

## Compression of 3D meshes based on a decomposition into singular vectors

El Mostafa RAJAALLAH<sup>1</sup> and Hakim ALLALI<sup>1</sup>

<sup>1</sup>(Laboratory LAVETE/ Faculty of Science and Technology - Settat / University Hassan 1st, Morocco)

**ABSTRACT :** Compression of 3D meshes is an important operation for many applications involving transfer and storage of 3D objects data in their process, such as research by content and navigation in 3D objects databases. Compression is to reduce the space needed to store and/or display the 3D mesh. To do that, we usually try to project it in a frequency space where information is less correlated. The approach suggested in this work is based on a decomposition into singular vectors of the laplacian transform of the adjacency matrix of the vertices of the 3D mesh. The obtained results show the invariance of this approach to a normalization and very encouraging performance semantically.

**Keywords** – 3D compression , singular vectors, 3D object.

### I. INTRODUCTION

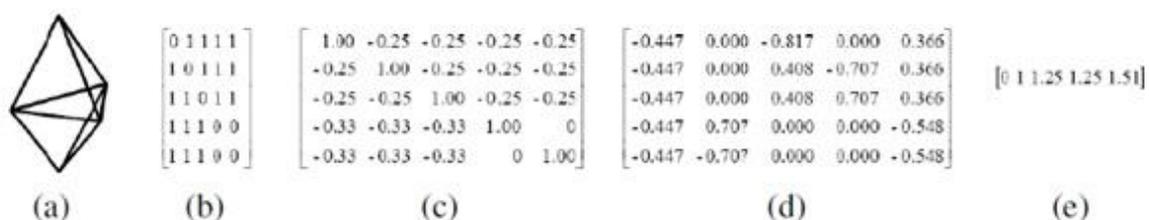
There are two types of compression: lossless and lossy. In the case of lossless compression, any applied processing would give the same results on the reconstructed object or the original object. But in most cases, we can get better compression rates by allowing a few losses that we try to minimize so that the geometric form of the object is not significantly altered.

Lossy compression is mostly used to compress multimedia data (audio, video and medical images). By contrast, lossless compression is usually required for data files such as text articles.

State of the art on compressing meshes has been suggested in literature, quoting for example the work of Alliez and al. 2005[1] and the thesis of Jonathan Delcourt 2010 [2].

The most used technique is that of Karni and Gotsman [3] (Figure 1). That lossless compression technique is based on the spectral decomposition of the mesh. The specter of the mesh is calculated based on the relation of connectivity between the vertices of the mesh. This calculation includes a decomposition into eigenvalues of the normalized Laplacian transform of the adjacency matrix of the vertices. This decomposition produces a sequence of eigenvalues and a corresponding sequence of eigenvectors of the matrix. The small eigenvalues correspond to low spatial frequencies and the big eigenvalues correspond to higher spatial frequencies. The projection of a vertex's coordinates on a normalized eigenvector produces a spectral mesh coefficient of the vertex.

Ben Chen and Gotsman [4] subsequently used a different transformation, for a class of the meshes with a precise connectivity, this time to get optimal bases, but time calculations remain high.



**Fig. 1: Spectral analysis of a 3D mesh with five vertices. (A) 3D mesh. (B) adjacency matrix. (C) laplacian matrix (D) eigenvectors (column) of the laplacian matrix and (e) associated eigenvalues. Image from [3].**

Khodakovskiy et al. [5][6] have observed the influence of a transformation technique to the original object, on the result of compression in comparison with methods that preserve the original connectivity.

In this work we present a method based on the same principle of the method [4], with a spectral analysis using the singular values and vectors to manage problems that may be related to pretreatment of 3D meshes.

## II. Compression based on a decomposition into singular vectors

Before detailing the compression approach based on a decomposition into singular values, we will present an overview of the approach based on an eigenvalues decomposition.

### 2.1. Definition of the Laplacian matrix

In the approach of compression based on a decomposition into eigenvectors, the specter of the studied mesh is generated by exploiting the connectivity relations between the vertices of the mesh, which includes a decomposition into eigenvalues of the normalized Laplacian transform of the adjacency matrix of the vertices.

The elements of the adjacency matrix are defined as follows:

$$a_{ij} = \begin{cases} 1, & \text{if vertices } i \text{ and } j \text{ are adjacent} \\ 0, & \text{if vertices } i \text{ and } j \text{ are not adjacent} \end{cases} \quad (1)$$

After generating the adjacency matrix "A", we calculate the normalized Laplacian transform of "A" using the formula:

$$L_N = I - D^{-1} * A \quad (2)$$

"D" being a diagonal valence matrix, its elements are defined as :

$$d_{ii} = \sum_j a_{ij} \quad (3)$$

In the approach based on singular values, we follow the same logic for the definition of the Laplacian matrix.

### 2.2. Decomposition into singular vectors

A practical way of putting into certitude the properties of a matrix is to decompose it into a product of simpler matrices having characteristics that are clearly identifiable and interpretable.

Firstly, we proceed with a decomposition into singular vectors of  $L_N$ , which could be written as:

$$L_N = U * W * V' \quad (4)$$

The matrix U contains a set of vectors of the orthonormal base of  $R^n$  (n being the number of vertices of the studied mesh) called "input vectors" or "analysis vectors". The columns of U (left singular vectors) are the eigenvalues of the matrix  $L_N * (L_N)'$ .

The matrix V contains a set of vectors of the orthonormal base of  $R^n$  called "output vectors". The columns of V (right singular vectors) are the eigenvectors of the matrix  $(L_N)' * L_N$ .

The diagonal matrix W contains in its diagonal coefficients the singular values of the matrix  $L_N$ , which must be sorted in descending order.

### 2.3. Decompression steps

Spectral compression methods are frequently constructed using various transforms to optimize the compression by lowering redundancies and compacting to best the energy of the objects.

For a spectral decomposition of a 3D mesh :

1. Construction of the adjacency matrix "A".
2. Calculation of the normalized Laplacian transform " $L_N$ " of "A".
3. Definition of the matrix "V".
4. Application of a first transformation by multiplying the vertex matrix by the transposed matrix of "V", do:  $\text{Trans\_Matrix} = V' * \text{Vertex}$ .
5. Definition of the level of the spatial frequency of the projection (the "keep").
6. Redefinition of "Trans\_Matrix" while considering the "keep".

7. Application of the second transformation by multiplying the matrix resulting from the 6<sup>th</sup> step by the matrix "V".

After these 7 steps, the vertex matrix of the compressed object can be defined as follows:

$$\text{Vertex\_Compression} = (V * \text{keep}(\text{Trans\_Matrix}))' \quad (5)$$

### III. Evaluating performance

To test the compression approach presented in this work, we used the object head of "nefertiti.off" in the following figure:

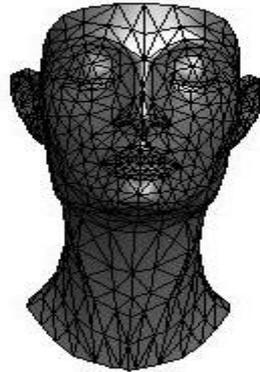


Fig. 2: Mesh representing the head of Nefertiti

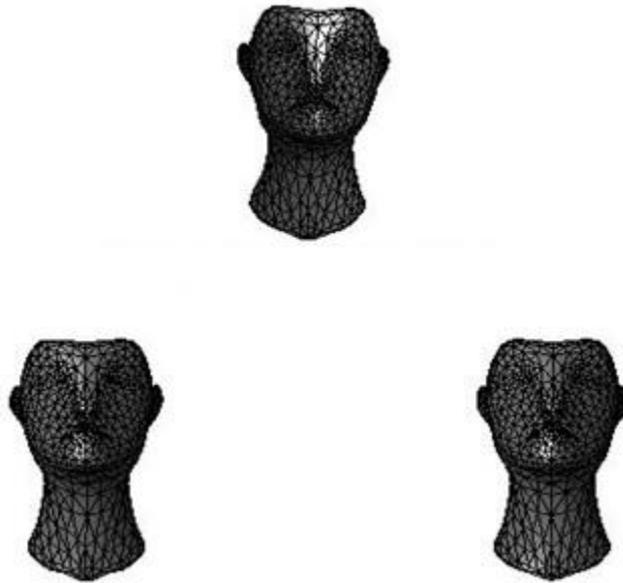
#### 3.1. Qualitative evaluation

Figure 3 shows the results obtained by the suggested approach with 3% as the high spatial frequency of projection.



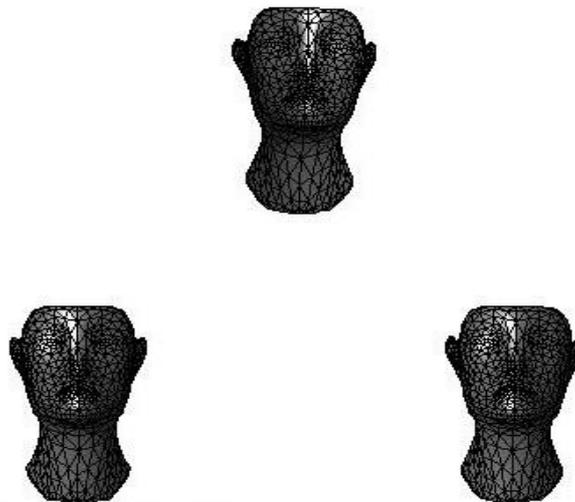
Fig. 3: Compression with 3% as the spatial frequency of projection

Figure 4 shows the results obtained by the suggested approach with 5% as the high spatial frequency of projection.



**Fig.4: Compression with 5% as the spatial frequency of projection**

Figure 5 shows the results obtained by the suggested approach with 7% as the high spatial frequency of projection.



**Fig. 5: Compression with 7% as the spatial frequency of projection**

In the next part we will proceed to a quantitative evaluation of the suggested approach.

### 3.2. Quantitative evaluation

For quantitative evaluation we used the following score:

$$\text{Score of compression} = (\text{Area of the initial object} / \text{Area of the compressed object}) [2]$$

Table 1 presents the scores of compression after normalization with the two methods, firstly the Continuous Principal Component Analysis (CPCA) [7], is a technique for normalization of a 3D object based on Principal Component Analysis (PCA) [8], and second is the technique of 3D Normalization Based on the Barycentric Coordinates [9].

Table 1. Scores of compression by using a decomposition into singular vectors

Level of the spatial frequency of projection	Score of the compression		
	Without normalization	With ACPC	Normalization with Barycentric Coordinates
3%	1,2591	1,2591	1,2591
5%	1,1993	1,1993	1,1993
7%	1,1656	1,1656	1,1656

The obtained results show the insensibility of the approach of compression, based on decomposition into singular vectors of the Laplacian transform of the adjacency matrix of vertices, to the normalization applied to the initial object.

#### IV. Conclusion

In this work we showed that the use of a compression based on a spectral decomposition exploiting the singular vectors shows a considerable improvement semantically, and in terms of the insensibility towards a normalization of 3D meshes.

The approach falls into the category of the techniques of lossy compression which is the most used to compress multimedia data (audio, video and still images). We will work in the future on the improvement of the quality of the compression while insuring an acceptable semantic quality with other existing methods.

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