Now, consider the equation (6), when dispersion $\theta_x = 0$ with boundary conditions $F_s(0) = S + \frac{r}{kR}$

[for this case there is no pollution upstream because of the absence of dispersion.]

$$-\frac{d(wRF_s(x))}{dx} + \gamma(S - F_s(x)) = 0$$

$$\frac{dF_s(x)}{dx} + \frac{\gamma}{wR}F_s(x) = \frac{\gamma S}{wR}$$

Appling Elzaki transform on both sides , we have

$$E\left[\frac{dF_s(x)}{dx}\right] + \frac{\gamma}{wR}E[F_s(x)] = E\left[\frac{\gamma S}{wR}\right]$$

$$\frac{T(u)}{u} - u.F_s(0) + \frac{\gamma}{wR}.T(u) = E\left[\frac{\gamma S}{wR}\right]$$

$$\frac{T(u)}{u} - u.\left[S + \frac{r}{kR}\right] + \frac{\gamma}{wR}.T(u) = E\left[\frac{\gamma S}{wR}\right],$$

where $F_s(0) = S + \frac{r}{kR}$ (by initial condition)

$$\frac{T(u)}{u} - \left[u.S + \frac{u.r}{kR}\right] + \frac{\gamma}{wR}.T(u) = \left[\frac{\gamma S}{wR}\right]E[1]$$ Since E [1] = u^2

$$T(u) = \left[\frac{uwR}{wR + \gamma u}\right]\left\{\left[\frac{\gamma S}{wR}\right]u^2 + u.S + \frac{u.r}{kR}\right\}$$

$$T(u) = S.\left[u^2 - \frac{u^2}{1 + \left(\frac{\gamma}{wR}\right)u}\right] + [S + \frac{r}{kR}]\left[\frac{u^2}{1 + \left(\frac{\gamma}{wR}\right)u}\right]$$

$$T(u) = S.u^2 + \frac{r}{kR}.\left[\frac{u^2}{1 + \left(\frac{\gamma}{wR}\right)u}\right]$$

Now, by appling inverse Elzaki transform on both sides, we have

$$E^{-1}[T(u)] = S.E^{-1}[u^2] + \frac{r}{kR}.E^{-1}\left[\frac{u^2}{1 + \left(\frac{\gamma}{wR}\right)u}\right]$$

$$F_s(x) = S.(1) + \frac{r}{kR}.e^{-\left(\frac{\gamma}{wR}\right)x}$$, Since $E^{-1}(u^2) = 1$

$$F_s(x) = S + \frac{r}{kR}.e^{-\left(\frac{\gamma}{wR}\right)x}$$

$$F_s(x) = \frac{r}{kR}.e^{-\left(\frac{\gamma}{wR}\right)x} + S\ .$$

This shows that the dissolved oxygen concentration downstream of the river.

If $x = 0$ then $F_s(x) = F_s(0) = \frac{r}{kR} + S$ and

If $x = \infty$ then $F_s(x) = F_s(\infty) = \frac{r}{kR}.e^{-\infty} + S$ (7)

This shows that in the downstream of the river dissolved oxygen being constant and its value is very close to saturated value S, these mathematical results are coincides with the actual facts in most of the cities in river water but at the upstream of the river, level of D.O. is very good.

III. FIGURES AND TABLES

Table-I: Specification limits for drinking river water

Parameters in p.p.m.	D.O.	B.O.D.	T.D.S	Sulphate	Chloride	Sodium	Nitrate	COD
Limit mg/l	7-11	7-9	0-500	0-192	0-142	0-60	0-20	0-70

Table-II: If we take the corresponding values for some parameters into consideration with help of pollution control board branch Nasik Maharashtra, then the solution of model-I and model-II are in full agreement with the solutions.

Parameters in p.p.m.	Different Sampling stations						
	S1	S2	S3	S4	S5	S6	S7
D.O.	3.0	3.9	4.0	4.6	4.9	5.9	7.2
B.O.D.	12.7	32.9	33.3	34.8	45.9	47.9	53.6
T.D.S.	398	458	475	600	875	914	1200
Sulphate	180	232	475	538	567	587	612
Chloride	24	41	56	62	67	77	88
Sodium	42	53	60	68	77	83	90
Nitrate	21.6	33	41	47	53	67	85
COD	69	91	114	122	168	186	193

Figure-1

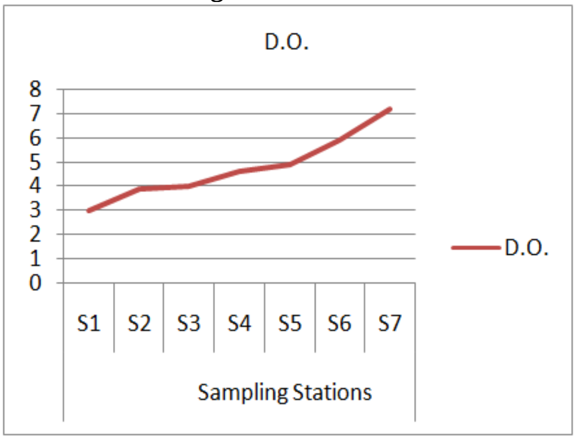


Figure-2

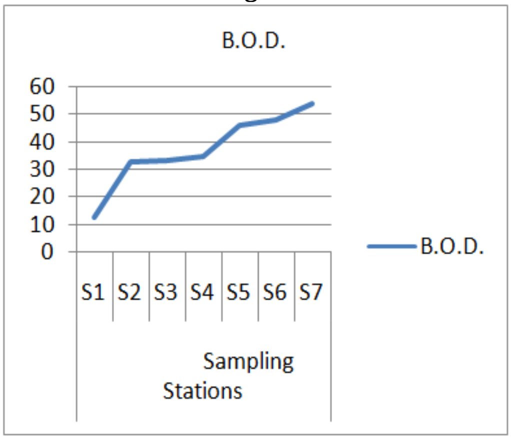


Figure-3

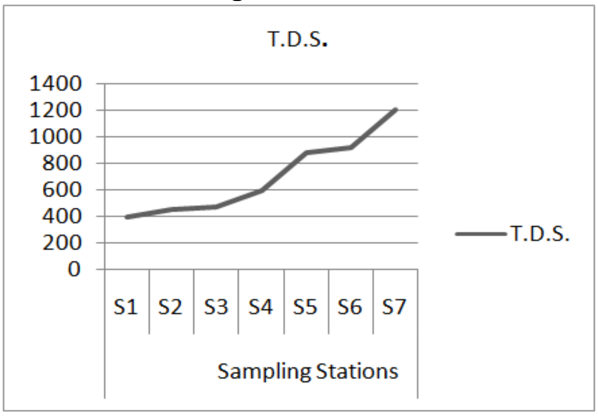


Figure-4

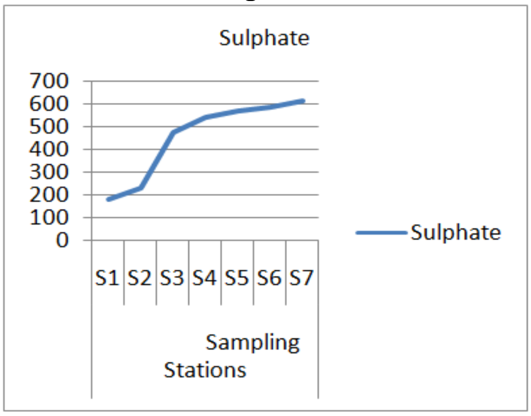


Figure-5

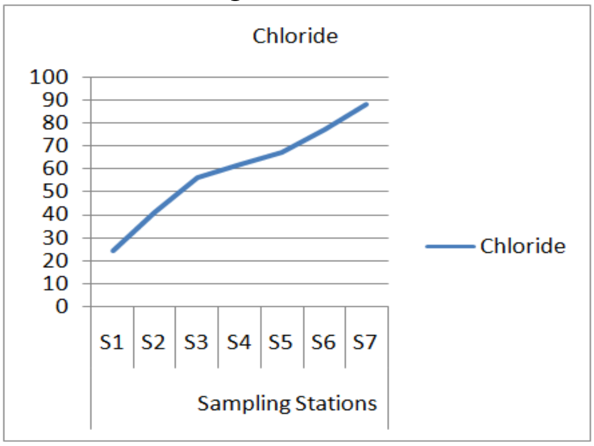


Figure-6

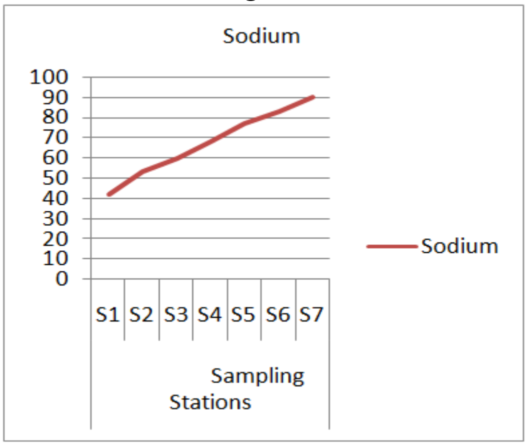


Figure-7

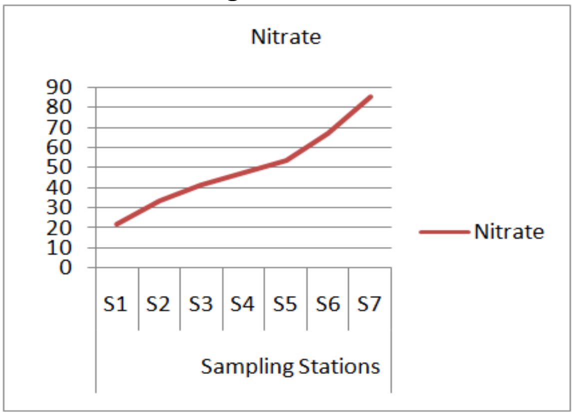
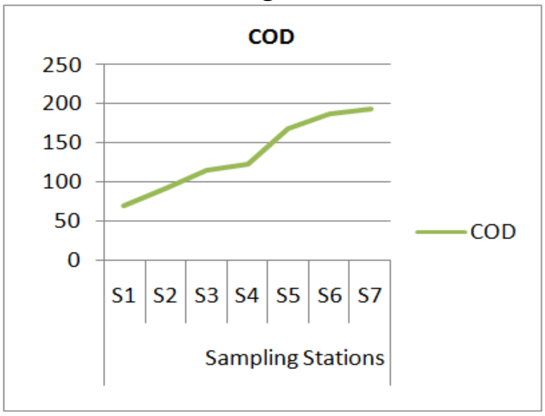


Figure-8



IV. CONCLUSION

Previously advection-diffusion model was used to get the solution related to problems of the river pollution. We tried Elzaki transform to get the solutions of advection-diffusion model. It is found that the solutions obtained by Elzaki transform are same as that of the solutions obtained by using advection-diffusion model. This reveals that the Elzaki transforms is also applicable for solving such type of problems. It may conclude that the new integral Elzaki transform is very convenient tool solving this type of mathematical model. The result shows that at $x = L$, the percentage of dissolved oxygen is found increased and it is in best agreement with the observations obtained using advection-diffusion model.

ACKNOWLEDGEMENTS

The author offers his grateful thanks to friends and the referee for their help and active guidance in the preparation of this revised paper.

REFERENCES

[1]. Tarig M Elzaki: The new integral transform “Elzaki Transform”, Global Journal of Pure and Applied Mathematics, 7(1), 2011, 57-64.

[2]. Application of new transform “Elzaki Transform” to Partial Differential Equations, Global Journal of Pure & Applied Mathematics, Tarig M. Elzaki and Salih M. Elzaki.

[3]. Tarig M. Elzaki, Salih M. Elzaki and Elsayed A. Elnour, on the new integral Transform “Elzaki Transform” fundamental properties investigation and applications.

[4]. B. Pimpunchat, W. L. Sweatman, G. C. Wake, W. Triampo and A. Parshotam: Mathematical Model for pollution in a river and its remediation by aeration, Applied Mathematical Letters, 22 (2009) 304.

[5]. Mathematical Model for the Concentration of Pollution and Dissolved Oxygen in the Diwaniya River (Iraq) by Saleem Azara Husain. American Journal of Scientific Research ISSN 2301- 2005, Issue-78, October, 2012.