

Appraisal of Queuing Behaviors in Intafact Beverage Transportation System

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ABSTRACT: *This study evaluates a queuing system generated in Intafact Beverages limited Onitsha, Anambra state of Nigeria. Information on arrivals and service times were collected from the transport unit and appraisal carried out using Erlang models. Single line, multiple server waiting line model were applied to evaluate the queuing performance, the model was used to estimate for different servers, the system utilization, average number of customers on the queue and in the system, the average time a customer spend on the queue and in the system and the probability of having zero customer in the system. The result shows that for two loading point used by the company, the system utilization is 95.7%, the average number of trucks on the line is 20.94, the average number of trucks in the system is 22.86, the average time in line is 1hour and 17mins, the average time in the system is 1hour and 28mins while for three loading points the system utilization is 63.8%, the average number of trucks on the line is 0.715, the average number of trucks in the system is 2.63, the average time in line is 2.4mins (i.e. 0.04hour), the average time in the system is approximately 9mins. Our results have shown that under the two loading points, the queuing system is heavy trafficked. Interestingly, this phenomenon could be eased in the company if additional loading points can be constructed to work in parallel to the existing two loading points.*

I. Introduction to the Study

Queues occur any time and mostly when the demand for service exceeds the capacity to provide it. Waiting in line (queue) is inevitable; it is a part of our everyday life [5, 6]. Delay and queuing problem are what we experience as we go about our daily activities like at the banking halls, ticketing office, in public transportation or traffic jam. But queue can also be seen in a more technical environment like manufacturing industries, computer networking and telecommunication. Hence, queuing theory is suitable to be applied in a manufacturing system since it is associated with queue or waiting line where trucks waiting to load products cannot be served immediately but have to queue (wait) for service for a long time.

The aim of the study is to evaluate queuing performance in Intafact Beverage Limited, to observe the best stable queuing system for their transportation.

II. Theoretical Background

The history of queuing theory began in 1909 in the research work of a Danish Engineer, A. K Erlang. [4] Experimented with fluctuating demand in telephone traffic. He developed models that accounted for callers that dropped due to frustration from waiting for an operator and for those who were patient enough to wait for their call to be connected. Eight years later he published a report addressing the delays in automatic dialling equipment. At the end of World War II, Erlang's early work was extended to more general problem and to business application.

Queuing models helps the analyst with a powerful tool for designing and evaluating the performance of queuing system [2]. In queuing system customers who have to wait in line can be either humans or inanimate objects, examples of object that must wait in line include machine waiting to be repaired, a customer order waiting to be processed, trucks waiting to be loaded or unloaded, etc. But for the purpose of this work, customers here are trucks waiting in line to load products.

Understanding waiting line or queue and learning how to manage them is one of the most important areas in operations management. It is the basic to creating schedules, job design, and inventory level. Whenever customers arrive at a service facility, some of them have to wait before they receive the service. It means that the customer has to wait for its turn in a line. Customers arrive at a service facility with several queue or single queue, and then customers choose a queue of a server according to some mechanism, may be shortest queue or shortest workload [1]. Sometimes, insufficiencies in service also occur due to an undue wait in services which may be because of lack experience of the new employee. Delay in service jobs beyond their due time may result in losing future business opportunities.

Queuing theory is the study of waiting in all these various situations. It is the mathematical study of waiting lines using model to show opportunities within arrival, service and departure process. Queuing model enable finding an appropriate balance between the cost of service and the amount of waiting. The ultimate goal of queuing analysis is to minimize two costs, which is service capacity cost and customer waiting cost.

Waiting line models require an arrival rate and service rate. The arrival rate specifies the average number of customers per time period, while the service rate specifies the average number of customers that can be served during a time period. If the number of customers (trucks) that can be served per time period is less than the average number of customers arriving, the waiting line will grow to infinite.

In waiting line system managers must decide what level of service to offer, a low level of service may be expensive but may incur high cost of customer dissatisfaction such as lost of future business and actual processing cost of complaint, while a high level of service will cost more to provide and will result in lower customer dissatisfaction, because of this trade-off, management must consider what is the optimal level of service to provide. However, since queue is part of our daily life, all we should hope to achieve is to minimize the inconveniences to some reasonable level.

The French mathematician S.D. Poisson (1781-1840) credits Poisson distribution with the pioneering work on queuing theory. The mathematician creates a distribution function to describe the probability of a prescribed outcome after repeated iterations of independent trials. Nevertheless, it was first applied in industrial setting by Erlang in 1909 in the context of telephone facilities. Thereafter, it has been extensively practiced or utilized in industrial setting or retail sector – operations management, and falls under the purview of decision sciences [10].

Kendall [7] was the pioneer who viewed and developed queuing theory from the perspective of stochastic processes. The literature on queuing theory and the diverse areas of its applications has grown tremendously [8, 9, 11]. In fact, [11] put forth a bibliography of books and survey papers on application of queuing in industrial settings.

The **research method** used in this work is the application of Erlang models to model the queuing system of the case study. This method was used in analyzing and development of the data results.

III. Model assumptions

The use of single line multiple channel queuing models were applied with the following assumptions:

1. Arrivals are served on a first-in, first-out (FIFO) basis, and every arrival waits to be served, regardless of the length of the line or queue.
2. That all servers work at the same average rate,
3. Arrivals into the system follow a Poisson process with parameter (λ) denoting the average number of arrivals in the system. Similarly, the service times are exponentially distributed and there are two servers (service points).
4. The trucks form a single line to be serviced by the two stations (loading points).

TABLE 1: Total Loading from January to June

Year 2012 (January to June)	LOADING POINTS OF THE TRUCKS			
	LOADING POINT 1		LOADING POINT 2	
	ARRIVAL RATE	DEPARTURE RATE	ARRIVAL RATE	DEPARTURE RATE
2012 January 2nd-6th TOTAL TRUCKS FOR THE WEEK	110	97	96	85
2012 January 9th-13th TOTAL TRUCKS FOR THE WEEK	104	88	92	88

2012 January 16th-20th TOTAL TRUCKS FOR THE WEEK	99	101	102	92
2012 January 23th-27th TOTAL TRUCKS FOR THE WEEK	98	88	85	85
2012 JAN/FEB 30TH- 3RDTOTAL TRUCKS FOR THE WEEK	96	92	92	91
2012 FEBUARY 6TH-10TH TOTAL TRUCKS FOR THE WEEK	90	104	104	102
2012 FEBUARY 13TH- 17TH TOTAL TRUCKS FOR THE WEEK	108	111	99	113
2012 FEBUARY 20TH- 24TH TOTAL TRUCKS FOR THE WEEK	77	84	80	90
2012 FEB/MARCH 27TH- 2ND TOTAL TRUCKS FOR THE WEEK	101	114	98	101
2012 MARCH 5TH-9TH TOTAL TRUCKS FOR THE WEEK	108	119	98	105
2012 MARCH 12TH-16TH TOTAL TRUCKS FOR THE WEEK	88	103	88	103
2012 MARCH 19TH-23RD TOTAL TRUCKS FOR THE WEEK	85	96	91	106
2012 MARCH 26TH-30TH TOTAL TRUCKS FOR THE WEEK	67	80	67	81
2012 April 2nd-6th TOTAL TRUCKS FOR THE WEEK	93	100	93	93
2012 APRIL 9TH-13TH TOTAL TRUCKS FOR THE WEEK	117	114	108	116
2012 APRIL 16TH-20TH TOTAL TRUCKS FOR THE WEEK	132	136	124	120
2012 APRIL 23RD-27H TOTAL TRUCKS FOR THE WEEK	121	124	110	132
2012 APRIL/MAY TOTAL TRUCKS FOR THE WEEK	131	139	120	139
2012 MAY 7TH-11TH TOTAL TRUCKS FOR THE WEEK	95	92	87	93
2012 MAY 14TH-18TH TOTAL TRUCKS FOR THE WEEK	67	75	72	82
2012 MAY 21TH-25TH TOTAL TRUCKS FOR THE WEEK	55	72	63	59
2012 MAY/JUNE 28TH- 1ST TOTAL TRUCKS FOR THE WEEK	61	59	58	54

2012 JUNE 4TH-8TH TOTAL TRUCKS FOR THE WEEK	63	73	74	95
2012 JUNE 11TH-15TH TOTAL TRUCKS FOR THE WEEK	58	66	63	59
2012 18TH-22ND TOTAL TRUCKS FOR THE WEEK	61	53	56	53
2012 JUNE 25TH-29TH TOTAL TRUCKS FOR THE WEEK	77	81	71	63

Source: Intafact Beverage Limited, Department of Transport/Utilities Onitsha Anambra State.

Parameters of the model used

$$(i) (P_0) = \left[\sum_{n=0}^{M-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^M}{M! \left(1 - \frac{\lambda}{M\mu}\right)} \right]^{-1} \tag{1}$$

$$(ii) L_q = \frac{\lambda \mu \left(\frac{\lambda}{\mu}\right)^M}{(M-1)!(M\mu - \lambda)^2} P_0 \tag{2}$$

$$(iii) \rho = \frac{\lambda}{M(\mu)} \tag{3}$$

$$(iv) (W_q) = \frac{L_q}{\lambda} \tag{4}$$

$$(v) (W_a) = \frac{1}{M\mu - \lambda} \tag{5}$$

$$(vi) (P_w) = \frac{W_q}{W_a} \tag{6}$$

$$(vii) L_s = L_q + R \tag{7.0}$$

$$\text{Or } L_s = W_s \times \lambda \tag{7.1}$$

$$(viii) W_s = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda} \tag{8.0}$$

$$W_s = \frac{L_s}{\lambda} \tag{8.1}$$

$$(ix) Q = M\mu \tag{9}$$

Where:

λ –Average arrival rate

μ –Average service rate

P_0 _ is the probability of having zero customers in the system.

L_q – Average number of customers in the queue.

L_s – Average number of customers in the system.

W_q –Average time a customer spends in queue

W_s –Average time a customer spends on the system

W_a –Average waiting time for an arrival not immediately served

ρ – system utilization

P_w –probability that an arrival will have to wait before service.

IV. Result and Analysis

The graphs below show the pattern of arrival rate and departure rate for the period of January to June. The number of trucks is on the vertical axis while months of the year are on the horizontal axis.

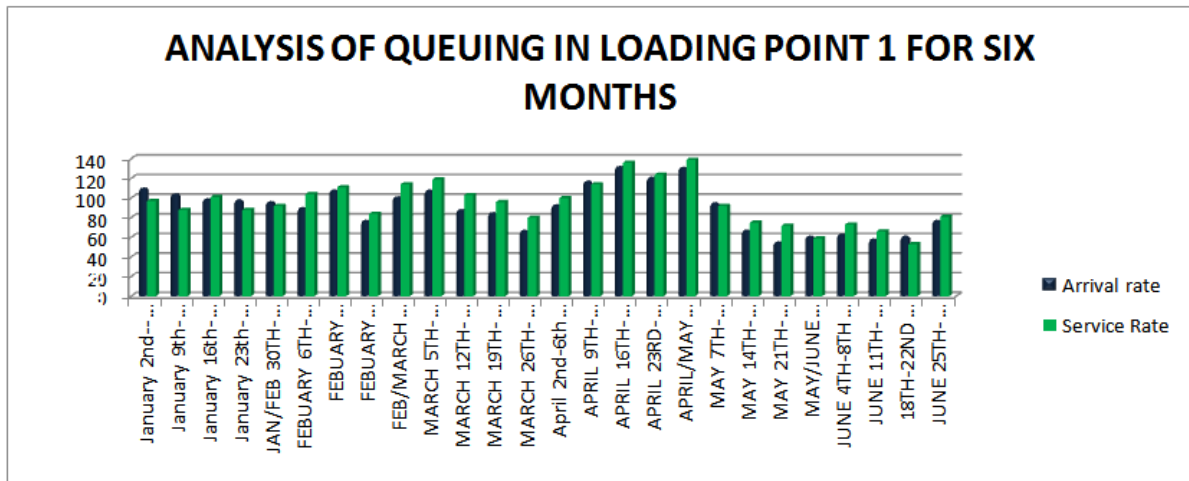


Figure 1: Analysis of Queuing in Loading Point 1 from January to June 2012

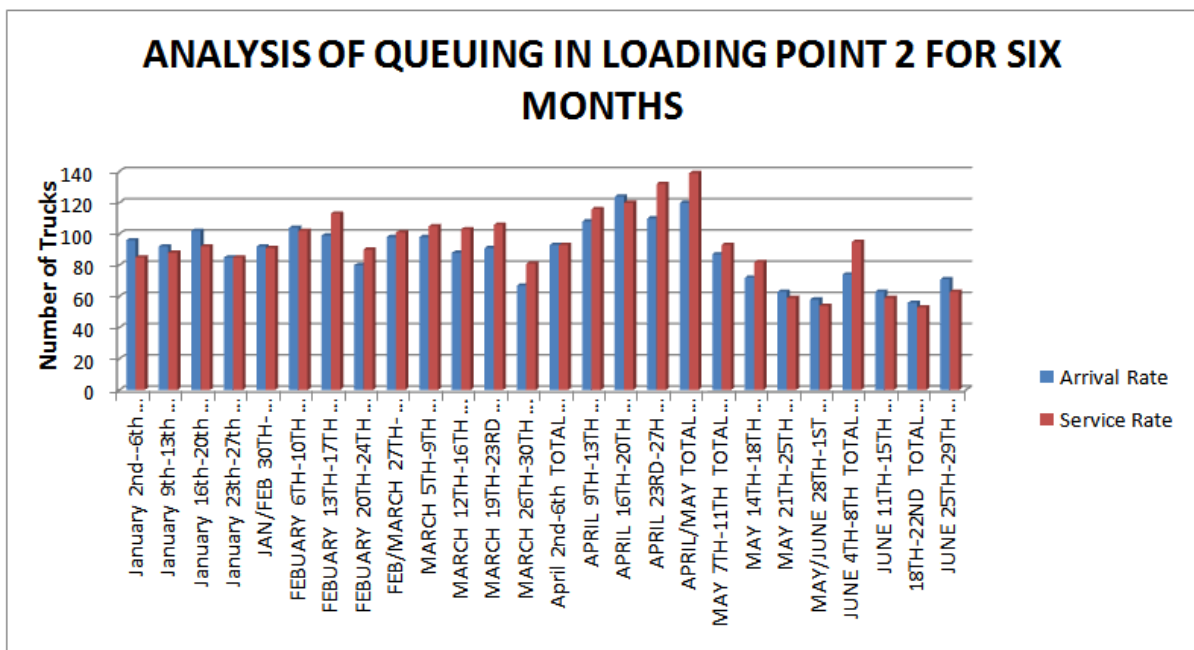


Figure 2: Analysis of Queuing in Loading Point 2 from January to June 2012.

Figure 1 and 2 are graphs for the analysis of queuing in loading point 1 and loading point 2 respectively. The graphs reveal the service pattern of the company; that is the rate at which the service channel rendered services to the arriving customers for the period of January to June.

From table 1

Average Trucks arrival rate for the loading points (λ) = 17.89615

Average Service Rate for the Loading points ($\bar{\mu}$) = 18.69615

The average number of Trucks being served(r)

$$R = \frac{\lambda}{\mu} \tag{10}$$

$$R = \frac{\lambda}{\mu} = \frac{17.89615}{18.69615} = 0.957210 \tag{11}$$

Note that $m(\mu) = \bar{\mu}$ which is the combined service rate and each server contributes service rate of $\frac{\bar{\mu}}{m}$ (i.e. 9.348077), while m is the number of servers.

Therefore the average service rate of each server (μ)= 9.348077.

It is assumed to be constant for all servers.

Considering two servers, $m=2$

System Utilization for each Channel

$$\rho = \frac{\lambda}{M(\mu)} = 17.89615/2(9.348077) = 0.9572 \tag{12}$$

Probability of Zero Units in the System (P₀)

$$(P_0) = \left[\sum_{n=0}^{M-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^M}{M!(1-\frac{\lambda}{M\mu})} \right]^{-1} \tag{13}$$

Number of customers, n=0, 1, 2, 3, 4,

$$(P_0) = \left[\sum_{n=0}^{M-1} \frac{(17.89615)^n}{9.348077^n n!} + \frac{(17.89615)^2}{2!(1-\frac{17.89615}{2 * 9.348077})} \right]^{-1}$$

$$(P_0) = 0.0228$$

$$(P_0) = 2.2\%$$

Average Number of Customers in Line

$$L_q = \frac{\lambda \mu (\frac{\lambda}{\mu})^M}{(M-1)!(M\mu-\lambda)^2} P_0 \tag{14}$$

$$L_q = \frac{17.89615 * 9.348077 (\frac{17.89615}{9.348077})^2}{(2-1)!(2 * 9.348077 - 17.89615)^2} * 0.0228$$

L_q=20.94, approximately 21 trucks are in queue

The Average Number of Customers in the System (waiting and /or being served)

$$L_s = L_q + R \tag{15}$$

$$\text{Or } L_s = W_s \times \lambda$$

Average waiting time for an arrival not immediately served (W_a)

$$(W_a) = \frac{1}{M\mu - \lambda} \tag{16}$$

The average time customers wait in line (W_q)

$$(W_q) = \frac{L_q}{\lambda} \tag{17}$$

Probability that an arrival will have to wait for service (P_w)

$$(P_w) = \frac{W_q}{W_a} \tag{18}$$

The average time spend in the system (waiting in line and service time) (W_s)

$$W_s = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda} \tag{19}$$

$$W_s = \frac{L_s}{\lambda}$$

$$\text{The system capacity} = M\mu \tag{20}$$

Table 2: Summarized the Performance of Queuing System for Different Servers

Number of Servers (Max 12)	M	2	3	4	5	6	7	8	9	10	11	12
System Utilization	p	0.957	0.638	0.4786	0.3829	0.3191	0.2735	0.2393	0.2127	0.1914	0.174	0.1595
Probability system is empty	P0	0.022	0.125	0.1431	0.1466	0.1473	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474
Probability Arrival must wait	Pw	0.936	0.405	0.1536	0.0509	0.0148	0.0038	0.0009	0.0002	3E-05	6E-06	9E-07
Average no in line	Lq	20.94	0.714	0.141	0.0316	0.0069	0.0014	0.0003	5E-05	8E-06	1E-06	2E-07
Average no in System	Ls	22.86	2.629	2.0554	1.946	1.9213	1.9158	1.9147	1.9145	1.9144	1.9144	1.9144
Average Time in Line	Wq	1.17	0.04	0.0079	0.0018	0.0004	8E-05	2E-05	3E-06	4E-07	7E-08	9E-09
Average Time in System	Ws	1.28	0.147	0.1149	0.1087	0.1074	0.1071	0.107	0.107	0.107	0.107	0.107
Average Waiting Time	Wa	1.25	0.099	0.0513	0.0347	0.0262	0.021	0.0176	0.0151	0.0132	0.0118	0.0106

V. Discussion of the Results

Table 2 shows the results of two to twelve servers of the aforementioned company. The results show the system utilization, probability of having zero customer(s) in the system, average waiting time in queue and in system and average number of customers both in line and in system.

The results show that the two loading points used in the case study has a system utilization of 95.7%, probability of having an empty system before queue is 2.2%, while the probability that the arrival must wait is 93.7%. The

average number of truck in the line is 20.94 while the average number of trucks in the system is 22.86 The average waiting time of the trucks in line is one hour seventeen minutes (i.e. 1:17mins), while the average waiting time it takes to serve each truck in the system is one hour twenty eight minutes (i.e. 1:28mins) and the average waiting time of the trucks not immediately served is one hour twenty five minutes (i.e. 1:25mins).

However, if the case study operates with three loading points, the system will make use of 63.8%, system utilization, probability of having an empty system is 12.5%, while the probability that the arrival must wait is 40.5%. The average number of trucks in the line is 0.715 while the average number of trucks in the system is 2.63. The average waiting time of the trucks in line is 2.4mins (i.e. 0.04hour) which is approximately 3mins, while the average waiting time it takes to serve each truck in the system is (0.147hour) which is 9mins approximately and the average waiting time of the trucks not immediately served is approximately 6mins (i.e. 0.099hour). The results reveal that increase in number of servers reduces system utilization and vice versa in the case investigated. These results reveals long queues and longer waiting time of trucks experienced at both service facilities.

Having observed the results of the queuing systems from server two to twelve, it was recommended that the best possible number of servers to be used in the case study is three (3). The results were based on [3] which says that the best value of system utilization should be greater than 0 but less than 0.8 i.e. 80%. This implies that for a good queuing performance, it is imperative to determine the best number of servers that gives the best economic system utilization value.

VI. Conclusion

A close examination of the case study reveals that there is an existence of long waiting line in Intafact beverages manufacturing industry transportation system. The queuing system is a single line multiple channel system. Based on the results of the analysis developed using single line multiple channel existing models, it reveals that the best suitable number of servers that will utilize the queuing system of the case study company is three (3) servers with system utilization of 63.8%. The results were based on [3] which says that the best value of system utilization should be greater than 0 but less than 0.8 i.e. 80%. This implies that for a good queuing performance, it is imperative to determine the best number of servers that will give the best economic system utilization in the case study company. The results and the techniques were therefore recommended to the case study company for evaluation of queuing system. Hence the objectives of the research work were achieved.

From our analysis, we concluded that adding one more loading point (server) to the existing two loading points in Intafact beverages limited Onitsha will help to reduce the time customers spend on queue and as well help to reduce the cost incurred from waiting.

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