American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-4, Issue-9, pp-181-187
www.ajer.org
Research Paper Open Access

Power Flow Analysis for Elastic Coupling Plates

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ABSTRACT: *Based on mobility power flow method, the elastic connection between plates was simulated by torsion spring uniformly distributed along the coupling boundary, and the continuity equation of coupling boundary was modified. A theoretical analysis model of elastic coupling plates was obtained. The influence of connection stiffness on the vibration characteristic of coupling structure was analyzed. Simulation result show that with the increase of connection stiffness, modal of sub plate makes less contribution to the input power, but the situation is opposite to the modal of coupling structure. The transfer power increases significantly with the increase of connection stiffness, and the stiffness which makes the transfer power converging at natural frequencies of sub plate is less than that of natural frequencies of coupling structure.*

Keywords: *elastic connection; power flow; substructure; mechanical mobility*

I. INTRODUCTION

As a common periodic support structure, slicing plate is convenient to be manufactured, disassembled and examined. It is widely used in engineering community. Such as the cover plate of elevator, warship deck, bridge and so on. Structural vibration is excited by mechanical movement or human trample in their daily use, which has a great influence on the Device reliability and threatens the security of usage. Scholars have used many methods to analyze the vibration characteristics of coupling plates. P. J. Shorter [1] established the energy flow model of coupling plates by Finite Element Method (FEM) and calculated the average response and input energy of each substructure. When using FEM to analyze the vibration characteristics of complex structure in high frequency, the problem of huge calculation comlpexity can't be avoided, so it is usually used in the research on low frequency vibration characteristics. Statistical Energy Analysis (SEA) [2-4] has less calculation comlpexity when the structure has enough modal. Some parameters, such as coupling loss factor, are required in the calculation process. SEA is usually used in the research on high frequency vibration characteristic. Mobility power flow method [6,7] has clear physical concepts. It combines the idea of mechanical mobility and power flow. The characteristics of energy distribution and power flow in the coupling structure can be represented validly. And there is no frequency limit in this method. With continuous development, Mobility power flow method is widely used in engineering community [8-11].

In the manufacturing process of slicing plates, sub plate is usually obtained first and then assembled together. So the connection between sub plates is different from rigid connection, and the connection stiffness should be considered. Based on mobility power flow method, the elastic connection between plates will be simulated by torsion spring uniformly distributed. By modifying the continuity equation of coupling boundary, a theoretical calculation model of elastic coupling plates can be obtained. Based on this model, the influence of connection stiffness on the vibration characteristic of coupling structure will be analyzed.

II. THEORITICAL MODEL

As shown in Fig.1, slicing plate is constituted by n sub plates. Four edges of each plate are all simply supported. The edge lengths of each plate are a_1, a_2, …,a_n. The width of the coupling boundary is b. Thickness, material density, Young modulus and Poisson's ratio of sub plate i are expressed as h_i, E_i, ρ_i and σ_i. The elastic connection between sub plates is simulated by torsion spring uniformly distributed. The stiffness value and amplitude of internal momenton boundary i are expressed as K_i and M_i. F_e represents the amplitude of external harmonic force which is applied on point (x_e, y_e).

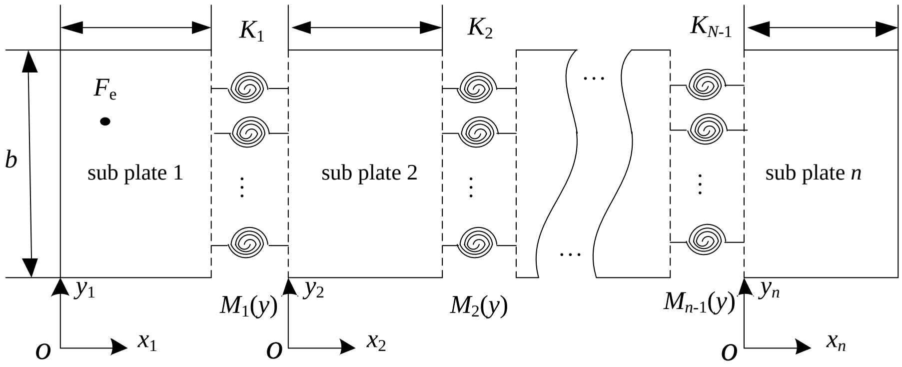

Fig.1 Schematic diagram of slicing plate

For the plate with all boundaries simply supported, its modal shape can be represented by trigonometric function. In the direction parallel to the coupling boundary, the point harmonic force and internal moment can be expanded in Fourier series. Based on the orthogonality of trigonometric function and the modal shape of the plate, terms containing y can be counteracted in the calculation progress so that the internal moment amplitude of each order can be obtained, concrete process can be referred to [12]. Subjected to p order sine distribution internal moment, the angular displacement of each sub plate on the coupling boundaries can be expressed as follows:

$$\begin{cases} \varphi_{1p}(1) = F_p Y_{Fp} + M_{1p} Y_1(1,1) \\ \varphi_{2p}(1) = -M_{1p} Y_2(1,1) + M_{2p} Y_2(2,1) \\ \varphi_{2p}(2) = -M_{1p} Y_2(1,2) + M_{2p} Y_2(2,2) \\ \varphi_{3p}(2) = -M_{2p} Y_3(2,2) + M_{3p} Y_2(3,2) \\ \qquad \vdots \\ \varphi_{(n-1)p}(n-1) = -M_{n-2}(x) Y_{n-1}(n-2,n-1) \\ \qquad\qquad + M_{n-1}(x) Y_{n-1}(n-1,n-1) \\ \varphi_{np}(n-1) = -M_{n-1}(x) Y_n(n-1,n-1) \end{cases} \tag{1}$$

Where $\varphi_{ip}(j)$ is the angular displacement of sub plate i on coupling boundary j subjected to p order coupling moment; F_p, M_{ip} are the amplitude of p order external force and internal moment; Y_{Fp} is the angular displacement mobility of sub plate subjected to p order external force, and it represents the mobility from emitting position to the points on coupling boundary; $Y_i(j, k)$ expresses the angular displacement mobility of sub plate i, and it represents the mobility from coupling boundary j to boundary k.

There is only internal moment along the coupling boundary under simply-supported boundary condition, so only continuous condition of angular displacement is essential. With elastic connection introduced, the continuity equation can be written as:

$$\begin{cases} K_1\left[\varphi_{1p}(1) - \varphi_2(1)\right] = M_{1p} \\ K_2\left[\varphi_{2p}(2) - \varphi_{3p}(2)\right] = M_{2p} \\ \qquad \vdots \\ K_{n-1}\left[\varphi_{(n-1)p}(n-1) - \varphi_{np}(n-1)\right] = M_{(n-1)p} \end{cases} \tag{2}$$

It can be written in matrix form as:

$$\left(\boldsymbol{K}\boldsymbol{Y}_p - \boldsymbol{E}\right)\boldsymbol{M}_p = \boldsymbol{K}\boldsymbol{\Omega}_p \tag{3}$$

Where $\boldsymbol{K} = diag\left\{K_1 \quad K_2 \quad \cdots \quad K_{n-1}\right\}$; $\boldsymbol{M}_p = \left[M_{1p} \quad M_{2p} \quad \cdots \quad M_{(n-1)p}\right]^{\mathrm{T}}$; $\boldsymbol{\Omega}_p = \left[-F_p Y_{Fp} \quad 0 \quad \cdots \quad 0\right]$; $\boldsymbol{E}$ is n-1 order unit matrix; $\boldsymbol{Y}_p$ expresses the mobility matrix of sub plates subjected to p order internal moment, it can be written as:

$$\boldsymbol{Y}_p = \begin{bmatrix} Y_1(1,1)+Y_2(1,1) & -Y_2(2,1) & \cdots & 0 \\ -Y_2(1,2) & Y_2(2,2)+Y_3(2,2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & -Y_{n-1}(n-1,n-2) \\ 0 & \cdots & -Y_{n-1}(n-2,n-1) & Y_{n-1}(n-1,n-1)+Y_n(n-1,n-1) \end{bmatrix} \quad (4)$$

$\boldsymbol{M_p}$ can be obtained according to equation (3). By substitution of $\boldsymbol{M_p}$ into equation (5), the amplitude and spatial distribution feature of each coupling moment can be obtained.

$$M_i(y) = \sum_{p=1}^{\infty} M_{ip} \sin\frac{p\pi y}{b} \quad (i=1,2,\cdots n-1) \quad (5)$$

The power flow from sub plate i to sub plate i+1 can be written as:

$$Q_i = \frac{1}{2}\sum_{y=0}^{b} \mathrm{Re}\left[M_i(y)\omega_i^*(y)\right]dy \quad (i=1,2,\cdots n-1) \quad (6)$$

Where Re expresses taking real part, '*' expresses taking conjugate complex; $\omega_i(y)$ is the angular velocity of sub plate i+1 on boundary i, it can be written as:

$$\omega_i(y)\begin{cases} \int_0^b\left[-M_i(y)Y_{i+1}(0,y|0,y_{i+1})+M_{i+1}(y)Y_{i+1}(a_{i+1},y|0,y_i)\right]dy & i=(1,2,\cdots n-2) \\ -\int_0^b M_{n-1}(y)Y_n(0,y|x_n,y_n)dy & i=n-1 \end{cases} \quad (7)$$

Where $Y_{i+1}(0, y|0, y_{i+1})$ is the angular velocity mobility of sub plate i+1 from point $(0, y)$ to point $(0, y_{i+1})$.

Input power of coupling plates can be obtained by:

$$Q_{\text{input}} = \frac{1}{2}\,\mathrm{Re}\left[F_e v_e^*(x_e, y_e)\right] \quad (8)$$

III. NUMERICAL STUDIES

3.1. Validation of the theoretical model.

The simulation example contains two sub plates which share the same parameters a=2m, b=1m, h=0.005m, E=2.16×10^{10}Pa, ρ=7900kg/m^3, η=0.01, Poisson's ratio σ=0.27; unit harmonic force is imposed on point of (0.6m, 0.2m) in sub plate 1. The calculation frequency domain ranges from 10Hz to 10k Hz.

As the mobility of sub plate is obtained by modal superposition method, the truncation order number is determined by upper frequency limit. In the simulation example, M and N share the same value. The Input power of slicing plate at 10 kHz with different truncation order number is shown in Fig.2. It shows that the input power converges when M, N=60, the maximum nature frequency of sub plate is 75.4 kHz which is much greater than 10k Hz.

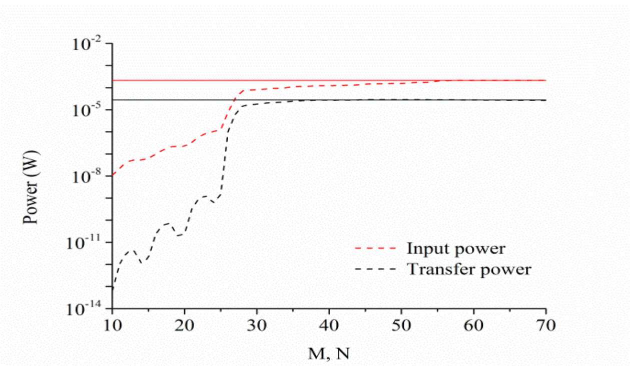

Fig.2 Convergence of input power

According to (2), when the connection stiffness tends to infinity, the angular displacements of nearby sub plates should tend to a same value as the amplitude of internal moment is a nonzero value. That is to say, the connection between sub plates tends to rigid connection. When $K_{max}=10^{10}$N/rad, the input power and transfer power is compared with that of rigid coupling plates which is obtained by reference [1], the results are shown in Fig.3 and Fig.4.It is seen from Fig.3 and Fig.4 that the rigid connection is a special case of elastic connection.

With the connection stiffness decreasing, the coupling degree between sub plates tends to be lower. When the connection stiffness tends to zero, the difference between angular displacements of nearby sub plates and the original structure will become two single plates with no connection. When $K_{min}=10^{-2}$N/rad, the input power is compared with that of single plate. Result is shown in Fig.5. It is seen from Fig.5 that except for small discrepancies in some peak value at natural frequencies, only small differences exist between the result of coupled plates and those of single plate. The reason for errors of some peak value lies in the fact that the difference between angular displacements of nearby sub plates is a finite value, so that the amplitude of internal moment can't tend to zero.

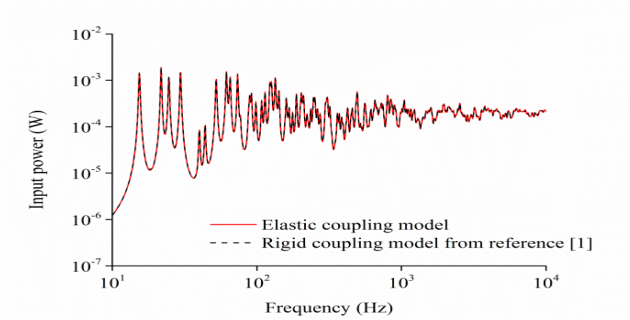

Fig.3 Curves of input power

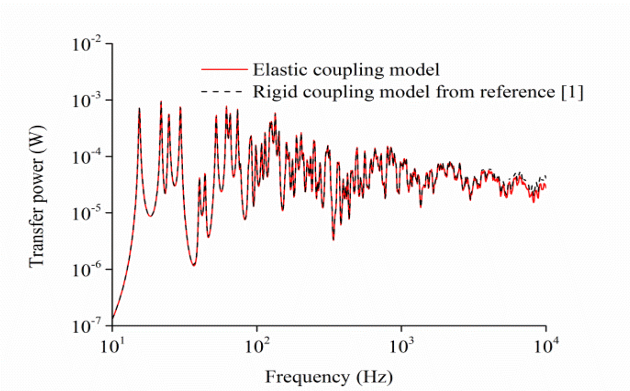

Fig.4 Curves of transfer power

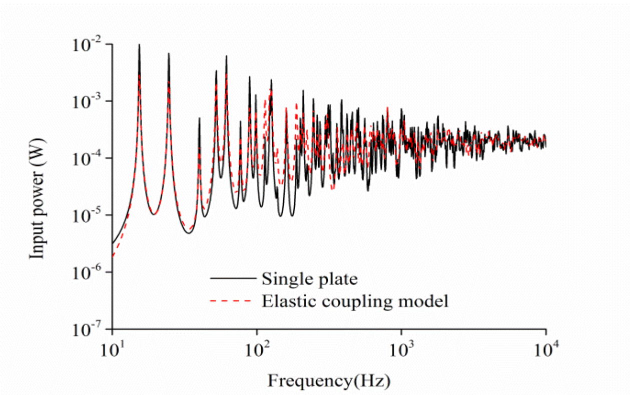

Fig.5 Input power of different model

3.2. Effect of connection stiffness.

In all calculations, the plates are assumed to be made up of steel, whose material property has been described above. Connection stiffness is changed as shown in Fig.6, $K_1=1\times10^2$N/rad, $K_2=1\times10^4$N/rad, $K_3=1\times10^6$N/rad, $K_4=1\times10^8$N/rad.

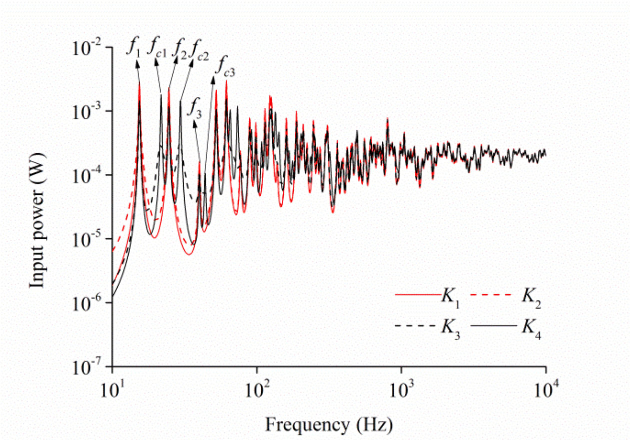

Fig.6 Curves of input power with different connection stiffness

It is seen from Fig.6 that with connection stiffness increasing, the trend of input power changes slightly but the peak value at f_1, f_2, f_3, which are the first three order nature frequencies of sub plate, decrease a little. At the same time, some new peaks appear at f_{c1}, f_{c2}, f_{c3}, which are the first three order nature frequencies of coupling structure.

Fig.7 shows the influence of stiffness on input power at those frequencies. It is seen from Fig.7 that with connection stiffness increasing, the trend of input power at natural frequencies of sub plate is totally different from those at natural frequencies of coupling structure. At the initial stage of stiffness increase, the modal of sub plate makes the main contribution to input power, but the degree becomes lower with the stiffness increasing, which makes the coupling degree sub plates stronger and the modal of coupling structure plays more important role in structure vibration. The input power at those frequencies converges when the value of stiffness increases to about 10^8N/rad, and then connection can be seen as rigid connection.

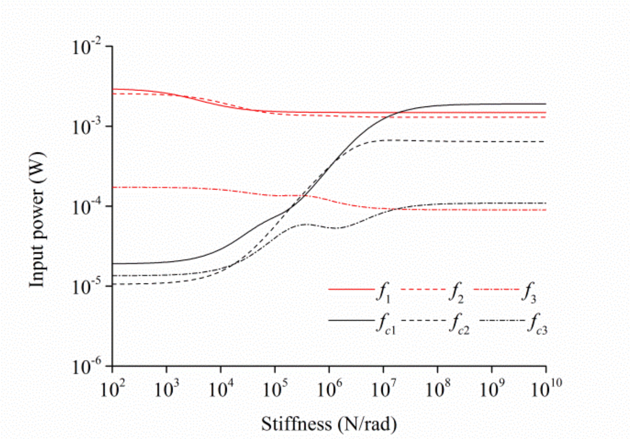

Fig.7 Curves of input power at different frequencies

The influence of stiffness on transfer power is shown in Fig.8. It can be seen from Fig.8 that transfer power increases significantly with stiffness increasing at frequency band of 10Hz to 10k Hz. And lower stiffness can restrain the vibration more effectively at high frequency. Some peaks also appear with the increase of stiffness. Transfer power at those frequencies is shown in Fig.9. Result shows that transfer power at natural frequencies of sub plate converges when stiffness increases to about 10^6N/rad, and transfer power at natural frequencies of coupling structure converges when stiffness increases to about 10^8N/rad.

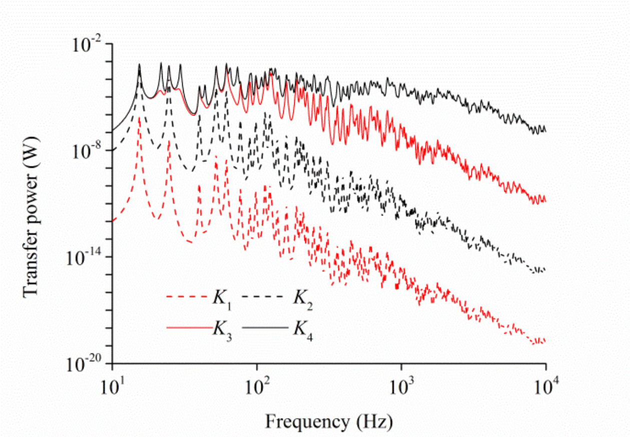

Fig.8 Curves of transfer power with different connection stiffness

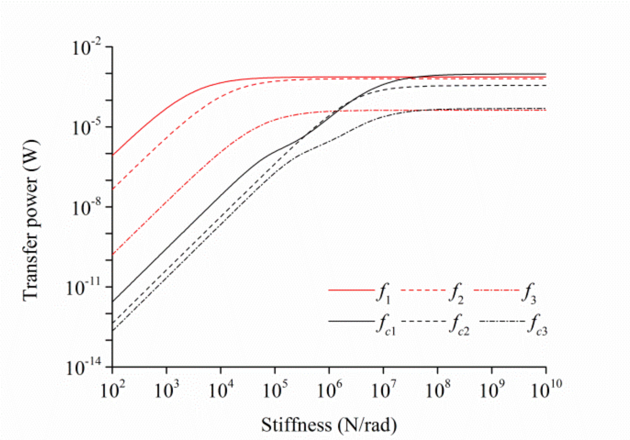

Fig.9 Curves of transfer power at different frequencies

IV. CONCLUSIONS

Based on mobility power flow method, the elastic connection between plates was simulated by torsion spring uniformly distributed, and the continuity equation of coupling boundary was modified. By introducing uniformly-distributed torsion spring and then modifying the continuity equation of coupling boundary, the elastic connection between plates can be well simulated. Based on the theoretical calculation model of elastic coupling plates, the influence of connection stiffness on the vibration characteristic of coupling structure is analyzed. Conclusions are shown as follows:

1. With the increase of connection stiffness, modal of sub plate makes less contribution to the input power but the situation is opposite to the modal of coupling structure.

2. The transfer power increases significantly with the increase of connection stiffness, and the stiffness which makes the transfer power converging at natural frequencies of sub plate is less than that of natural frequencies of coupling structure.

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