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Analytical Effects of Torsion on Timber Beams

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Abstract : *This study investigates the analytical effect of torsion on a rectangular cross-sectional timber beam. It reviewed the theoretical concept of both isotropic and anisotropic elastic behavior of the rectangular beam to torsional load. Finite difference method was used to evaluate the torsional parameters* $K_1, K_2, K_3, \beta, K_3$ *and* K_4 *for isotropic and anisotropic elastic material behavior and compared with the result obtained from the analytical values from previous research. The finite difference codes and equations together with the associated boundary conditions are employed to approximate the stress function and torsional parameter derivatives in the differential equation of analytical method. MATLAB programming was used to seek solution to the finite difference equations formulated. It was observed that the values obtained by finite difference method are approximate to the analytical values. However, the findings through ANOVA Test, revealed that there was no significant difference between the finite difference method and the analytical values (*$F_{calculated}$ $(4.747) > F_{critical}$ (0.00167) *at* 0.05 *level of significant and a strong value of correlation of* 0.998 *was obtained. The finite difference method was capable of predicting the stress functions and torsional parameters for both isotropic and anisotropic material behavior of the rectangular beam cross-section.*

Keywords: *Torsion, Finite difference method, Timber, isotropic, anisotropic, analytical*

I. Introduction

All structures are designed to satisfy the requirement of strength, rigidity and stability which is essentially for their reliability and safe operation. Structural elements can be deformed under the action of external forces, that is, their shape and dimension can change to the extent of warping and distorting. Torsional load is a load that subjects a structural member to couples or moments that twist the member spirally (Onouye and Kane, 2007) under the action of torque. . Torsion may be induced in a structural timber beam in various ways during transfer of load in a structural system. Torsion can be induced in a structural beam when the beam is subjected to external transverse load in such a manner that the resultant force acts a distance away from the shear centre axis of the beam. Torsion can also be induced in a structural timber beam due to monolithic and to satisfy the compatibility condition between members that are joined. In other words, torsion happens because of integrity and continuity of members and also under the effect of external loads in timber structures. Shear centre is defined as the point in the cross-section through which the transverse load must pass to produce bending without twisting. The determination of the torsional moment for any loading in torsion and shear requires knowledge of location of the centre of shear in addition to the geometric centroid (Ziegler, 1995). If the timber beam is subjected to two opposite turning moments, it is said to be in pure torsion (Rajput, 2004), it will exhibit the tendency of shearing off at every cross-section which is perpendicular to longitudinal axis. The induced torsional moment or torque tends to twist the beam to give a rotational displacement. When this occurs, the beam undergoes deformation by warping and distorting, that is changes in dimension, shape, or both simultaneously. If the timber beam is not properly designed against these torsional shear stresses, a sudden fragile fracture can occur, leading to failure of the beam at torsional cracking loads. The consequences of torsional effect on timber beams' cross-section and shape are; reduction in the reliability and safe operations of the structure under serviceability. Structural timber beams subjected to torsion are of different shapes such as solid saw lumber, I- shaped joists, glue laminated timber joists and open web trusses (Khokhar, 2011)

The effects of torsional loading can be classified into uniform and non-uniform. When torsional load is applied to a structural member, its cross-section may warp in addition to twisting. If the member is allowed to warp freely, then the applied torque is resisted entirely by torsional shear stresses (called St Venent's torsional shear stress). If the member is not allowed to warp freely, the applied torque is resisted by St Venant's torsional shear stress and warping torsion (Hoogenboom, 2006). This behavior is called non-uniform torsion. Warping of the cross-section does not allow a plane section to remain as plane after twisting and this phenomenon is predominant in thin walled section. Timber beams must be designed against all induced stresses. Through the design of wooden structures, we try to avoid torsion of structural beams with different shape and constructive measures, and carry all loads through preferably bending. If we can succeed in doing that, we will have only tensile and /or compressive normal stresses to which wood corresponds better. Wood is a natural material which varies in mechanical properties. Knowledge of these properties is obtained through experimentation either in the employment of the wood in practice or by means of special testing apparatus in the laboratory. The shear modulus and shear strength are fundamental mechanical properties that are used in general timber design, compared to other engineering materials, timber has a relatively low shear stiffness and strength in comparison to its modulus of elasticity and so shear deformation contributes a more significant portion of flexural deflection (Khokhar and Zhang, 201).

Problem statement

The properties of timber can be obtained, theoretically through analytical model and numerical model. The problems of elasticity usually require solution of certain partial differential equations with a given boundary conditions. Only in simple boundary cases can these equations be treated in a rigorous manner. Very often, we cannot obtain a rigorous solution and must resort to approximate methods. It is necessary to realize that analytical solution requires quite a lot of theoretical knowledge due to its mathematical exactness and it's convenient only for the simpler cases. Analytical approach has been rarely used due to its complex procedure. Finding exact analytical solutions in general more complicated cases is usually very difficult, sometimes even impossible. However, the use of finite difference equations with the specified values of the independent variables, then lead to a system of simultaneous algebraic equation that can be solved by computer.

Aims and objectives of this study is to: understand the basic behavior of timber beams subjected to torsional loading, review the theoretical concept of torsion in timber beams, evaluate the analytical and numerical methods of determining the torsional parameters; stress functions and shear stresses in two-directions of a rectangular timber section and compare the results obtained from the two methods.

The scope and limitations of the study: This study is based on the evaluation of torsion problems in timber beams using analytical and numerical models (Finite difference) to determine the stress functions, shear stresses induced in a solid rectangular cross-section. The study also discussed the isotropic and anisotropic behavior /response of timber beams to torsional effects. The study uses MATLAB computer programming to seek for the solution of the finite difference equations derived and compared it with the analytical value

II. LITERATURE REVIEW

Wood, as a structural material is subjected to various types of loading conditions. If no external forces act upon a timber beam, its particles assume certain relative positions, and it has what is called its natural shape and size. If sufficient external force like torque is applied the natural shape and size will be changed. This distortion or deformation of the material is known as the Torsional strain. The ability of the timber beam material to withstand a twisting load, the ultimate strength of the timber material subjected to torsional loading, and the maximum torsional stress that the material sustains before rupture depends on its Torsional strength (Record, 2004). The design of a timber joists mainly depends upon its stiffness and strength properties. Stiffness is the property by means of which a body acted upon by external forces tends to retain its natural size and shape, or resists deformation.

Theory of elasticity

The classical theory of elasticity is based on an idealized 'Hookean' solid for which stress is directly proportional to strain and the deformation is completely recoverable after release of the force that produces the deformation. Furthermore, if the relationship between the applied stresses and the deformation can be assumed linear, then the material is said to be linear elastic. This is independent from any other assumption regarding the relationship between displacements and strain (Leitao, 1994). **Saint Venant,** 1855 was the first to provide the correct solution to the problem of torsion of bars subjected to moment couples at the ends. He made certain assumptions about the deformation of the twisted bar and then show that this solution satisfied the equations of equilibrium and the boundary conditions. From the uniqueness of solutions of the elasticity equations, it follows that the assumed forms for the displacements are the exact solution to the torsional problems.

Saint-Venant Torsion Theory for non-circular cross-section

In order to develop the torsional behavior of non-circular cross section, Saint-Venant made the following assumptions (Sadd, 2005 and Sadda, 1993): The member is straight, has constant cross section without taper and; The load is pure torque and produced by the shear stresses distributed over the end cross sections; Each cross section of member rotates approximately as rigid body and rotation of each cross section varies linearly along the longitudinal direction; Angle of twist must be small for small deformation and that warping must be small and the same for each cross section; The member must be homogeneous, **isotropic** and linearly elastic.

The torsion problem for a rectangular bar can be solved in terms of either the warping function or the stress function. Later in 1903, Ludwig Prandtl suggested that τ_{xz} and τ_{yz} can be taken as a magnitude of a slope of stress function surface to their perpendicular planes and Equations can be written as:

$$\tau_{xz} = \theta \frac{\partial \phi}{\partial y};\ \tau_{yz} = -\theta \frac{\partial \phi}{\partial x} \tag{2.1}$$

The governing equation for the problem is a Poisson equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2G\theta \tag{2.2}$$

Where $\emptyset$ is the stress function, represents surface over the cross-section of the torsion member. τ_{xz} And τ_{yz} represent shear stresses in relative planes to applied torque, angle of twist per unit length of member (θ).

The stress function can be determined by using an elastic membrane analogy approach (Boresi and Schmidt, 2003). Torque-rotation relationship can further be simplified by:

$$\frac{T}{\theta} = Gk_1(d)(b)^3 \tag{2.3}$$

The maximum shear stress (τ_{max}) in the cross-section can be obtained as:

$$\tau_{max} = \frac{T}{k_2(d)(b)^2} \tag{2.4}$$

Torsion Theory of Anisotropic Bars

It has long been recognized that deformation behavior of many materials depends upon orientation, that is, the stress-strain response of a sample taken from the material in one direction will be different if the sample were taken in a different direction. The term anisotropic is generally used to describe such behavior (Sadd, 2005). Wood is highly anisotropic due mainly to the elongated shapes of wood cells and the oriented structure of the cell walls. In addition, anisotropy results from the differentiation of sizes of cell throughout a growth season and in part from a preferred direction of certain cell types (August, 2008), thus, knowledge of stress distributions in anisotropic materials is very important for proper use of these high-performance materials in structural applications. Applying anisotropic theory to wooden beams of rectangular cross-section under torsion is definitely more complex than just assuming isotropic behavior. The two shear Moduli in LR and LT plane have fundamental influence on the results for shear strength. For the orthotropic rectangular member bar, the stress function or governing equation can be written as (Lekhnitskii, 1981):

$$\left(\frac{G_{LT}}{G_{LR}}\right)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2G_{LT}\theta \tag{2.5}$$

G_{LT} and G_{LR} are the shear modulus in longitudinal-Tangential (LT) plane (longer side) and longitudinal-Radial (LR) plane (short side), respectively of the member). The maximum shear stresses can be obtained on the centre of either the short side or long side as:

$$\tau_{max}(short\ side) = \frac{T}{(d)(b)^2}k_3; \tag{2.6}$$

$$\tau_{max}(long\ side) = \frac{T}{(d)(b)^2\left(\sqrt{\frac{G_{LT}}{G_{LR}}}\right)}; \tag{2.7}$$

$$\beta = \frac{32}{\pi^4}C^2\sum_{K=1,3,5,...}^{\infty}\frac{1}{K^4}\left(1-\frac{2}{K\pi}C\ tanh\frac{K\pi}{2C}\right) \tag{2.8}$$

$$K_3 = \frac{8}{\pi^2\beta}C\sum_{K=1,3,5,...}^{\infty}\frac{(1)^{(K-1)/2}}{K^2}tanh\frac{K\pi}{2C} \tag{2.9}$$

$$k_4 = \frac{C}{\beta}\left[1-\frac{8}{\pi^2}\sum_{K=1,3,5,...}^{\infty}\frac{1}{K^2cosh\frac{K\pi}{2C}}\right]; \tag{2.10}$$

$$\text{Where } C = \frac{d/b}{\sqrt{G_{LT}/G_{LR}}} \quad (2.11)$$

Numerical method for Determination of Torsion effects with Finite Difference Method (FDM)

The basic idea of the finite difference method is to represent the governing differential equations and the associated boundary conditions with finite differential equations. The finite differential equations are employed to approximate the derivatives in the differential equations. Combinations of the values of unknown functions at the specified locations of the independent variable form the finite difference quotients. The finite difference equations with the specified values of the independent variables then lead to a system of simultaneous algebraic equations that can be solved by computer. The basic finite difference expression follows logically from the fundamental rules of calculus.

For the empirical study of numerical model on torsion, Stefan et al (2012) and Hsieh (2007) respectively, have researched on the numerical determination of torsional parameters and shear stresses of a rectangular timber bar using finite difference and finite element methods.

III. Research Methodology

The Numerical model will be used in this study to determine the stress function and torsional parameters of a rectangular timber beams subjected to torsional effects. Both isotropic and anisotropic nature of timber beam will be considered in the numerical model. By means of finite difference method, approximate stress function, torsional parameters and shear stresses in rectangular structural members are evaluated using **MATLAB** computer programming and checked against the analytical values.

The analyses of torsion of a rectangular bar section using finite difference technique

The torsion problem to be modeled is one with rectangular boundary. Such cross-section will be embedded in a basic rectangle of length b, in the y-direction and width a, in the x-direction. The length and width are such that b is a fraction (or multiple of a.) Then, the length of the cross-section in the x-direction is 1. The origin of the x, y coordinate system is assumed to be at the centroid of the basic rectangle.

a) The Grid: The rectangle is partitioned into a grid, uniform in the x- and y –directions respectively, but not necessarily with equal x and y increments as shown in figure 3.1 below.

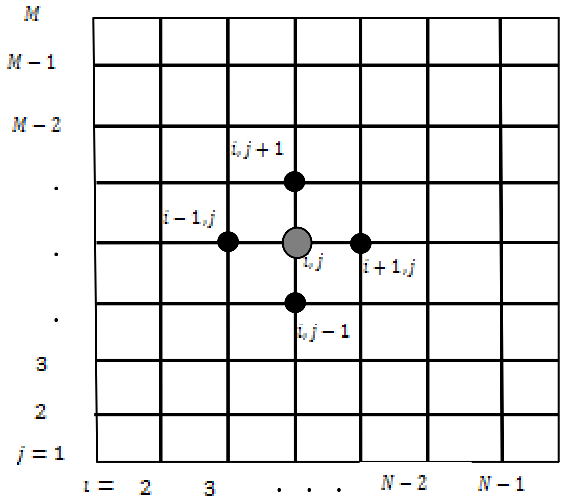

Figure 3.1, the finite difference grid for the basic rectangle R domain. The circles depict the computational molecule (of equation below).

The N grid points in x- and M grid points in y- respectively, have coordinates

$$\left[-\frac{a}{2} = x_1, x_2, x_3, \ldots, x_N = \frac{a}{2}\right] \text{ with } dx = x_i - x_{i-1};\ i = 2, \ldots, N.$$

$$\left[-\frac{b}{2} = y_1, y_2, \ldots, y_M = \frac{b}{2}\right] \text{ with } dy = y_j - y_{j-1};\ j = 2, 3, \ldots, M.$$

b) The finite difference equations: The interior constitutes all the grid points with indexes $i = 2, \ldots, N-1$ and $j = 2, \ldots, M-1$. the values of $\emptyset$ are specified on the boundary point $j = 1$ and M for $i \in [1, \ldots, N]$ and $i = 1$ and N for $j \in [1, \ldots, M]$. Let the net function $f_{i,j} = \emptyset(x_i, y_j)$ be the numerical values at the grid points of the sought after solution $\emptyset(x, y)$. The equations for $f_{i,j}$ are found by substituting the governing Poisson's equations, evaluated at each point $i, j \in R$ domain by finite difference formulae respectively.

For Isotropic nature of timber beam:

The governing equation for the problem is a Poisson equation as in (2.2): $\frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial y^2} = -2G\theta$,

And substituting it with the finite difference equations and codes gives

$$\frac{f_{i-1,j} - 2f_{i,j} + f_{i+1,j}}{dx^2} + \frac{f_{i,j-1} - 2f_{i,j} + f_{i,j+1}}{dy^2} = -2$$

Assuming that $dx = dy$, Resulting into:

$$f_{i-1,j} + f_{i+1,j} + f_{i,j-1} + f_{i,j+1} - 4f_{i,j} = -2dx^2$$

The solution of the finite difference equation is obtained by rewriting the above equation as:

$$f_{i,j} = \frac{1}{4}(f_{i+1,j} + f_{i-1,j} + f_{i,j-1} + f_{i,j+1} + 2dx^2);$$

For Anisotropic nature of timber beam:

The governing equation for the problem is a Poisson equation as in (2.5) is: $(\frac{G_{LT}}{G_{LR}})\frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial y^2} = -2G_{LT}\theta$.

Substituting it with the finite difference equations and codes gives:

$$\frac{g(f_{i-1,j} - 2f_{i,j} + f_{i+1,j})}{dx^2} + \frac{e(f_{i,j-1} - 2f_{i,j} + f_{i,j+1})}{dy^2} = -2$$

$where\ g = \frac{1}{G_{LR}};\ e = \frac{1}{G_{LT}};$ Considering equal division points: $dx = dy$

$$f_{(i,j)} = \frac{1}{2(g+e)}[gf_{(i-1,j)} + gf_{(i+1,j)} + ef_{(i,j-1)} + ef_{(i,j+1)} + 2dx^2]$$

The iteration process:

By starting the iteration process with $f_{i,j} = 0$ everywhere, the boundary conditions are automatically satisfied. This process is considered in both isotropic and anisotropic behavior respectively.

For k=2:k
For j=2:M-1
For i=2:N-1
$f_{i,j}{}^{k} = q_{(i,j)}[f_{i-1,j}{}^{k} + f_{i+1,j}{}^{k-1} + f_{i,j-1}{}^{k} + f_{i,j+1}{}^{k-1} + 2dx^2$
End
End
End

A fixed number of passes (K) equal to the product of number of grid points N and M in the x- and y-direction respectively, which is enough for the system to converge.

The numerical computation of the stresses and other torsional parameters are obtained from the numerical stress function $f_{(i,j)}$ using the following formulas:

The stresses are evaluated using the following:

i. For the bottom and top of the rectangle, $j = 1$ and $j = M; i = 1$ to N $respectively.$

$$\tau_{xz,i,1} = [\frac{\partial \emptyset}{\partial y}]_{i,1} = [\frac{-3f_{i,1}+4f_{i,2}-f_{i,3}}{2dy}];\ \tau_{xz,i,M} = [\frac{\partial \emptyset}{\partial y}]_{i,M} = [\frac{f_{i,M-2}-4f_{i,M-1}+3f_{i,M}}{2dy}]$$

$where\ \tau_{yz,} = 0,$ On these two boundaries

ii. For the left and right ends of the rectangle; $i = 1, and\ i = N,\ \ j = 1\ to\ M,\ respectively$

$$\tau_{yz,1,j} = -[\frac{\partial \emptyset}{\partial y}]_{1,j} = -\left[\frac{-3f_{1,j,j}+4f_{2,j}-f_{3,j}}{2dx}\right];\ \tau_{yz,N,j} = -[\frac{\partial \emptyset}{\partial y}]_{N,j} = -[\frac{f_{N-2,j}-4f_{N-1,j}+3f_{N,j}}{2dx}]$$

iii. The torsional parameters for anisotropic rectangular cross-section K_3 and K_4 are obtained as follows:

$$K_3 = \tau_{xz} * a * b^2;\ K_4 = \tau_{xz} * a * b^2/(\sqrt{\frac{G_{LT}}{G_{LR}}})$$

<<

IV. PRESENTATION OF RESULTS

To validate the finite difference method formulated, three examples will be evaluated for verification of the results. Analytical and numerical (finite difference method) models developed will be employed to evaluate the values on rectangular timber beam of isotropic and anisotropic material behavior of timber. In order to provide the accuracy and efficiency of the finite difference method, the results obtained from the numerical model program will be compared with those of the analytical method which is available from previous works of $(Boresi\ \&\ Schmidt,\ 2003)$ and (Hsiehk, 2007).

Example 1: Torsional Parameters for Isotropic material behavior of Rectangular Cross Sections. The verification example is from textbook $(Boresi\ \&\ Schmidt, 2003)\ and\ (Hsiehk, 2007).$ The value of the torsional parameter from the previous researcher is compared with the value obtained from the Finite difference method (FDM). The values of k_1 and k_2, depends on cross section/ aspect ratio of rectangular bar.

Table 4.1 evaluation of K_1 and K_2value between analytical and finite difference method (FDM) results for isotropic behavior which depend on the ratio of the sides of the rectangular cross-section of the beam (y-z direction)

b/a	1.0	1.5	2.0	2.5	3.0	4.0	6.0	10.0
K_1 $(Boresi\ \&\ Schmidt)$	0.141	0.196	0.229	0.249	0.263	0.281	0.299	0.312
K_1 (FDM)	0.1404	0.1962	0.2288	0.2493	0.2635	0.2808	0.2988	0.3055
K_2 $(Boresi\ \&\ Schmidt)$	0.208	0.231	0.246	0.256	0.267	0.282	0.299	0.312
K_2 (FDM)	0.2082	0.2314	0.2461	0.2557	0.2668	0.2819	0.2988	0.3124

Example 2: Torsional coefficients for anisotropic rectangular cross-section with two orthogonal shear properties in different directions of timber beam axis. The values of β, k_3 and k_4, depends on aspect ratio of the cross section and shear Modulus of rectangular bar

Table 4.2 evaluation of $\beta\ value,\ K_3\ value\ and\ K_4\ value$ between analytical (Lekhnitskili 1981) and finite difference method results for anisotropic behavior depending on the ratio of shear Moduli and the ratio of the sides of the rectangular cross-sectional beam

C	1.0	1.5	2.0	2.5	3.0	4.0	5.0	10	20
β $(Lekhnitskili)$	0.141	0.196	0.229	0.249	0.263	0.281	0.291	0.312	0.323
β (FDM)	0.1407	0.1968	0.2285	0.2478	0.2626	0.2812	0.2902	0.3101	0.3215

K_3 $(Lekhnitskili)$	4.804	4.330	4.068	3.882	3.742	3.550	3.430	3.202	3.098
K_3 (FDM)	4.8023	4.3071	4.0783	3.8819	3.7584	3.5745	3.4897	3.2128	3.0662
K_4 $(Lekhnitskili)$	4.804	3.767	3.234	2.975	2.538	2.644	2.548	2.379	2.274
K_4 (FDM)	4.8023	3.7948	3.2184	2.9785	2.5260	2.6249	2.5716	2.3609	2.2805

Example 3: The shear stress distribution of full-sized specimen measuring $44 * 140 * 1372\ mm^2$ under torsion for all three structural composites lumber (SCL) material; laminated veneer lumber (LVL), parallel strand lumber (PSL) and laminated strand lumber (LSL) by (Gupta and Siller, 2005) . A constant torque value of $300Nm$ was applied to each sample. Shear Moduli of the materials are: LSL (GRL = 318 Mpa, GTL = 782 Mpa), LVL (GRL = 407 Mpa, GTL = 593 Mpa) and PSL (GRL = 310 Mpa, GTL = 398 Mpa)

Table 4.3: Comparing the longitudinal tangential and radial shear stresses between Gupta and Siller (2005) experimental work, and finite difference method for both isotropic and anisotropic.

	Isotropic				anisotropic			
	(Gupta and siller)		Finite Difference Method		(Gupta and siller)		Finite Difference Method	
material	Shear stress (Mpa)				Shear stress (Mpa)			
	$long\ \tau_{LT-I}$	$short\ \tau_{LR-I}$	$long\ \tau_{LT-I}$	$short\ \tau_{LR-I}$	$long\ \tau_{LT-A}$	$short\ \tau_{LR-A}$	$long\ \tau_{LT-A}$	$short\ \tau_{LR-A}$
LSL	11.59	8.83	11.692	8.7364	12.69	6.43	12.498	6.327
LVL	7.66	5.83	7.712	5.5045	7.96	4.90	7.769	4.46
PSL	6.65	5.02	7.150	5.1637	6.82	4.55	6.787	4.775

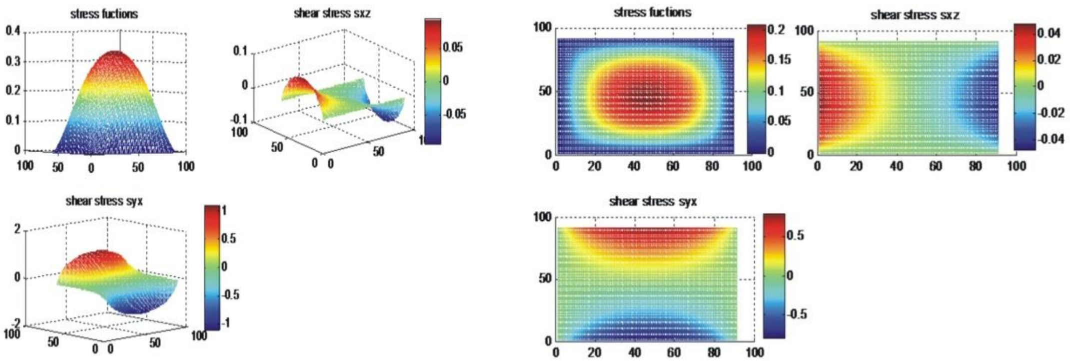

Figure 4.1 stress distribution from numerical solution for isotropic behavior: $a = 1, N = 41, M = 71$ (a) shear stress (syz) distribution in the $y - direction$ (b) shear stress (sxz) distribution in the $x - direction$ e (c) the stress function contour for rectangular section $xy -$ view and (d) $xz\ and\ yz -$ view (from MATLAB)

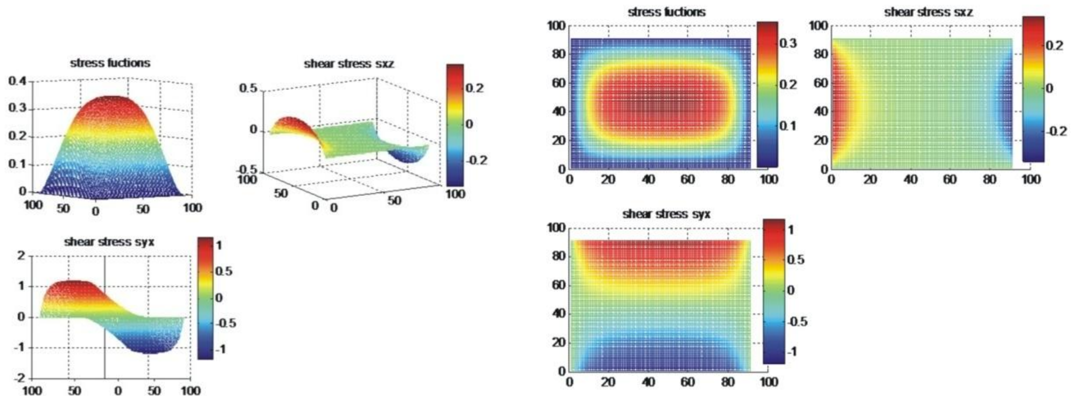

Figure 4.3 stress distribution for anisotropic material behavior of timber subjected to torsional loading, $a = 1, N = 41, M = 71$ (from MATLAB)

V. DISCUSSION ON FINDINGS

The torsional parameters for rectangular cross-sectional timber beam $K_1, K_2, \beta, K_3, K_4$ were determined using the finite difference method of numerical analysis with MATLAB programming and compared with the analytical model from previous research work for both isotropic and anisotropic material behavior of timber beam.

Tables 4.1 present the torsional parameters K_1 *and* K_2 for isotropic material behavior of rectangular cross-section for both analytical and finite difference method (FDM). The result indicates that finite difference method values are not exact, but approximately to the values obtained from analytical calculation. Observations indicated that, through ANOVA Test; there was evidence at 0.05 levels of significance that there was no significant difference between analytical and numerical values, statistically. The result shows that $F_{critical}$ $(4.747) >$ $F_{calculated}(0.00167)$ with variance of 0.00153 for K_1. $F_{critical}$ $(4.747) > F_{calculated}(0.000000849)$, variance of 0.000841 for K_2.

Tables 4.2 present the torsional parameters β, K_3 *and* K_4 respectively, for anisotropic material behavior of rectangular cross-section for analytical and finite difference methods. It was observed that the values obtained by finite difference method are not exact, but approximate when compared with the analytical calculated values. From the ANOVA Test conducted, there was indication at 0.05 levels of significance that there was no significant difference between analytical and finite difference methods statistically. $F_{critical}$ $(4.60) >$ $F_{calculated}(0.0000169)$, variance of 0.001804 for β. AVOVA Test conducted for K_3, K_4 also confirmed that there was no significant difference between analytical method and finite difference method. It was found that the torsional parameters, obtained using the two methods, have a very strong correlation as R^2 was found about 0.998.

Table 4.3 presents the shear stress distribution of structural composite lumber (LVL, LSL and PSL) for both isotropic and anisotropic behavior to torsional load. Solving the problem for various materials, it was observed that some values of longitudinal tangential and radial shear stresses matched the experimental work of Gutpa and Siller (2005) for both isotropic and anisotropic material behavior, but the process is not convergence. The values obtained from finite difference method developed for tangential and radial shear stresses are approximate to that of Gupta and Siller (2005).

Comparing the isotropic and anisotropic shear stress distribution in figures 4.1 and 4.3, the shear stress distribution in isotropic timber beam behavior depends only on the ratio of the rectangular cross-sections $(^b/_a)$ thereby having a uniform stress distribution with equal number of iteration (passes) on the longitudinal tangential and radial directions as shown in figure 4.1. However, in the anisotropic timber beam behavior, shear stresses are not only dependent on the rectangular side ratio, but also on the ratio of the two shear Moduli in longitudinal tangential (LT) and radial (LR) directions as shown in figure 4.2. In other words, the two shear Moduli in LT *and* LR plane have fundamental influence on the results for shear strength. The higher the aspect ratio, more anisotropic the material behaves.

Convergence of the finite difference method using MATLAB

The values of torsional parameters depend on the distance between the points laying on the boundary and the neighbor point inside the section domain (dx and dy). Smaller values lead to better estimation of the torsional parameters and an increase size of unknown system. The utilization of variable distance between mesh points (dx and dy), leads to a more accurate estimate of the torsional parameters, using smaller number of grid point. This is in line with the results gotten by **Stefan et al** in the literature review.

VI. Conclusion

The finite difference model developed in this study is capable of predicting the torsional shear stress distributions and torsional parameters of rectangular timber beam subjected to torsional load, for both isotropic and anisotropic material behavior. The finite difference method allows the study of stress distribution for sections and boundary conditions in which an analytical approach is difficult. The knowledge of stress distributions in anisotropic material behavior is very important for proper use of those high-performance materials in structural application. The isotropic material behavior neglected those effects of directional-dependent behavior of wood, thus resulting in a material that behaves the same in all directions. Good agreement was found between the finite difference method and previous results.

Recommendations

The study was limited to finite difference method. Further research needs to be conducted to evaluate the torsional parameters using finite element model or any other numerical models. Further study should be carried out on other shapes like L-shaped, I-shaped and open web trusses. Another area that needs further study is on the best way of making the finite difference method converge easily using MATLAB programming.

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