American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-4, Issue-11, pp-163-170
www.ajer.org
Research Paper Open Access

Strain Based Finite Element for Cylindrical Shell under Sinusoidal Loads

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ABSTRACT: *A cylindrical triangular finite element is developed in this paper. The element has only five essential nodal degrees of freedom (Three general external degrees of freedom and two rotations) at each of the three corner nodes. The displacement fields of the element satisfy the exact requirements of rigid body modes of motion. Shallow shell formulation is used and the element is based on an independent strain assumption insofar as it is allowed by the compatibility equations. A cylindrical shell problem for which a previous solution exists is first analyzed using the new element to validate the program and to compare the results. The element is then used in the analysis of cylindrical shell subjected to uniformly distributed load varying sinusoidal along its length in addition to symmetric sinusoidal edge loads present along its longitudinal boundaries. The distribution of various components of stresses is obtained and the effect of radius-length ratio is also presented to give the designer an insight for the behavior of such structures.*

Keywords -*Strain-based, cylindrical triangular finite element.*

I. INTRODUCTION

Considerable attention has been given applying the finite element method of analysis to curved structures. The early work on the subject was presented by Grafton and Strome (1963) who developed conical segments for the analysis of shells of revolution. Jones and Strome (1966) modified the method and used curved meridional elements which were found to lead to considerably better results for the stresses.

Further research led to the development of curved rectangular as well as cylindrical shell elements (Connor *et al*, 1967; Bogner *et al*, 1967; Cantin *et al*, 1968 and Sabir *et al*, 1972). However, to model a shell of arbitrary or triangular shape by the finite element method, a triangular shell element is needed. Thus many authors have been occupied with the development of curved triangular shell elements and consequently many elements (Lindberg *et al*, 1970 and Dawe, 1975) resulting in an improvement of the accuracy of the results. However, this improvement is achieved at the expense of more computer time as well as storage to assemble the overall structure matrix.

Meanwhile, at the United Kingdom, a simple alternative approach has been used to the development of curved elements. This approach is based on determining the exact terms representing all the rigid body modes together with the displacement functions representing the straining of the element by assuming independent strain functions insofar as it is allowed by the compatibility equations.

This approach has successfully employed in the development of curved shell elements (Ashwell *et al*, 1971, 1972; Sabir *et al*, 1975, 1982, 1983, 1987; El-Erris, 1994, 1995 and Mousa, 1994, 1998, 2012). These elements were found to yield faster convergence with other available finite elements.The strain-based approach is employed in the present paper to develop a new triangular strain-based cylindrical element having only five degrees of freedom at each corner node.

The new element is first tested by applying it to the analysis of a clamped barrel vault for which a previous solution exists.

The work is then extended to the analysis of a cylindrical shell subjected to uniformly distributed load varying sinusoidal along its length and symmetric sinusoidal edge loads present along its longitudinal boundaries.The distribution of the various components of stresses is obtained to give designers an insight into the behavior of such structures.

II. THEORETICAL CONSIDERATION OF THE DISPLACEMENT FUNCTIONS FOR THE NEW CYLINDRICAL ELEMENT

In a system of curvilinear coordinates, the simplified strain-displacement relationship for the cylindrical shell element shown in Fig. (1) can be written as:

$$\varepsilon_x = \frac{\partial_u}{\partial_x} + \frac{w}{R} \ , \ \varepsilon_y = \frac{\partial_v}{\partial_y} \ , \ \gamma_{xy} = \frac{\partial_u}{\partial_y} + \frac{\partial_v}{\partial_x} and$$
$$K_x = \frac{\partial^2 w}{\partial x^2} \ , \ K_y = \frac{\partial^2 w}{\partial y^2} \ , \ K_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \tag{1}$$

Where u,v and w are the displacement in the x,y and z axes, $\varepsilon_x, \varepsilon_y$ are the in-plane direct axial and circumferential strains and γ_{xy} is the in-plane shearing strain. AlsoK_x, K_y, K_{xy} are the mid-surface changes of curvatures and twisting curvature respectively and R is the principle radii of curvature.

Equation (1) gives the relationships between the six components of the strain and three displacement u,v, and w. Hence, for such a shell there must exist three compatibility equations which can be obtained eliminating u,v and w from equation (1).

This is done by a series of differentiations of equation (1) to yield the following compatibility equations:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} + \frac{K_y}{R} = 0$$
$$\frac{\partial K_{xy}}{\partial x} - 2\frac{\partial K_y}{\partial y} = 0 \tag{2}$$
$$\frac{\partial K_{xy}}{\partial y} - 2\frac{\partial K_x}{\partial x} = 0$$

To keep the triangular element as simple as possible, and to avoid the difficulties associated with internal and non-geometric degrees of freedom, the essential five degrees of freedom at each corner node are used, namely u,v,w, $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$. Thus, the triangular cylindrical element, to be developed has a total of fifteen degrees of freedom and 15x15 stiffness matrix.

To obtain the rigid body components of the displacement field, all the strains, as given by equations (1), are set to zero and the resulting partial differential equations are integrated. The resulting equations for u, v, and w are given by:

$$u_1 = -\frac{a_1 x}{R} - a_2\left(\frac{x^2}{2R}\right) - \frac{a_3 xy}{R} + a_4 + a_6 y$$
$$v_1 = -a_3\left(\frac{x^2}{2R}\right) + a_5 - a_6 x\frac{\partial K_{xy}}{\partial y} - 2\frac{\partial K_x}{\partial x} \tag{3}$$
$$w_1 = a_1 + a_2 x + a_3 y$$

Where $u_1, v_1, and\ w_1$ are the rigid body components of the displacement fields u, v, and w, respectively, and are expressed in terms of the six independent constants$a_1, a_2, \dots . a_6$.

Since the element has fifteen degrees of freedom, the final displacement fields should be in terms of fifteen constants. Having used six for the representation of the rigid body modes, the remaining nine constants are available for expressing the straining deformation of the element. These nine constants can be apportioned among the strains in several ways, for the present element we take:

$$\varepsilon_x = a_7 - \frac{1}{R}\left(a_{12}\frac{y^2}{2} + a_{13} x\frac{y^2}{2} + a_{14}\frac{y^3}{6}\right)$$
$$\varepsilon_y = a_8$$
$$\gamma_{xy} = a_9 \tag{4}$$
$$K_x = a_{10} + a_{11} xy$$
$$K_y = a_{12} + a_{13} x + a_{14} y$$
$$K_{xy} = a_{15} + [a_{11} x^2 + 2\, a_{13} y]$$

In which the un-bracketed independent constants terms in the above equations were first assumed. The linking bracketed terms are then added to satisfy the compatibility equation (2).

Equations (4) are then equated to the corresponding expressions, in terms of u,v and w from equations (1) and the resulting equations are integrated to obtain:

$$
\begin{aligned}
u_2 &= a_7 x + \frac{a_9 y}{2} + \frac{a_{10} x^3}{6R} + \frac{a_{11} x^4 y}{24R} + a_{15}\left(\frac{x^2 y}{4R}\right) \\
v_2 &= a_8 y + \frac{a_9 x}{2} - \frac{a_{11} x^5}{120R} - a_{15}\left(\frac{x^3}{12R}\right) \\
w_2 &= -\frac{a_{10} x^2}{2} - \frac{a_{11} x^3 y}{6} - \frac{a_{12} y^2}{2} - \frac{a_{13} x y^2}{2} - \frac{a_{14} y^3}{6} - \frac{a_{15} x y}{2}
\end{aligned}
\tag{5}
$$

The complete displacement functions for the element are the sum of corresponding expressions in equations (3) and (5). The rotation about the x and y-axes respectively, are given by:

$$
\begin{aligned}
\varphi y &= -\frac{\partial w}{\partial x} = -a_2 + a_{10} x + \frac{a_{11} x^2 y}{2} + \frac{a_{13} y^2}{2} + \frac{a_{15} y}{2} \\
\varphi x &= -\frac{\partial w}{\partial y} = -a_3 + \frac{a_{11} x^3}{6} - a_{12} y - a_{13} x y - \frac{a_{14} y^2}{2} - \frac{a_{15} x}{2}
\end{aligned}
\tag{6}
$$

The complete displacement functions are the sums of corresponding expressions from equations (3) and (5).

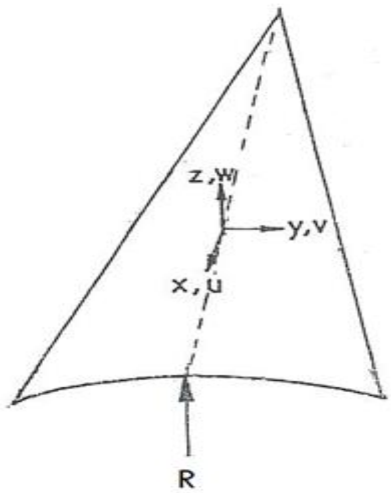

Fig. 1. Coordinate system for a triangular cylindrical shell element

Having derived the final expression for the displacement functions, it is now possible to obtain the elemental stiffness [K] for the cylindrical shell element in the usual manner using the following equations:

$$K = [C^{-1}]^T \left\{ \iint B^T B \, dx dy \right\} [C]^{-1} \tag{7}$$

where B and D are the strain and rigidity matrices, respectively, and C the matrix relating the nodal displacement to the constants a_1 to a_{15}. B can be calculated from equations (1), (3), (4) and D is given by substituting the matrices B and D into equation (5). The integration within the bracketed terms of equation (5) are carried out explicitly and the rest are computed to obtain the stiffness matrix [K].

III. PATCH CONVERGENCE TEST

This test is to be considered which is frequently used to test the performance of the shell elements is that of Scordelis-Lo Roof having the geometry as shown in Fig. (2). The shell has the following dimensions and material properties: thickness, t= 0.03 m, R= 3m, L = 6m, α= 40°, modulus of elasticity, E= 3 Pa, Poisson's ratio, μ= 0.0, density, ρ = 0.625 Pa. The straight edges are free while the curved edges are supported on rigid diagrams along their plan considering the symmetry of the problem only one quarter of the roof is analyzed.

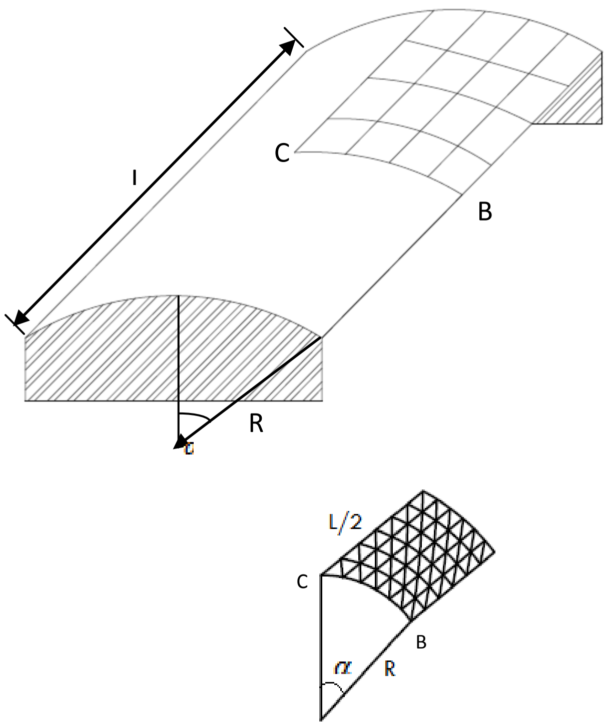

Fig. 2. Geometry and finite element mesh of the roof

The results obtained by the new present element for the vertical displacement at the midpoint B of the free edge and the center C of the roof are compared to other kinds of shells elements (Batoz *et al*, 1992 and Hamadi *et al*, 2000). The analytical solution of this problem is based on the shallow shell theory is given by Scordelis and Lo (1969). Convergence curves (Fig 3 and Fig 4) show that the convergence of the present element faster convergence than the other. Then the new present element would be more efficient to use it in the analysis of proposed cylindrical shell under loads.

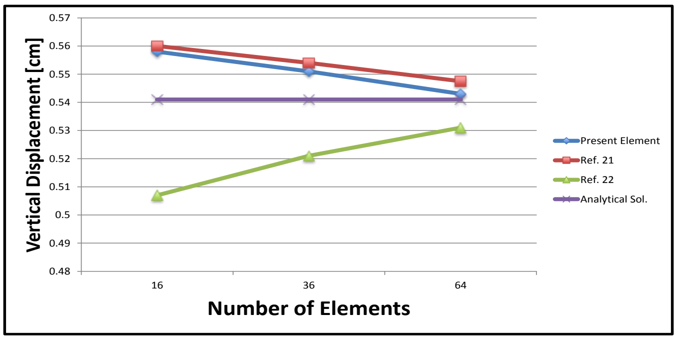

Fig. 3Convergence curve for the deflection, w, at point C

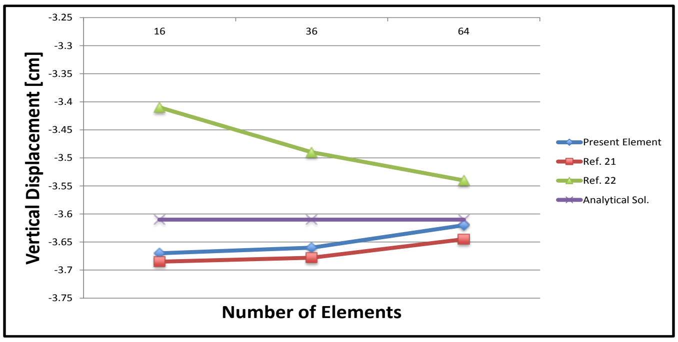

Fig. 4Convergence curve for the deflection, w, at point B

IV. PROBLEM CONSIDERED

An open barrel cylindrical shell is analyzed under uniform surface load varying sinusoidally in addition to the sinusoidal symmetric edge loads present along the longitudinal boundaries. The geometric properties of the shell is shown in Fig.(5). The shell has the following geometry and properties; L=26.67 m, R=8 m, $\alpha = 45^{\circ}$, thickness t=80 mm, E=25 $x\,10^{6}\frac{kN}{m^{2}}$, $\mu = 0.2$ and the applied load q = 3.25 $\frac{kN}{m^{2}}$, the distribution of this load is based on the following equation: $q\sin\left(\frac{\pi x}{L}\right)$

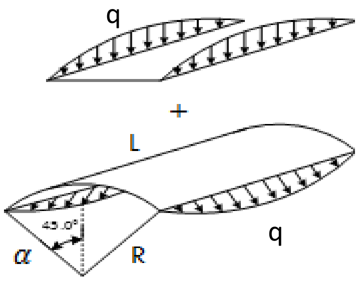

Fig. 5. Geometry and loads on the shell

Figs. (6-10) show the stress resultant for the problem considered. It is seen that the obtained results on the basis of the proposed present element very closely agree with these obtained from the analysis based on classic flexure theory procedure discussed by Chandrasekaran *et al* (2009).

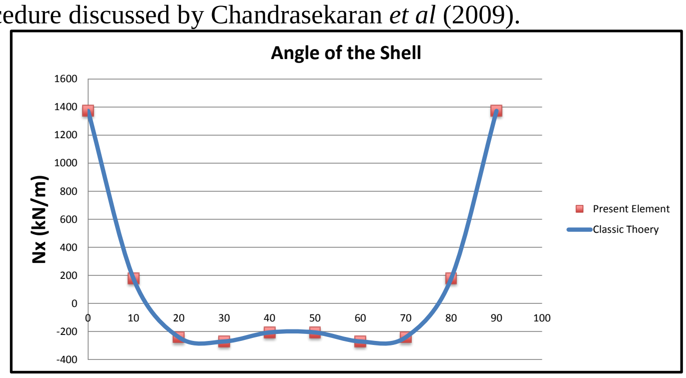

Fig. 6Axial stress resultant Nx, at distance L/2

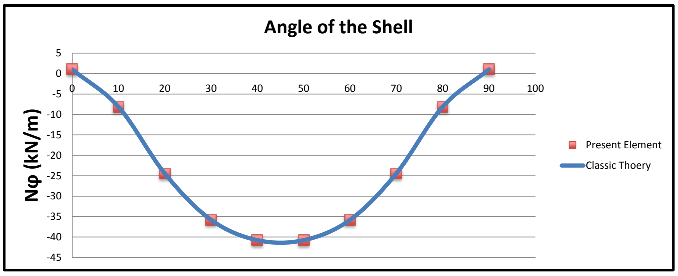

Fig. 7Axial stress resultant Nφ, at distance L/2

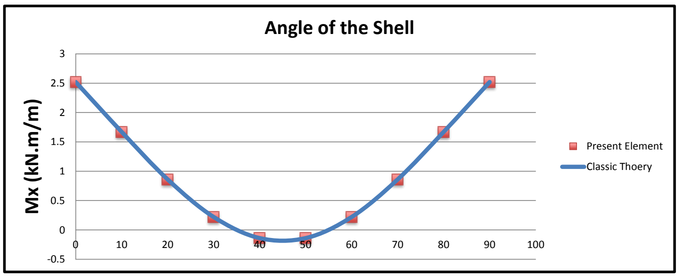

Fig. 8Bending stress resultant Mx, at distance L/2

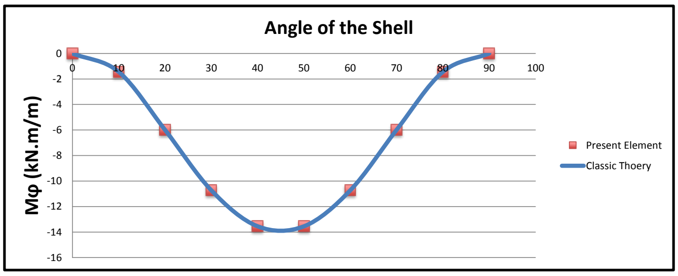

Fig. 9Bending stress resultant Mφ, at distance L/2

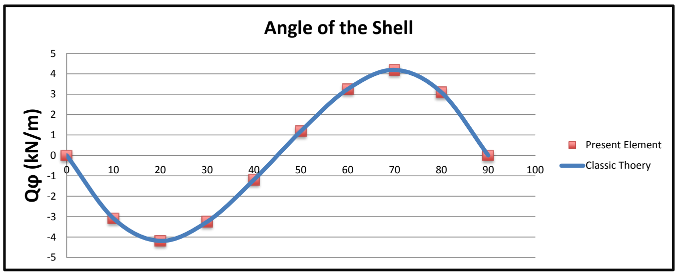

Fig. 10Stress resultant Qφ, at distance L/2

The work is extended to study the effect of radius length ratio on behavior of the cylindrical shell problem. Figs. (11-15) show the stresses resultant for different radius length ratio (R/L = 0.3, 0.4, 0.5, 0.6). It's clear that the various stress resultants are decreased by increasing the radius length ratio.

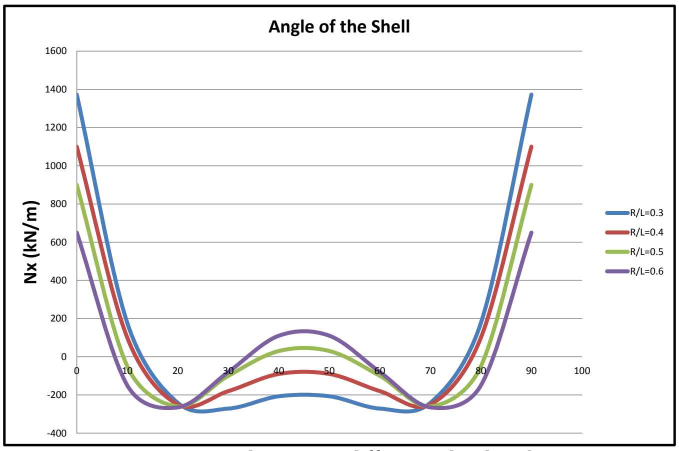

Fig. 11Stress resultant Nx, at different radius length ratio

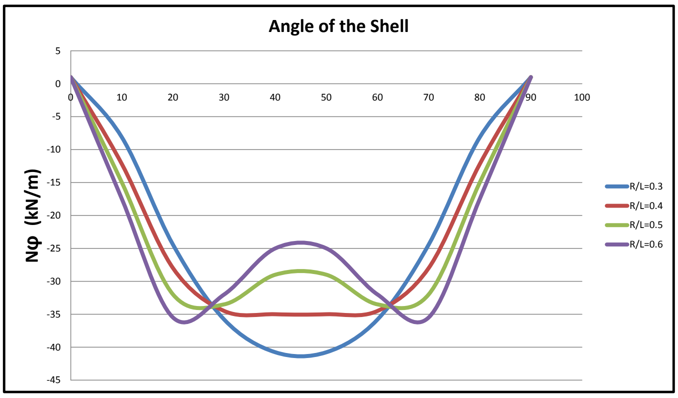

Fig. 12 Stress resultant Nφ, at different radius length ratio

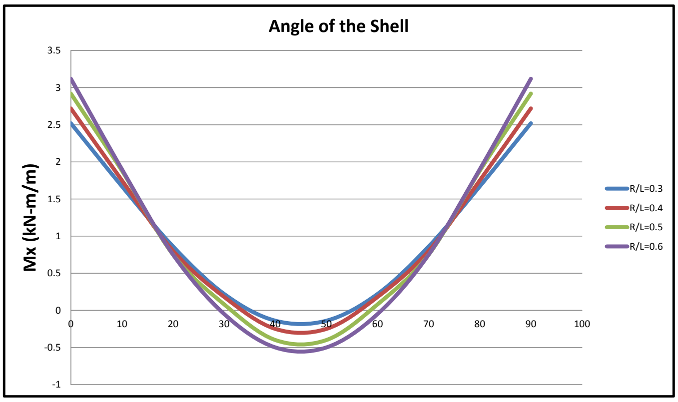

Fig. 13 Stress resultant Mx, at different radius length ratio

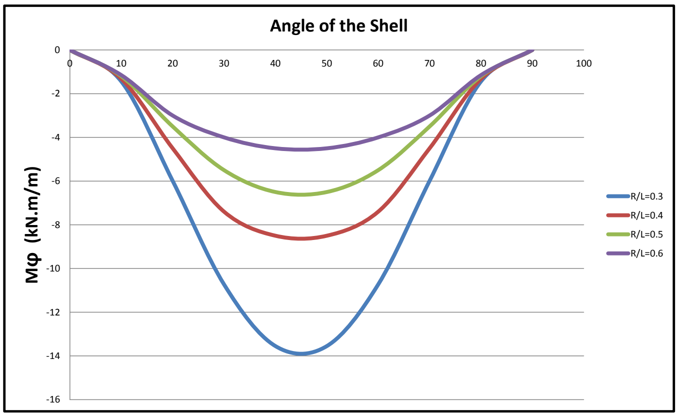

Fig. 14 Stress resultant Mφ, at different radius length ratio

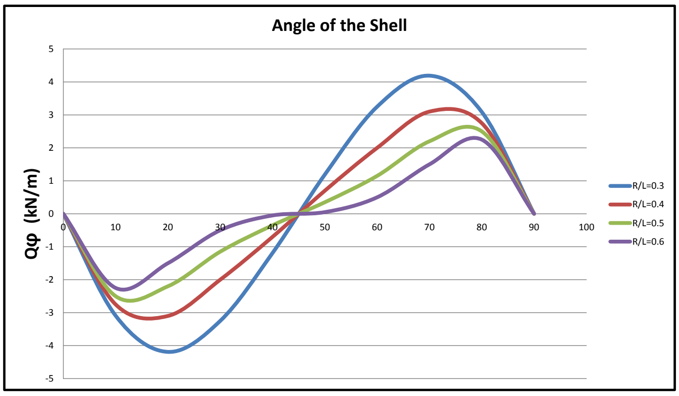

Fig. 15 Stress resultant Qφ, at different radius length ratio

V. CONCLUSION

A new triangular strain-based finite cylindrical element is developed using shallow shell formulation. The element has the essential five degrees of freedom at each corner node.

The developed triangular cylinder finite element is first applied to analysis of a clamped barrel vault. The results for the deflections is presented and shows that the developed element has a very good agreement results. This element is then used to analyze a cylindrical shell subjected to uniformly distributed load varying sinusoidal along its length and symmetric cylindrical edge sinusoidal load presented along its longitudinal boundaries.

The effect of the radius length ratio on the various stresses components is presented.

It's seen the various stresses components is decreased by increasing the radius length ratio. This gives the designers an insight for the behavior of such structures.

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