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Epq model for deteriorating items under three parameter weibull distribution and time dependent ihc with shortages

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ABSTRACT:- *In this paper, we have analysed a production inventory model for deteriorating items with time-dependent holding cost. Three parameter Weibull distribution is assumed for time to deterioration of items. Shortages are allowed to occur. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.*

KEYWORDS: - *Deterioration, Production, Shortages, Three parameter Weibull distribution, Time dependent holding cost*

I. INTRODUCTION

Deterioration is the damage caused due to spoilage, dryness, etc. Deterioration of an item is a realistic situation associated with an inventory system. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) have remarkable deterioration overtime. It has been observed that the failure of many such items can be expressed by Weibull distribution.

Manna and Chiang [1] developed an EPQ model for deteriorating items with ramp type demand. Teng and Chang [2] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Jain et al. [3] developed an economic production quantity model with shortages by incorporating the deterioration effect and stock dependent demand rate. Roy and Chaudhary [4] developed two production rates inventory model for deteriorating items when the demand rate was assumed to be stock dependent. In the research of Sana et al. [5] shortages are allowed to occur at the end of a cycle. With the consideration of time varying demand and constant deteriorating rate, the optimal production inventory policy was studied. Raman Patel [6] developed a production inventory model for Weibull deteriorating items with price and quantity dependent demand and varying holding cost with shortages.

Both Skouri and Papachristos [7] and Chen et al. [8] developed a production inventory model in which the shortages are allowed at the beginning of a cycle. In contrast, in Manna and Chaudhari [9] shortages are allowed but occur at the end of each cycle. Goyal's [10] production inventory problem of a product with time varying demand, production and deterioration rates in which the shortages occur at the beginning of the cycle.

Kirtan Parmar and U. B. Gothi [11] have developed an order level inventory model for deteriorating items under quadratic demand with time dependent IHC. Authors also [12] have developed a deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration where shortages are allowed with partial backlogging. Authors also [13] have developed an EPQ model of deteriorating items using three parameter Weibull distribution with constant production rate and time varying holding cost. Authors [14] also have developed an inventory model of deteriorating items using two parameter Weibull distribution with linear time dependent demand and IHC.

The rate of deterioration-time relationship for the three-parameter Weibull distribution is shown in Figure – 1. The figure shows that the three-parameter Weibull distribution is most suitable for items with any initial value of the rate of deterioration and for items, which start deteriorating only after a certain period of time (Begum at el. [15]). The probability density function for three parameter Weibull distribution is given by

$$f(t) = \alpha\beta(t-\mu)^{\beta-1} e^{-\alpha(t-\mu)^{\beta}} \qquad ; t \geq \mu \ \ (0 < \alpha < 1 \text{ and } \beta, \mu > 0)$$

The instantaneous rate of deterioration θ(t) (i.e. for hazzard rate) of the non-deteriorated inventory at time t, can be obtained from $\theta(t) = \dfrac{f(t)}{1-F(t)}$, where $F(t) = 1 - e^{-\alpha(t-\mu)^{\beta-1}}$ is the cumulative distribution function (c.d.f.) for the three parameter Weibull distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t) = \alpha\beta(t-\mu)^{\beta-1}$.

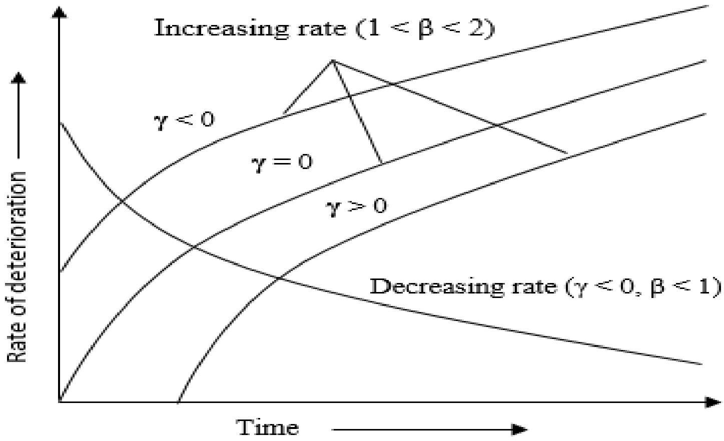

Figure - 1: Hazzard rate relationship for 3 parameter Weibull distribution

II. NOTATIONS

We use the following notations for the mathematical model

1. Q(t) : Inventory level of the product at time t (≥ 0).
2. R : Demand rate.
3. θ(t) : Rate of deterioration per unit time.
4. A : Ordering cost per order during the cycle period.
5. p : Selling price per unit.
6. k : Rate of production per unit time.
7. Q_1 : The maximum inventory level at time t = μ.
8. Q_2 : The maximum inventory level during shortage period.
9. C_h : Inventory holding cost per unit per unit time.
10. C_d : Deterioration cost per unit per unit time.
11. T : Duration of a cycle.
12. TC : Total cost per unit time.

III. ASSUMPTIONS

1. The demand rate of the product is a linear function of price and quantity which is $R = R(p, Q(t)) = [a - p + \rho Q(t)]$ (where a , p, $\rho > 0$).
2. As soon as a unit is produced it is available for to meet with demand.
3. As soon as the production is terminated the product starts with deterioration.
4. Holding cost is a linear function of time and it is $C_h = h + rt$ (h, r > 0)
5. Replenishment rate is infinite and instantaneous.
6. Shortages occur and they are completely backlogged.
7. Repair or replacement of the deteriorated items does not take place during a given cycle.
8. The second and higher powers of ρ and α are neglected in the analysis of the model.

IV. MATHEMATICAL MODEL AND ANALYSIS

Here, we consider a single commodity deterministic production inventory model with a time dependent demand rate $R(p, Q(t)) = [a - p + \rho\, Q(t)]$. Initial, inventory level is zero. At time t = 0, the production starts and simultaneously supply also starts. The production stops at $t = \mu$ when the maximum inventory level Q_1 is reached. In the interval $[0, \mu]$ the inventory is built up at a rate k – R and there is no deterioration in this interval. In the interval $[\mu, t_2]$ deterioration starts and the inventory depleted at the rate R. The inventory is finitely decreasing in the time interval $[\mu, t_2]$ until inventory level reaches zero. It is decided to backlog the demands up to Q_2 level which occur during stock-out time. Thereafter, shortages can occur during the time interval $[t_2, t_3]$, and all of the demand during the period $[t_2, t_3]$ is completely backlogged. Thereafter, Production is started at the rate k – (a – p) so as to clear the backlog, and the inventory level reaches to 0 (i.e. the backlog is cleared).

The pictorial presentation is shown in the Figure – 2.

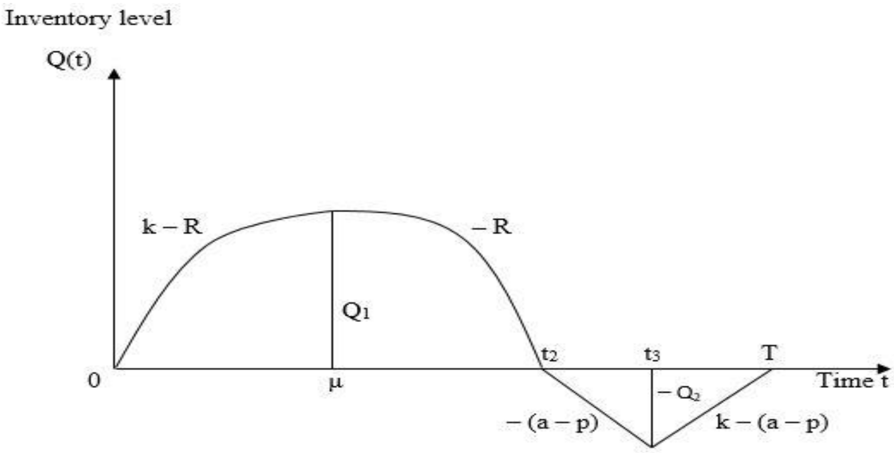

Figure - 2: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of Q(t) over the period (0, T) are given by

$$\frac{dQ(t)}{dt} = k - [a - p + \rho Q(t)], \qquad (0 \le t \le \mu) \tag{1}$$

$$\frac{dQ(t)}{dt} + \alpha\beta(t-\mu)^{\beta-1} Q(t) = -[a - p + \rho Q(t)], \qquad (\mu \le t \le t_2) \tag{2}$$

$$\frac{dQ(t)}{dt} = -(a-p), \qquad (t_2 \le t \le t_3) \tag{3}$$

$$\frac{dQ(t)}{dt} = k - (a-p), \qquad (t_3 \le t \le T) \tag{4}$$

Under the boundary conditions $Q(0) = 0$, $Q(\mu) = Q_1$, $Q(t_2) = 0$, $Q(t_3) = -Q_2$ and $Q(T) = 0$ solutions of equations (1) to (4) are given by

$$Q(t) = (k - a + p)\left(t - \frac{\rho t^2}{2}\right), \qquad (0 \le t \le \mu) \tag{5}$$

$$Q(t) = \left[1 - \rho(t-\mu) - \alpha(t-\mu)^{\beta}\right]Q_1 + (a-p)\left[(\mu - t)(1-\rho t) + \frac{\rho}{2}\left(\mu^2 - t^2\right) + \frac{\alpha\beta}{\beta+1}(t-\mu)^{\beta+1}\right], \quad (\mu \le t \le t_2) \tag{6}$$

$$Q(t) = (a-p)(t_2 - t), \qquad (t_2 \le t \le t_3) \tag{7}$$

$$Q(t) = (k-a+p)(t-T), \qquad (t_3 \le t \le T) \tag{8}$$

Putting $Q(t_2) = 0$ in equation (6), we get

$$Q_1 = \frac{(p-a)\left[(\mu - t_2)(1-\rho t_2) + \frac{\rho}{2}\left(\mu^2 - t_2^2\right) + \frac{\alpha\beta}{\beta+1}(t_2-\mu)^{\beta+1}\right]}{1-\rho(t_2-\mu) - \alpha(t_2-\mu)^{\beta}} \tag{9}$$

Taking $Q(t_3) = -Q_2$ in equation (7), we get

$$Q_2 = (a-p)(t_3 - t_2) \tag{10}$$

Equation (7) and (8) coincide at $t = t_3$ hence

$$(a-p)(t_2-t_3) = (k-a+p)(t_3-T)$$

$$\Rightarrow T = \frac{kt_3 - (a-p)t_2}{k-a+p} \tag{11}$$

The total cost comprises of the following costs

1) The ordering cost
$$OC = A \tag{12}$$

2) The deterioration cost during the period $[\mu, t_2]$

$$DC = C_d\left\{Q_1 - \int_{\mu}^{t_2}\left[a-p+\rho Q(t)\right]dt\right\}$$

$$\Rightarrow DC = C_d\left\{Q_1 - \left[(a-p)(1+\rho\mu) + \rho Q_1\right](t_2-\mu) + \frac{\rho(a-p)\left(t_2^2-\mu^2\right)}{2}\right\} \tag{13}$$

3) The inventory holding cost during the period $[0, t_2]$

$$IHC = \int_0^{\mu}(h+rt)Q(t)dt + \int_{\mu}^{t_2}(h+rt)Q(t)dt$$

$$= \int_0^{\mu}(h+rt)(k-a+p)\left(t - \frac{\rho t^2}{2}\right)dt + \int_{\mu}^{t_2}(h+rt)\left\{\begin{aligned}&\left[1-\rho(t-\mu)-\alpha(t-\mu)^{\beta}\right]Q_1\\&+(a-p)\left[(\mu-t)(1-\rho t)+\frac{\rho}{2}\left(\mu^2-t^2\right)+\frac{\alpha\beta}{\beta+1}(t-\mu)^{\beta+1}\right]\end{aligned}\right\}dt$$

$$\Rightarrow IHC = (k-a+p)\left[\frac{h\mu^2}{2}+\left(r-\frac{h\rho}{2}\right)\frac{\mu^3}{3}-\frac{r\rho\mu^4}{8}\right]+\left\{\begin{array}{l}(h+r\mu)\left\{\begin{array}{l}\left[(t_2-\mu)-\frac{\rho}{2}(t_2-\mu)^2-\frac{\alpha(t_2-\mu)^{\beta+1}}{\beta+1}\right]Q_1\\+(a-p)\left[\frac{\rho}{6}(t_2-\mu)^3-\frac{(t_2-\mu)^2}{2}+\frac{\alpha\beta(t_2-\mu)^{\beta+2}}{(\beta+1)(\beta+2)}\right]\end{array}\right\}\\+r\left\{\begin{array}{l}\left[\frac{(t_2-\mu)^2}{2}-\frac{\rho(t_2-\mu)^3}{3}-\frac{\alpha(t_2-\mu)^{\beta+2}}{\beta+2}\right]Q_1\\+(a-p)\left[\frac{\rho(t_2-\mu)^4}{8}-\frac{(t_2-\mu)^3}{3}+\frac{\alpha\beta(t_2-\mu)^{\beta+3}}{(\beta+1)(\beta+3)}\right]\end{array}\right\}\end{array}\right\} \tag{14}$$

4) The shortage cost per cycle

$$SC = -C_s\left[\int_{t_2}^{t_3}Q(t)dt+\int_{t_3}^{T}Q(t)dt\right]$$

$$\Rightarrow SC = C_s\left[\frac{(a-p)(t_2-t_3)^2}{2}+\frac{(k-a+p)(t_3-T)^2}{2}\right] \tag{15}$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{T}(OC+IHC+DC+SC)$$

$$TC = \frac{1}{\left[\frac{kt_3-(a-p)t_2}{k-a+p}\right]}\left\{\begin{array}{l}A+(k-a+p)\left[\frac{h\mu^2}{2}+\left(r-\frac{h\rho}{2}\right)\frac{\mu^3}{3}-\frac{r\rho\mu^4}{8}\right]\\+\left\{\begin{array}{l}(h+r\mu)\left\{\begin{array}{l}\left[(t_2-\mu)-\frac{\rho}{2}(t_2-\mu)^2-\frac{\alpha(t_2-\mu)^{\beta+1}}{\beta+1}\right]Q_1\\+(a-p)\left[\frac{\rho}{6}(t_2-\mu)^3-\frac{(t_2-\mu)^2}{2}+\frac{\alpha\beta(t_2-\mu)^{\beta+2}}{(\beta+1)(\beta+2)}\right]\end{array}\right\}\\+r\left\{\begin{array}{l}\left[\frac{(t_2-\mu)^2}{2}-\frac{\rho(t_2-\mu)^3}{3}-\frac{\alpha(t_2-\mu)^{\beta+2}}{\beta+2}\right]Q_1\\+(a-p)\left[\frac{\rho(t_2-\mu)^4}{8}-\frac{(t_2-\mu)^3}{3}+\frac{\alpha\beta(t_2-\mu)^{\beta+3}}{(\beta+1)(\beta+3)}\right]\end{array}\right\}\end{array}\right\}\\+C_d\left\{Q_1-[(a-p)(1+\rho\mu)+\rho Q_1](t_2-\mu)+\frac{\rho(a-p)\left(t_2^2-\mu^2\right)}{2}\right\}\\+C_s\left[\frac{(a-p)(t_2-t_3)^2}{2}+\frac{(k-a+p)(t_3-T)^2}{2}\right]\end{array}\right\} \tag{16}$$

μ^*, t_2^* and t_3^* are the optimum values of μ, t_2 and t_3 respectively, which minimize the cost function TC and they are the solutions of the equations $\frac{\partial TC}{\partial \mu}=0$, $\frac{\partial TC}{\partial t_2}=0$, $\frac{\partial TC}{\partial t_3}=0$, satisfying the sufficient condition $H > 0$, at μ^*, t_2^* and t_3^* where

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_2} & \frac{\partial^2 TC}{\partial \mu \partial t_3} \\ & \frac{\partial^2 TC}{\partial t_2^2} & \frac{\partial^2 TC}{\partial t_2 \partial t_3} \\ & & \frac{\partial^2 TC}{\partial t_3^2} \end{vmatrix} \text{ is Hessian determinant.} \quad (17)$$

V. NUMERICAL EXAMPLE

Let us consider the following example to illustrate the above developed model, taking A = 200, α = 0.05, β = 2, ρ = 0.04, a = 100, p = 0.05, k = 300, Cs = 8, h = 5, r = 0.05 and C_d = 25 (with appropriate units).

The optimal values of μ, t_2 and t_3 are μ^* = 0.2922609055, t_2^* = 0.7971385166, t_3^* = 1.162469779 units and the optimal total cost per unit time TC = 292.1188781 units.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for the cycle length T and total cost per time unit TC with respect to the changes in the values of the parameters A, α, β, ρ, a, p, k, Cs, h, r and C_d.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed. The last column of the **Table – 1** gives the % changes in TC as compared to the original solution for the relevant costs.

Table – 1: Partial Sensitivity Analysis

Parameter	Values	μ	t_2	t_3	TC	% changes in TC
A	160	0.26086836	0.71820154	1.04433609	260.777187	–10.73
	180	0.27698628	0.75890706	1.10516826	276.870453	– 5.22
	220	0.30681595	0.83326276	1.21676237	306.646292	4.97
	240	0.32074754	0.86756457	1.26845115	320.548910	9.73
α	0.04	0.29132373	0.80636505	1.17056312	291.212773	– 0.31
	0.06	0.29315442	0.78854655	1.15495940	292.983715	0.30
	0.07	0.29400802	0.78051774	1.14796489	293.810738	0.58
	0.08	0.29482499	0.77299146	1.14142941	294.602984	0.85
β	1.7	0.29381028	0.78922481	1.15645351	293.636073	0.52
	2.2	0.29145691	0.80188926	1.16623813	291.333361	– 0.27
	2.4	0.29079177	0.80625594	1.16979361	290.684723	– 0.49
	2.7	0.28999578	0.81214901	1.17471824	289.910361	– 0.76
ρ	0.030	0.29176671	0.79774869	1.16300147	292.056120	– 0.02
	0.035	0.29201336	0.79744468	1.16273660	292.087416	– 0.01
	0.044	0.29245960	0.79689204	1.16225493	292.144166	0.01
	0.048	0.29265887	0.79664422	1.16203886	292.169558	0.02
a	110	0.31389982	0.78652619	1.12527284	297.961556	2.00
	120	0.33629005	0.77961771	1.09466062	302.867847	3.68
	130	0.35965081	0.77605652	1.06968258	305.253654	4.50
	140	0.38422542	0.77564139	1.04969128	306.826258	5.03
p	0.040	0.29228224	0.79712589	1.16242888	292.125497	0.002
	0.045	0.29253072	0.79713220	1.16244933	292.122188	0.001
	0.055	0.29225024	0.79714483	1.16249024	292.115569	– 0.001
	0.060	0.29223957	0.79715115	1.16251069	292.112258	– 0.002

Parameter	Values	µ	t_2	t_3	TC	% changes in TC
k	240	0.38956686	0.86409821	1.20463323	272.291802	– 6.79
	270	0.33376102	0.82541901	1.17989319	283.437558	– 2.97
	330	0.26011077	0.77551601	1.14955528	299.081800	2.38
	360	0.23443121	0.75842588	1.13961106	304.795673	4.34
Cs	6.4	0.27842610	0.76252505	1.19759872	278.307925	– 4.73
	7.2	0.28585872	0.78115463	1.17819763	285.728022	– 2.19
	8.8	0.29783754	0.81101420	1.14946209	297.685233	1.91
	9.6	0.30274153	0.82318024	1.13852528	302.579872	3.58
h	4.0	0.34291138	0.90648539	1.24976742	274.488318	– 6.04
	4.4	0.32041369	0.85911048	1.21176409	281.981824	– 3.47
	4.6	0.31036926	0.83734779	1.19439906	285.498196	– 2.27
	4.8	0.30100918	0.81672156	1.17799505	288.874283	– 1.11
r	0.040	0.29250414	0.79771582	1.16297647	292.062414	– 0.02
	0.045	0.29238242	0.79742695	1.16272292	292.090659	– 0.01
	0.055	0.29213959	0.79685053	1.16221705	292.147071	0.01
	0.060	0.29201847	0.79656299	1.16196474	292.175239	0.02
C_d	24.0	0.29209044	0.79873637	1.16386135	291.953929	– 0.06
	24.5	0.29217588	0.79793452	1.16316289	292.036604	– 0.03
	26.0	0.29242970	0.79556377	1.16109933	292.282240	0.06
	27.0	0.29259685	0.79401151	1.15974944	292.444046	0.11

VII. GRAPHICAL PRESENTATION

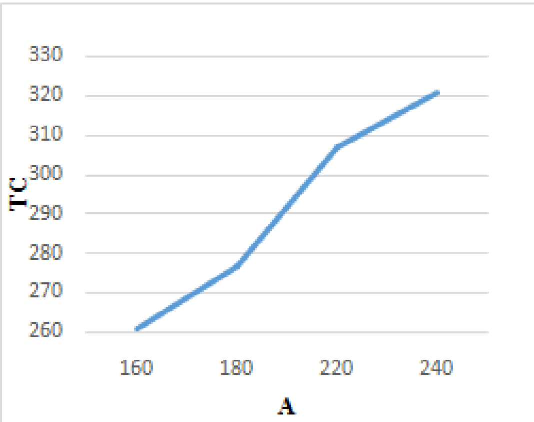

Figure – 3

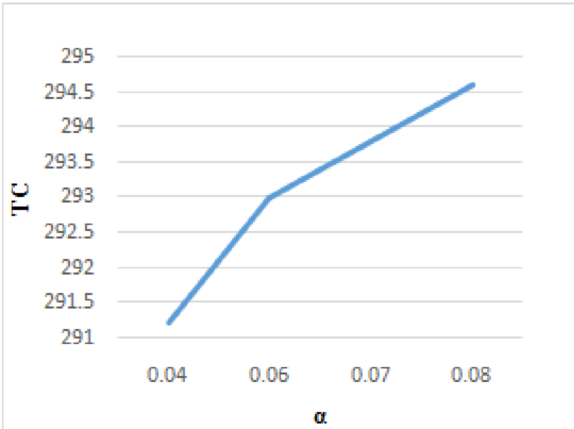

Figure – 4

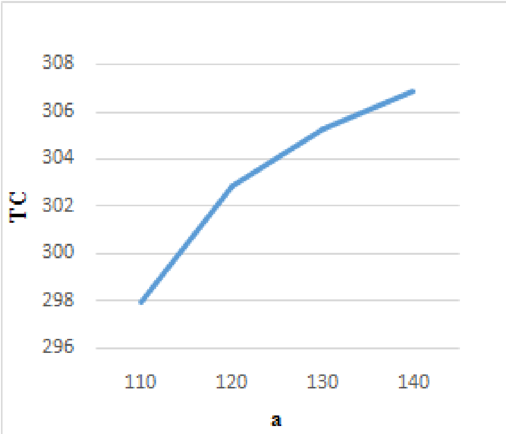

Figure – 5

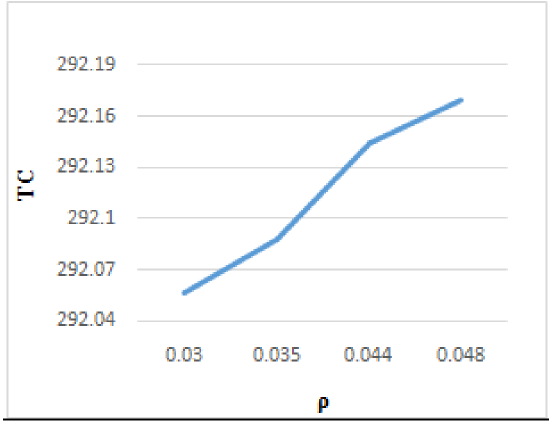

Figure – 6

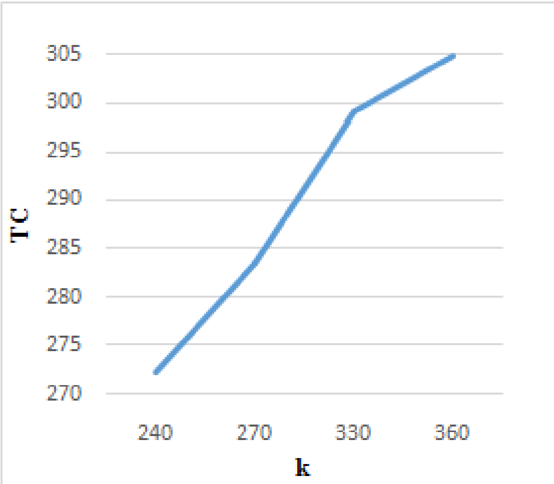

Figure – 7

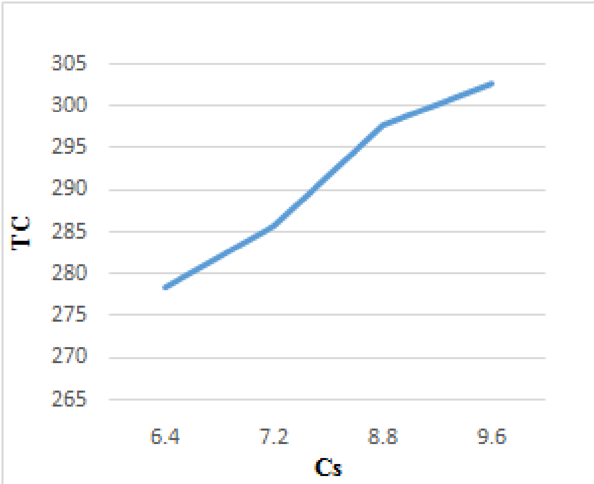

Figure – 8

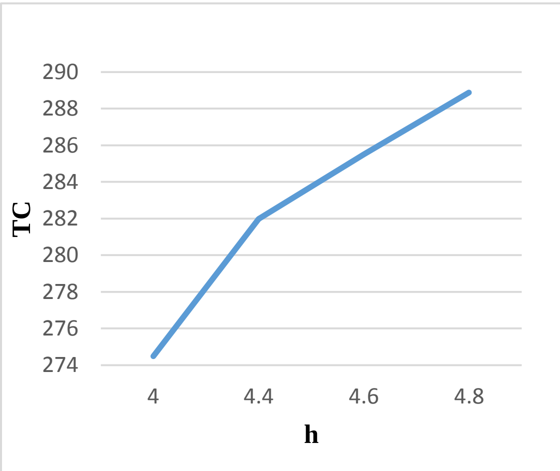

Figure – 9

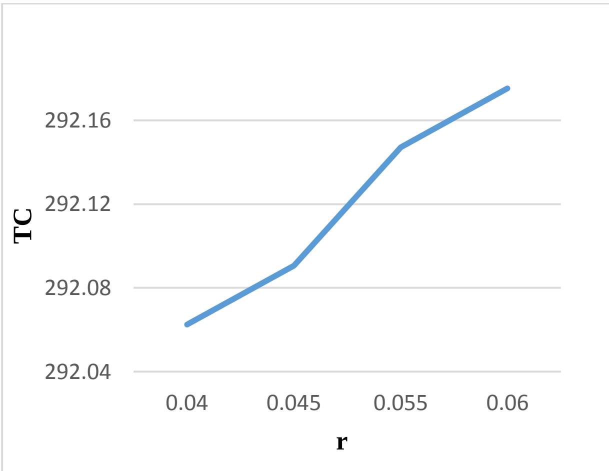

Figure – 10

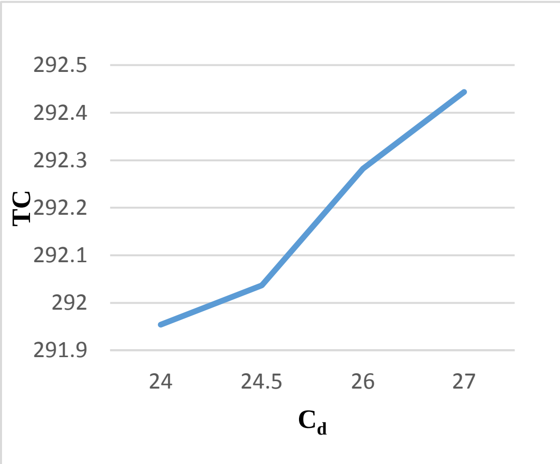

Figure – 11

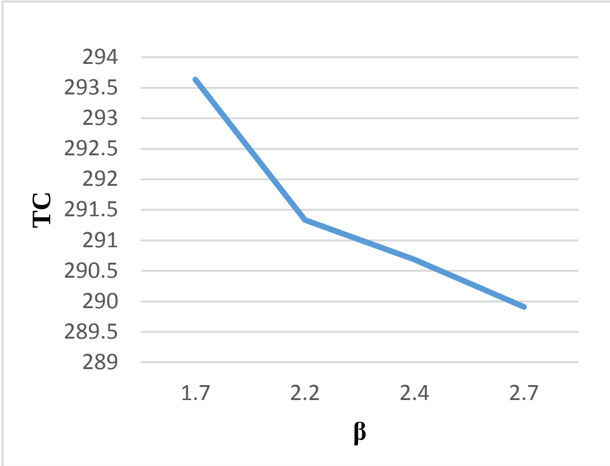

Figure – 12

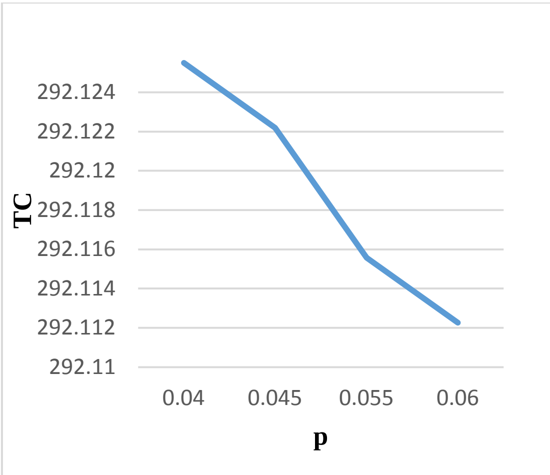

Figure – 13

VIII. CONCLUSIONS

- It is observed from **Figure No – 3 to 11** that when the values of parameters A, α, a, ρ, k, h, r, Cs, C_d increase the average total cost (TC) also increases.
- It is observed from **Figure No – 12 to 13** that when the values of parameters β and p increase then the average total cost (TC) has reverse effect.
- It is observed from **Figure No – 3, 5, 7, 8, 9** that the total cost per time unit (TC) is highly sensitive to changes in the values of A, a, k, Cs, h.
- From **Figure No – 4, 12** that the total cost per time unit (TC) is moderately sensitive to changes in the values of α, β.
- From **Figure No – 6, 10, 11, 13** that the total cost per time unit (TC) less sensitive to changes in the values of ρ, r, C_d, p.

IX. ACKNOWLEDGEMENT

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