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Free Vibration Analysis of an Alround-Clamped Rectangular Thin Orthotropic Plate Using Taylor-Mclaurin Shape Function

D.O Onwuka[1], O.M Ibearugbulem[2], A.C Abamara[3], C.F Njoku[4], S.I Agbo[5]
[1, 2, 4, 5]*(Department of Civil Engineering, Federal University of Technology, Oweeri Nigeria)*
[3]*(Federal Ministry of Transport, Abuja, Nigeria)*

ABSTRACT:*A comprehensive free vibration analysis of an alround-clamped rectangular thin or thotropic plate, was carried out using Taylor- Mclaurin shape function, and Ritz method. The Taylor-Mclaurin shape function truncated at the fourth term satisfied all the boundary conditions of the alround-clamped thin orthotropic plate. The shape function was substituted into the total energy functional, which was subsequently minimized. From the minimized equation, the natural frequency equation for the clamped plate, was derived. The resulting equation was used to calculate fundamental natural frequencies of the clamped plate for various aspect ratios, p and different combinations of flexural rigidity ratios, φ. The fundamental frequencies for a clamped plate vibrating in the first mode are given in Tables*1-5, *for different flexural rigidities, φ and aspect ratios varying from 0.1 to 2 at increments of 0.1. The average percentage difference in the values of natural frequency for the flexural rigidity ratios, φ1, φ2, and φ3, are 1.532%,1.367% and 1.425% for different values of the aspect ratio, $p = \frac{b}{a}$; and 1.149%, 1.506% and……for different values of the aspect ratio,$p = \frac{a}{b}$.*
These average percentage differences indicate that the formulated deflection function for the clamped plate, is a very good approximation to the exact deflection function of the free vibration of a clamped rectangular thin orthotropic plate.
Keywords:*Clamped rectangular plate, free vibration analysis, natural frequency, orthotropic vibrating plate, Taylor-Mclaurin method, shape function.*

I. INTRODUCTION

Orthotropic plates are commonly used in the fields of structural engineering and are considered to be fundamental structural elements in aerospace, naval and ocean structures [1], [2],[3].The governing equation for free vibration of thin rectangular plates, isa fourth order differential equation and the determination of the exact solution of a clamped plate by direct integration, is not possible. Some of the works on this subject were carried out using other approaches such as numerical and variational methods, which are approximate methods. [4] Used the Rayleigh-Ritz method and made some useful contributions. [5] Presented and used Rayleigh- Ritz method and decomposition technique, to evaluate the upper and lower bounds of vibration frequencies for an alround-clamped rectangular orthotropic plate. [6], first used a method based on superposition of the appropriate Levy type of solutions, for the analysis of rectangular plates. Gorman, further applied this method by Timoshenko and Krieger, to the free vibration analyses of isotropic plate, [7], thereafter to clamped orthotropic plate [8], then to free orthotropic plate, [9]. And finally to point supported or thotropic plates [10]. Before now, it was believed that the exact solution of free vibration of an alround-clamped orthotropic plate was not achievable until [11] used novel separation of variable to obtain the exact solutions for free vibrations of rectangular thin orthotropic plates with all combinations of simply supported and clamped boundary conditions. As a matter of fact, the equation by [12], was used for calculating the radian natural frequency of a deformed orthotropic vibrating plate. One of the plate cases considered, is the alround-clamped plate.Solutions for an alround-clamped plate, were obtained for the first time, even though, it was originally believed it was not obtainable. He alsovalidated the results from his work by extensive comparison with results from finite element method and other numerical methods available in literature. The new exact solution provided values for other researchers that used approximate methods, to compare their results with. It is noteworthy, that none of the researchers, has used the Taylor-Mclaurin series in Rayleigh-Ritz method, to evaluate approximate solutions of alround-clamped orthotropic rectangular thin plates, and the object of this study is to fill that gap.

II. MATHEMATICALFORMULATION

2.1. Governing Differential Equation of a Thin Plate in Vibration.

In 2001, [13] derived the fourth-order homogenous partial differential equation governing the undamped, free, linear vibration of plates as follows:

$$D\nabla^2\nabla^2 w(x,y,t) + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \quad (1)$$

$$D\nabla^4 w(x,y,t) + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \quad (2)$$

where

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 dy^2} + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \quad (3)$$

D = flexural rigidity of the plate
$w(x,y,t)$ = deflection of the plate
x = Cartesian co-ordinate in the horizontal direction
y = Cartesian co-ordinate in the vertical direction
t = thickness of the plate.
ρ =density of the material
m= mass

2.2. Truncated Taylor Maclaurin Series.

[14]Expanded the general shape function using the Taylor-Maclaurin series and obtained equation (4)

$$w = w(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{F^{(m)}(x_0)F^{(n)}(y_0)}{m!\,n!}(x-x_0)^m.(y-y_0)^n \quad (4)$$

where $F^{(m)}(x_0)$ is the m^{th}partial derivative of the function, w, with respect to x and $F^{(n)}(y_0)$ is the n^{th}partial derivative of the function, w with respect to y. And m! and n! are the factorials of m and n respectively, while x0 and y0 are the points of origin. He truncated the infinite series at m= n =4 and gave shape function as:

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} I_m J_n\, x^m.y^n \quad (5)$$

Transforming the x-y co-ordinate system to R-Q coordinate system, yielded

$R = \frac{x}{a}$ and $Q = \frac{y}{b}$, where R and Q are dimensionless quantities.

Since x = aR and y – bQ and letting $a_m = I_m.a^m$ and $b_n = J_n.b^n$, Equation (5) reduces to Equation(6).

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} a_m b_n\, R^m Q^n \quad (6)$$

The function given by Equation (6) can be further expanded in the following form:

$$w(R,Q) = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (7)$$

Wherea_i and b_i (i= 0,1,2,3 and 4) are unknown constants of the shape function series.
Here, the Equation (7) is truncated at M=N=4.

2.3. Boundary Conditions for an Alround Clamped Plate.

Consider a rectangular plate: which is clamped on all edges as shown in Fig 1:

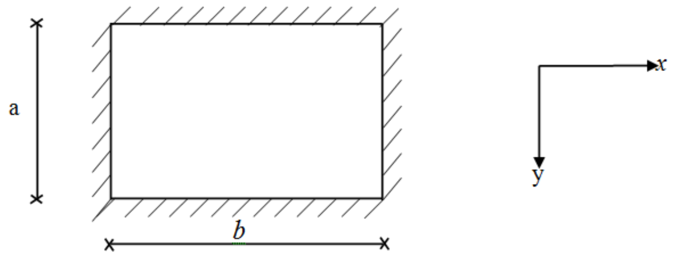

Fig 1: Alround claped rectangular plate

From Fig 1, the boundary conditions for the orthotropic rectangular plate clamped on all 4 edges and represented by CCCC are:

$$W(R=0) = 0 \quad (8)$$

$$W(R=1) = 0 \quad (9)$$

$W(Q = 0) = 0$ (10)
$W(Q = 1) = 0$ (11)
$W'^R(R = 0) = 0$ (12)
$W'^R(R = 1) = 0$ (13)
$W'^Q(Q = 0) = 0$ (14)
$W'^Q(Q = 1) = 0$ (15)

whereW'^R and W'^Q are the first derivatives of the displacement functions with respect to the R and Q directions respectively.

Substituting successively, the boundary conditions, namely, $W(R = 0) = 0$; $W'^R(R = 0) = 0$,$W(Q = 0) = 0$, and $W'^Q(Q = 0) = 0$ into the Equation (7), yields respectively.

$a_0 = 0$ (16)
$a_1 = 0$ (17)
$b_0 = 0$ (18)
$b_1 = 0$ (19)

And, substituting the other boundary conditions, viz, $W(Q = 1) = 0$ and $W'^Q(Q = 1) = 0$ into Equation (7) one after the other, gives respectively:

$a_2 + a_3 + a_4 = 0$ (20)
$2a_2 + 3a_3 + 4a_4 = 0$ (21)

Solving simultaneously yields

$a_2 = a_4$, and $a_3 = 2a_4$ (22)

Also, substituting the boundary conditions, $W(Q = 1) = 0$ and $W'^Q(Q = 1) = 0$ one after the other, into Equation (7) and solving the simultaneous equation gives:

$b_2 = b_4$, and $b_3 = -2b_4$

Substituting the constants, a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, and b_4 into Equation (7) gives the deflection function, w, as follows:

$W = A\,(R^2-2R^3+R^4)\,(Q^2-2Q^3+Q^4)$ (23a)
$\;=AH$ (23b)

where $A=a_4b_4$ and $H= (R^2-2R^3+R^4)\,(Q^2-2Q^3+Q^4)$

As a matter of fact, 'A' is the amplitude of the deflected shape while 'H' is the deflected shape.

III. NATURAL FREQUENCY EQUATION, Λ, FOR A VIBRATING ORTHOTROPIC PLATE.

3.1. Formulation of the Natural Frequency Equation, λ, for a Free Vibrating Alround-clamped Rectangular Orthotropic Plate.

Using a deflection function technique based on the work of [14], derived the equation for the fundamental frequency of a vibrating continuum. This was achieved by employing the principle of conservation of energy, in which the strain and kinetic energies of the continuum, were derived from the first principles using the theory of elasticity. The expressions were subsequently substituted into the potential energy functional, and then minimized to determine the fundamental frequency, that is, at mode M=N=1.

Then, the fundamental frequency is made the subject of the equations after substituting the aspect ratios p=a/b and p=b/a, as the case may be.

First, the strain energy, U, is given as:

$$U = \frac{D_x}{2b^2}\int_0^1\int_0^1\left[\frac{\varphi_1}{p^3}\left(\frac{\partial^2 w}{R^2}\right)^2 + 2\,\frac{\varphi_2}{p}\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2 + p\varphi_3\left(\frac{\partial^2 w}{\partial Q^2}\right)^2\right]\partial R\,\partial Q \quad (24)$$

And the kinetic energy, K.E, is given as:

$$K.E = \frac{pb^2\lambda^2\rho t}{2}\int_0^1\int_0^1 W^2\,\partial R\,\partial Q \quad (25)$$

And total potential energy functional represented by the symbol, π is expressed by Equation (26)

$$\Pi = U - K.E \quad (26)$$

Substituting Equation (24) and (25) into Equation (26), yields, Equation (27)

$$\Pi = \frac{D_x}{2b^2}\int_0^1\int_0^1\left[\frac{\varphi_1}{p^3}\left(\frac{\partial^2 w}{R^2}\right)^2 + 2\frac{\varphi_2}{p}\left(\frac{\partial^2 w}{\partial R\,\partial Q}\right)^2 + p\varphi_3\left(\frac{\partial^2 w}{\partial Q^2}\right)^2\right]\partial R\,\partial Q - \frac{pb^2\lambda^2\rho t}{2}\int_0^1\int_0^1 W^2\,\partial R\,\partial Q \quad (27)$$

Since the deflection function, W = AH, the Equation (27) becomes:

$$\Pi = \frac{D_x A^2}{2b^2}\int_0^1\int_0^1\left[\frac{\varphi_1}{p^3}\left(\frac{\partial^2 H}{\partial R^2}\right)^2 + 2\frac{\varphi_2}{p}\left(\frac{\partial^2 H}{\partial R\,\partial Q}\right)^2 + p\varphi_3\left(\frac{\partial^2 H}{\partial Q^2}\right)^2\right]\partial R\,\partial Q - \frac{pA^2b^2\lambda^2\rho t}{2}\int_0^1\int_0^1 H^2\,\partial R\,\partial Q \quad (28)$$

Minimizing this Equation (28) gives:

$$\frac{\partial \Pi}{\partial A} = \frac{D_x A}{b^2}\int_0^1\int_0^1\left[\frac{\varphi_1}{p^3}\left(\frac{\partial^2 H}{\partial R^2}\right)^2 + 2\frac{\varphi_2}{p}\left(\frac{\partial^2 H}{\partial R\,\partial Q}\right)^2 + p\varphi_3\left(\frac{\partial^2 H}{\partial Q^2}\right)^2\right]\partial R\,\partial Q - pAb^2\lambda^2\rho t\int_0^1\int_0^1 H^2\,\partial R\,\partial Q = 0 \quad (29)$$

At this point, the natural frequency squared,λ^2 is made the subject of the Equation (29). For the aspect ratio, p=a/b,λ^2 can be expressed in terms of φ and b as follows:

$$\lambda^2 = \frac{\frac{D_x A}{b^4 \rho t}\int_0^1\int_0^1[\frac{\varphi_1}{p^4}(\frac{\partial^2 H}{\partial R^2})^2+2\frac{\varphi_2}{p^2}(\frac{\partial^2 H}{\partial R\,\partial Q})^2+p\varphi_3(\frac{\partial^2 H}{\partial Q^2})^2]\,\partial R\,\partial Q}{\int_0^1\int_0^1 H^2\,\partial R\,\partial Q} \quad (30)$$

In terms of a and b, Equation (30) becomes:

$$\lambda^2 = \frac{\frac{D_x}{a^4 \rho t}\int_0^1\int_0^1[\varphi_1(\frac{\partial^2 H}{\partial R^2})^2 + \frac{2\varphi_2 a^2(\frac{\partial^2 H}{\partial R\,\partial Q})^2}{b^2} + \frac{\varphi_3 a^4(\frac{\partial^2 H}{\partial Q^2})^2}{b^4}]\,\partial R\,\partial Q}{\int_0^1\int_0^1 H^2\,\partial R\,\partial Q} \quad (31)$$

In terms of p and a, the same expression for λ^2,becomes:

$$\lambda^2 = \frac{\frac{D_x}{a^4 \rho t}\int_0^1\int_0^1[\varphi_1(\frac{\partial^2 H}{\partial R^2})^2+2\varphi_2 p^2(\frac{\partial^2 H}{\partial R\,\partial Q})^2+p\varphi_3 p^4(\frac{\partial^2 H}{\partial Q^2})^2]\,\partial R\,\partial Q}{\int_0^1\int_0^1 H^2\,\partial R\,\partial Q} \quad (32)$$

Similarly, for aspect ratio p = b/a, the expression for natural frequency squared,λ^2 in terms of a and p, is given as follows:

$$\lambda^2 = \frac{\frac{D_x A}{a^4 \rho t}\int_0^1\int_0^1[\varphi_1(\frac{\partial^2 H}{\partial R^2})^2 + \frac{2\varphi_2(\frac{\partial^2 H}{\partial R\,\partial Q})^2}{p^2} + \frac{\varphi_3 a^4(\frac{\partial^2 H}{\partial Q^2})^2}{p^4}]\,\partial R\,\partial Q}{\int_0^1\int_0^1 H^2\,\partial R\,\partial Q} \quad (33)$$

Where t, is the plate thickness, a and b are the length and width of the plate respectively.

3.2. Use of Rayleigh – Ritz Method to Determinethe Natural Frequency ,λ, of an Alround-Clamped Plate in Free Vibration.

Let the partial differentials of the deflection functions, W, expressed in terms of dimensionless parameters R and Q, be as follows:

$$W'^R = \frac{\partial W(R,Q)}{\partial R} \quad (34)$$

$$W''^R = \frac{\partial^2 W(R,Q)}{\partial R^2} \quad (35)$$

$$W'^Q = \frac{\partial W(R,Q)}{\partial Q} \quad (36)$$

$$W''^Q = \frac{\partial^2 W(R,Q)}{\partial Q^2} \quad (37)$$

$$W''^{RQ} = \frac{\partial^2 W(R,Q)}{\partial R\,\partial Q} \quad (38)$$

where W=AH are as defined earlier.

The parameters,W''^R, W''^Q, W''^{RQ}, their squares and double integrals ,are evaluated with respect to R and Q as follows:

$$\int_0^1\int_0^1 (W''^R)^2 = 2R\,\partial Q = A^2(0.8)(0.00159) = 0.00127A^2 \quad (39)$$

$$\int_0^1\int_0^1 (W''^Q)^2\,\partial R\,\partial Q = A^2(0.00159)(0.8) = 0.00127A^2 \quad (40)$$

$$\int_0^1\int_0^1 (W''^{RQ})^2 = 2R\,\partial Q = A^2(0.01905)(0.01905) = 0.00036A^2 \quad (41)$$

$$\int_0^1\int_0^1 W^2\,\partial R\,\partial Q = (0.0015873)^2 = 0.000002519526329A^2 \quad (42)$$

Substituting these values into the natural frequency squared, λ2, equation, gives

$$\lambda^2 = \frac{\frac{D_x}{b^4 \rho t}[\frac{\varphi_1}{p^4}*0.00127 + \frac{2\varphi_2}{p^2}*0.00036 + \varphi_3*0.00127]}{0.000002519526329} \quad \text{for p= a/b} \quad (43)$$

This expression further reduces to Equation (44)

$$\lambda^2 = \frac{D_x}{b^4 \rho t}[\frac{\varphi_1}{p^4} * 503.9683 + \frac{\varphi_2}{p^2} * 285.714 + \varphi_3 * 503.9683] \quad (44)$$

Re-arranging the equation in terms of a and p, gives:

$$\lambda^2 = \frac{D_x}{a^4 \rho t}[\varphi_1 * 503.9683 + \varphi_2 * 285.714p^2 + \varphi_3 * 503.9683p^4] \quad (45)$$

Substituting a/b in place of p into Equation (32) yields the expression for λ^2in termsof a and b.

$$\lambda^2 = \frac{\frac{D_x}{a^4 \rho t}\int_0^1\int_0^1[\varphi_1(\frac{\partial^2 H}{\partial R^2})^2 + \frac{2\varphi_2 p^2(\frac{\partial^2 H}{\partial R\,\partial Q})^2}{b^2} + \frac{p\varphi_3 p^4(\frac{\partial^2 H}{\partial Q^2})^2}{b^4}]\,\partial R\,\partial Q}{\int_0^1\int_0^1 H^2\,\partial R\,\partial Q} \quad (46)$$

Therefore, the natural frequency squared, λ^2, equation in terms of aand b is given by:

$$\lambda^2 = \frac{\frac{D_x}{b^4 \rho t}[\varphi_1*0.00127 + \frac{2\varphi_2 a^2}{b^2}*0.00036 + \frac{\varphi_3 a^4}{b^4}*0.00127]}{0.000002519526329} \quad (47)$$

This can be simplified further to Equation (48)

$$\lambda^2 = \frac{D_x}{b^4\rho t}\left[\varphi_1 * 503.9683 + \frac{\varphi_2 a^2}{b^2} * 285.714 + \frac{\varphi_3 a^4}{b^4} * 503.9683\right] \quad (48)$$

When aspect ratio, p=b/a, (i.e. reciprocal of a/b), is substituted into the expression, then Equation (45) becomes:

$$\lambda^2 = \frac{D_x}{a^4\rho t}\left[\varphi_1 * 503.9683 + \frac{\varphi_2}{p^2} * 285.714 + \frac{\varphi_3}{p^4} * 503.9683\right] \quad (49)$$

IV. RESULTS AND DISCUSSION

The equation for natural frequency, λ, of a free vibrating rectangular orthotropic plate clamped on all edges, can be obtained from the square roots of Equation (45) and Equation (49) for aspect ratios, p=a/b and p= b/a respectively.

$$\lambda = \sqrt{\frac{D_x}{a^4\rho t}\left[\varphi_1 * 503.9683 + \varphi_2 * 285.714p^2 + \varphi_3 * 503.9683p^4\right]} \quad (50)$$

$$\lambda = \sqrt{\frac{D_x}{a^4\rho t}\left[\varphi_1 * 503.9683 + \frac{\varphi_2}{p^2} * 285.714 + \frac{\varphi_3}{p^4} * 503.9683\right]} \quad (51)$$

These equations were used to calculate the natural frequencies of a free vibrating thin rectangular orthotropic plate clamped on all edges for various aspect ratios, p=b/a and p= a/b and various combinations of flexural rigidities, φ_1,φ_2 and φ_3. The results are shown in Tables 1-5. In addition, graphs of natural frequencies, λ, against aspect ratios, p=b/a, were plotted for various combinations of flexural rigidity,φ_1,φ_2 and φ_3 (see Fig 2)The values of natural frequencies, λ, obtained in this work, were compared withboth the corresponding exact solutions by [11] and solutions by Kantorovich in Tables 1-5.

As shown in Tables 1-3, the percentage differences between the natural frequencies obtained in this work and those of [11], range from 1.367 percent to1.532 percent for different aspect ratios, p=b/a.Both results tend to converge as the aspect ratio, p, increase, but diverges as the aspect ratio, p decreases. Similar comparison indicates higher convergence between the results of the present work and those of Kantorovich.

Table 1: Flexural rigidities, φ_1=φ_2 =φ_3=1, and aspect ratio p = $\frac{b}{a}$

S/N	P	λ^2	λ_1(Pred)	λ_2 (Liu)	λ_3 (Kant)	$\frac{\lambda1-\lambda2}{\lambda1} * 100\%$	$\frac{\lambda1-\lambda3}{\lambda1} * 100\%$
1	0.1	5068758	2251.390				
2	0.2	322627	568.003				
3	0.3	65896.88	256.704				
4	0.4	21975.94	148.243				
5	0.5	9710.317	98.541	97.542	98.324	1.014	0.220
6	0.6	5186.263	72.016				
7	0.7	3186.051	56.445				
8	0.8	2180.788	46.699				
9	0.9	1624.829	40.309				
10	1.0	1293.651	35.967	35.112	35.999	2.377	0.089
11	1.1	1084.313	32.929				
12	1.2	945.4211	30.748				
13	1.3	849.4831	29.146				
14	1.4	780.9278	27.945				
15	1.5	730.5016	27.028				
16	1.6	692.4748	26.315				
17	1.7	663.1716	25.752				
18	1.8	640.1596	25.301				
19	1.9	621.7848	24.936				
20	2.0	606.8948	24.635	24.358	24.581	1.124	0.219

λ_1(Pred) – Result obtained from the formulated

λ_2 (Liu) – Liu and Xing (2008) solution

λ_3 (Kant) – Kantorovich solution

Table 2: Flexural rigidities, φ_1= 1, φ_2 = 0.5, φ_3=1, and aspect ratio p = $\frac{b}{a}$

S/N	P	λ^2	λ_1(Pred)	λ_2 (Liu)	λ_3 (Kant)	$\frac{\lambda1-\lambda2}{\lambda1} * 100\%$	$\frac{\lambda1-\lambda3}{\lambda1} * 100\%$
1	0.1	5054473	2248.215				
2	0.2	319055.6	564.850				
3	0.3	64309.58	253.593				
4	0.4	21083.09	145.200				
5	0.5	9138.889	95.598	94.725	95.391	0.913	0.217
6	0.6	4789.438	69.206				

7	0.7	2894.507	53.801				
8	0.8	1957.574	44.244				
9	0.9	1448.462	38.059				
10	1.0	1150.794	33.923	33.174	33.917	2.208	0.018
11	1.1	966.2491	31.085				
12	1.2	846.2148	29.090				
13	1.3	764.9524	27.658				
14	1.4	708.0416	26.609				
15	1.5	667.0096	25.827				
16	1.6	636.6713	25.232				
17	1.7	613.7401	24.774				
18	1.8	596.0679	24.415				
19	1.9	582.2122	24.129				
20	2.0	571.1806	23.899	23.681	23.848	0.912	0.213

λ_1(Pred) – Result obtained from the formulated
λ_2 (Liu) – Liu and Xing (2008) solution
λ_3 (Kant) – Kantorovich solution

Table 3: Flexural rigidities, $\varphi_1 = 1$, $\varphi_2 = 0.5$, $\varphi_3 = 0.5$, and aspect ratio p = $\frac{b}{a}$

S/N	P	λ^2	λ_1(Pred)	λ_2 (Liu)	λ_3 (Kant)	$\frac{\lambda 1-\lambda 2}{\lambda 1} * 100\%$	$\frac{\lambda 1-\lambda 3}{\lambda 1} * 100\%$
1	0.1	2534631	1592.053				
2	0.2	161565.5	401.952				
3	0.3	33200.42	182.210				
4	0.4	11239.96	106.019				
5	0.5	5107.143	71.464	70.524	71.371	1.315	0.130
6	0.6	2845.115	53.340				
7	0.7	1845.01	42.954				
8	0.8	1342.378	36.638				
9	0.9	1064.399	32.625				
10	1.0	898.8095	29.980	29.329	29.986	2.171	0.020
11	1.1	794.1405	28.180				
12	1.2	724.6947	26.920				
13	1.3	676.7257	26.014				
14	1.4	642.4481	25.347				
15	1.5	617.2349	24.844				
16	1.6	598.2215	24.459				
17	1.7	583.5699	24.157				
18	1.8	572.0639	23.918				
19	1.9	562.8765	23.725				
20	2.0	555.4316	23.568	23.399	23.504	0.717	0.272

λ_1(Pred) – Result obtained from the formulated
λ_2 (Liu) – Liu and Xing (2008) solution
λ_3 (Kant) – Kantorovich solution

Table 4: Flexural rigidities, $\varphi_1 = 1, \varphi_2 = 0.648088$, $\varphi_3 = 3.117304$ and aspect ratio p = $\frac{a}{b}$

S/N	P	λ^2	λ_1(Pred)	λ_2 (Liu)	λ_3 (Kant)	$\frac{\lambda 1-\lambda 2}{\lambda 1} * 100\%$	$\frac{\lambda 1-\lambda 3}{\lambda 1} * 100\%$
1	0.1	505.9771	22.494				
2	0.2	513.8887	22.669				
3	0.3	533.3587	23.095				
4	0.4	573.8134	23.954				
5	0.5	648.4492	25.465	25.104	25.424	1.418	0.161
6	0.6	774.2333	27.825				
7	0.7	971.9031	31.175				
8	0.8	1265.967	35.580				
9	0.9	1684.702	41.04				
10	1.0	2260.159	47.541	46.741	47.481	1.683	0.126
11	1.1	3028.155	55.029				
12	1.2	4028.282	63.469				
13	1.3	5303.899	72.828				
14	1.4	6902.137	83.079				
15	1.5	8873.897	94.201	93.378	93.980	0.874	0.235
16	1.6	11273.85	106.178				
17	1.7	14160.44	118.998				
18	1.8	17595.88	132.649				
19	1.9	21646.15	147.126				
20	2.0	26381	162.422	161.51	161.95	0.562	0.291

λ_1(Pred) – Result obtained from the formulated
λ_2 (Liu) – Liu and Xing (2008) solution
λ_3 (Kant) – Kantorovich solution

Table 5: Flexural rigidities, $\varphi_1 = 1, \varphi_2 = 0.232019, \varphi_3 = 0.070766$, and aspect ratio $p = \frac{b}{a}$

S/N	P	λ^2	λ_1(Pred)	λ_2 (Liu)	λ_3 (Kant)	$\frac{\lambda 1-\lambda 2}{\lambda 1} * 100\%$	$\frac{\lambda 1-\lambda 3}{\lambda 1} * 100\%$
1	0.1	504.6348	22.464				
2	0.2	506.677	22.509				
3	0.3	510.2234	22.588				
4	0.4	515.4879	22.704				
5	0.5	522.7701	22.864	22.757	22.780	0.468	0.367
6	0.6	532.4551	23.075				
7	0.7	545.0138	23.346				
8	0.8	561.0025	23.685				
9	0.9	581.0632	24.105				
10	1.0	605.9233	24.616	24.358	24.564	01.158	0.211
11	1.1	636.396	25.227				
12	1.2	673.38	25.950				
13	1.3	717.8598	26.793				
14	1.4	770.9051	27.765				
15	1.5	833.6715	28.873	28.289	28.869	2.023	0.014
16	1.6	907.400	30.123				
17	1.7	993.4175	31.517				
18	1.8	1093.136	33.063				
19	1.9	1208.054	34.757				
20	2.0	1339.754	36.603	35.735	36.618	2.371	0.041

λ_1(Pred) – Result obtained from the formulated
λ_2 (Liu) – Liu and Xing (2008) solution
λ_3 (Kant) – Kantorovich solution

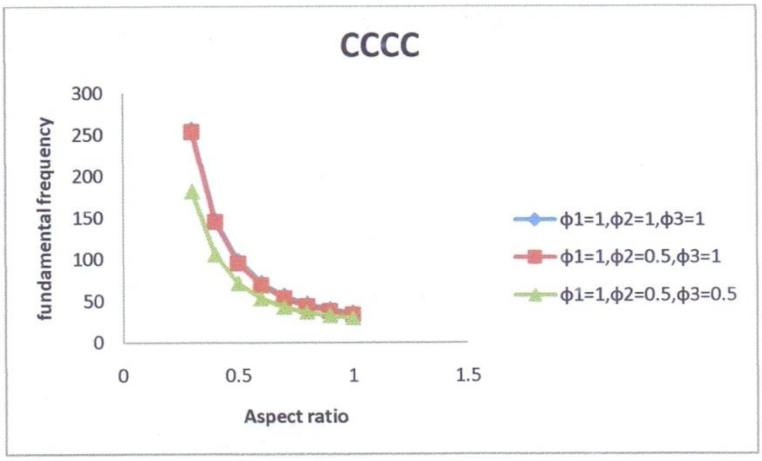

Figure2.

V. CONCLUSION

The fully restrained or (clamped) (CCCC) plates, yielded higher natural frequencies than alround- simply supported (SSSS) plates. Besides, the convergence of the graph/results, present solution with the exact solution, of plates, shows that the present solution approximates closely than the exact solution.

It can also be concluded that the use of Taylor series (which overcomes the deficiencies or limitations of other methods), is more effective in approximating deformed shape of alround clamped thin rectangular orthotropic plate undergoing free vibration. Thus, fully restrained free vibrating plates can simply be analyzed using the newly developed method in this work.

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