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Effect of two temperature and anisotropy in an axisymmetric problem in transversely isotropic thermoelastic solid without energy dissipation and with two temperature

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ABSTRACT: *The present study is concerned with the thermoelastic interactions in a two dimensional homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperatures in the context of Green - Naghdi model of type-II. The Laplace and Hankel transforms have been employed to find the general solution to the field equations. Concentrated normal force, normal force over the circular region and concentrated thermal source and thermal source over the circular region have been taken to illustrate the application of the approach. The components of displacements, stresses and conductive temperature distribution are obtained in the transformed domain. The resulting quantities are obtained in the physical domain by using numerical inversion technique. Numerically simulated results are depicted graphically to show the effect of two temperature and anisotropy on the components of normal stress, tangential stress and conductive temperature.*

KEYWORDS: *Transversely isotropic, thermoelastic, Laplace transform, Hankel transform, concentrated and distributed sources*

I. INTRODUCTION

During the past few decades , widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation , these theories are referred to as generalized theories. Thermoelasticity with two temperatures is one of the non classical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is a thermal dependence.

In a series of papers, Green and Naghdi [6] –[8] provided sufficient basic modifications in the constitutive equations and proposed three thermoelastic theories which are referred to as GN theories of Type-I, II, and III . GN Theory of Type-I is a theory describing behaviour of a thermoelastic body which relies on entropy balance rather than entropy inequality. The novel quantity is a thermal displacement variable. GN theory of Type-II allows heat transmission at finite speed without energy dissipation. This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier’s law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. GN theory of Type-III includes the previous two models as special cases and admits dissipation of energy in general. This theory was pursued by many authors. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef [25,27,29] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation and obtained the variational principle. Chen and Gurtin [2], Chen et al. [3] and [4] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermodynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Several researchers studied various problems involving two temperature.e.g. (Warren and Chen [24] ,Quintanilla [16], Youssef AI-Lehaibi [26] and Youssef AI -Harby [27] , Kaushal, Kumar and Miglani [12] , Kumar, Sharma and Garg [14],Sharma and Marin[18],Sharma and Bhargav [17], Sharma, Sharma and Bhargav [22],Sharma and Kumar[19]). The axisymmetric problems has been studied during the past decade by many authors.e.g. (Kumar and Pratap [10]. Sharma and Kumar [15] , Kumar and Kansal [13], Kumar,Kumar and Gourla[11],Sharma,Kumar and Ram[21]). Inspite of these studies no attempt has been made to study the axisymmetric deformation in transversely isotropic medium with two temperature and without energy dissipation.

In the present investigation, a two dimensional axisymmetric problem in transversely isotropic thermoelastic solid without energy dissipation and with two temperature is investigated . The components of normal stress, tangential stress and conductive temperature subjected to concentrated normal force , normal force over the circular region and concentrated thermal source along with thermal source over the circular region are obtained by using Laplace and Hankel transforms. Numerical computation is performed by using a numerical inversion technique and the resulting quantities are shown graphically.

II. BASIC EQUATIONS

Following Youssef [28] the constitutive relations and field equations in absence of body forces and heat sources are:

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \qquad (1)$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\ddot{u}_i \qquad (2)$$

$$K_{ij}\varphi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T} \qquad (3)$$

where

$$T = \varphi - a_{ij}\varphi_{,ij} \qquad (4)$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij} \qquad (5)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad i,j = 1,2,3 \qquad (6)$$

Here

$C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the thermodynamic temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the thermal conductivity, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

III. FORMULATION OF THE PROBLEM

We consider a homogeneous transversely isotropic , thermoelastic body initially at uniform temperature T_0. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about z –axis. As the problem considered is plane axisymmetric, the field component $v = 0$, and $u , w, and\ \varphi$ are independent of θ. We have used appropriate transformation following Slaughter[23] on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid without energy dissipation and with two temperature and restrict our analysis to the two dimensional problem with $\vec{u} = (u, 0, w)$, we obtain

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{1}{r^2}u\right)+c_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right)+c_{44}\frac{\partial^2 u}{\partial z^2}+c_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right)-\beta_1\frac{\partial}{\partial r}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}=\rho\frac{\partial^2 u}{\partial t^2} \quad (7)$$

$$(c_{13}+c_{44})\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}\right)+c_{44}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+c_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3\frac{\partial}{\partial z}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}=\rho\frac{\partial^2 w}{\partial t^2} \quad (8)$$

$$K_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+K_3\frac{\partial^2\varphi}{\partial z^2}=T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u}{\partial r}+\beta_3\frac{\partial w}{\partial z}\right)+\rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\} \quad (9)$$

Constitutive relations are

$$t_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}-\beta_1 T$$

$$t_{zr}=2c_{44}e_{rz}$$

$$t_{zz}=c_{13}e_{rr}+c_{13}e_{\theta\theta}+c_{33}e_{zz}-\beta_3 T$$

$$t_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz}-\beta_1 T \quad (10)$$

where

$$e_{rz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right),\ e_{rr}=\frac{\partial u}{\partial r},\quad e_{\theta\theta}=\frac{u}{r},\quad e_{zz}=\frac{\partial w}{\partial z},\ T=\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}$$

$$\beta_{ij}=\beta_i\delta_{ij}\ ,\qquad K_{ij}=K_i\delta_{ij}$$

$$\beta_1=(c_{11}+c_{12})\alpha_1+c_{13}\alpha_3,\qquad \beta_3=2c_{13}\alpha_1+c_{33}\alpha_3$$

In the above equations we use the contracting subscript notations $(1\to 11,2\to 22,3\to 33,4\to 23,5\to 31,6\to 12)$ to relate c_{ijkl} to c_{mn}

To facilitate the solution, the following dimensionless quantities are introduced

$$r'=\frac{r}{L},\quad z'=\frac{z}{L},\ t'=\frac{c_1}{L}t,\ u'=\frac{\rho c_1^2}{L\beta_1 T_0}u,\ w'=\frac{\rho c_1^2}{L\beta_1 T_0}w,\ T'=\frac{T}{T_0},\ t'_{zr}=\frac{t_{zr}}{\beta_1 T_0}$$

$$t'_{zz}=\frac{t_{zz}}{\beta_1 T_0},\ \varphi'=\frac{\varphi}{T_0},\ a'_1=\frac{a_1}{L},\ a'_3=\frac{a_3}{L} \quad (11)$$

in equations (7)-(9) and after that suppressing the primes and applying the Laplace and Hankel transforms defined by

$$\hat{f}(r,z,s)=\int_0^\infty f(r,z,t)e^{-st}dt \quad (12)$$

$$\tilde{f}(\xi,z,s)=\int_0^\infty \hat{f}(r,z,s)rJ_n(r\xi)\,dr \quad (13)$$

on the resulting quantities, we obtain

$$\left(-(\xi^2+s^2)+\delta_2 D^2\right)\tilde{u}-\xi\delta_1 D\tilde{w}+(\xi(1-a_1\xi)-a_3\xi D^2)\,\tilde{\varphi}=0 \quad (14)$$

$$\delta_1\left(\frac{-\xi^2+1}{\xi}\right)D\tilde{u}+\left(\delta_3 D^2-(\delta_2\xi^2+s^2)\right)\tilde{w}-\left(\frac{\beta_3}{\beta_1}(\xi^2 a_1+1)D-D^3\right)\tilde{\varphi}=0 \quad (15)$$

$$(\delta_4 s^2\xi)\tilde{u}-\delta_5 D\tilde{w}+\left(\left(\frac{\delta_6 a_3}{L}+\frac{K_3}{K_1}\right)D^2-\xi^2+\delta_6 s^2(1+\xi^2)\right)\tilde{\varphi}=0 \quad (16)$$

where $\delta_1=\frac{c_{13}+c_{44}}{c_{11}}$, $\frac{c_{44}}{c_{11}}=\delta_2$, $\frac{c_{33}}{c_{11}}=\delta_3$, $\delta_4=\frac{\beta_1^2 T_0}{K_1}$, $\delta_5=\frac{\beta_3\beta_1 T_0}{K_1}s^2$, $\delta_6=\frac{\rho C_E C_1^2}{K_1}$

The solution of the equations (14)-(16), using the radiation condition that $\tilde{u}$, $\tilde{w}$, $\tilde{\varphi} \to 0$ as $z \to \infty$,yields

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z} \tag{17}$$

$$\tilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z} \tag{18}$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z} \tag{19}$$

where $\pm\lambda_i$ ($i = 1,2,3$) are the roots of the equation

$$AD^6 + BD^4 + CD^2 + E = 0 \tag{20}$$

where A , B, C,D, and E are listed in appendix A and the values of coupling constants d_i and l_i , are given in appendix B and A_i ,*i=1,2,3* being arbitrary constants.

IV. APPLICATIONS

Mechanical forces/ Thermal sources acting on the surface

The boundary conditions are

(i) $t_{zz}(r, z, t) = -P_1(r, t)$

$(ii) t_{zr}(r, z, t) = 0$

$$(iii) \frac{\partial \varphi}{\partial z} = P_2(r, t) \tag{21}$$

$P_1(r, t)$, $P_2(r, t)$ are well behaved functions

Here $P_2(r, t)$ =0 corresponds to plane boundary subjected to normal force and $P_1(r, t)$ = 0 corresponds to plane boundary subjected to thermal point source.

Case 1. **Concentrated normal force/ Thermal point source**

When plane boundary is subjected to concentrated normal force/ thermal point force, then $P_1(r, t), P_2(r, t)$ take the form

$$(P_1(r, t), P_2(r, t)) = \left(\frac{P_1 \delta(r)\delta(t)}{2\pi r}, \frac{P_2 \delta(r)\delta(t)}{2\pi r}\right) \tag{22}$$

P_1 is the magnitude of the force applied , P_2 is the magnitude of the constant temperature applied on the boundary and $\delta(r)$ is the Dirac delta function.

Using the equations (10), (11) in the boundary conditions (21) and applying the transforms defined by(12) and (13) and substitute the values of $\tilde{u}$, $\tilde{w}$, $\tilde{\varphi}$ from (17)-(19) in the resulting equations, we obtain the expressions for the components of displacement, stress , and conductive temperature in case of concentrated normal force which are given in appendix C and in case of thermal point source are these are obtained by replacing Δ_i by Δ_i^* and P_1 with P_2, as listed in appendix D

Case II: Normal force over the circular region/ Thermal source over the circular region

Let a uniform pressure of total magnitude P_1/ constant temperature P_2 applied over a uniform circular region of radius *a* is obtained by setting

$$P_1(r, t), P_2(r, t) = \left(\frac{P_1}{\pi a^2} H(a - r)\delta(t), \frac{P_2}{\pi a^2} H(a - r)\delta(t)\right) \tag{23}$$

where $H(a - r)$ is the Heaviside unit step function.

Making use of dimensionless quantities defined by (11) and then applying Laplace and Hankel transforms defined by (12)-(13) on (23) ,we obtain

$$(\widetilde{P_1}(\xi,s),\widetilde{P_2}(\xi,s)) = \left(\frac{P_1}{\pi a}\frac{J_1(a\xi)}{\xi},\frac{P_2}{\pi a}\frac{J_1(a\xi)}{\xi}\right)$$

The expressions for the components of displacements, stress and conductive temperature are obtained by replacing $\frac{P_1}{2\pi}$ with $\frac{P_1}{\pi a}\frac{J_1(a\xi)}{\xi}$ and by replacing $\frac{P_2}{2\pi}$ with $\frac{P_2}{\pi a}\frac{J_1(a\xi)}{\xi}$ in equations (C.1)-(C.5) and in (D.1)-(D.5) respectively

V. PARTICULAR CASES

(i) If $a_1 = a_3 = 0$, from equations (C.1) $-$ (C.5) and from (D.1) – (D.5) we obtain the corresponding expressions for displacements, stresses and temperature change in thermoelastic medium without energy dissipation.

(ii) If we take $a_1 = a_3 = a$, $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $K_1 = K_3 = K$ in equations (C.1) $-$ (C.5) and (D.1) – (D.5) , we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic thermoelastic solid without energy dissipation.

VI. INVERSION OF THE TRANSFORMS

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (26)-(30) These expressions are functions of z, the parameters of Laplace and Hankel transforms s and ξ ,respectively ,and hence are of the form $\tilde{f}(\xi,z,s)$. To get the function $f(r,z,t)$ in the physical domain, first we invert the Hankel transform using

$$\hat{f}(r,z,s) = \int_0^\infty \xi \tilde{f}(\xi,z,s) J_n(\xi r)d\xi \qquad (24)$$

Now for the fixed values of ξ, z and r the $\hat{f}(r,z,s)$ in the expression above can be considered as the Laplace transform $\hat{g}(s)$ of $g(t)$. Following Honig and Hirdes [9] , the Laplace transform function $\hat{g}(s)$ can be inverted.

The last step is to calculate the integral in equation (24). The method for evaluating this integral is described in Press et al. [15]. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

VII. NUMERICAL RESULTS AND DISCUSSION

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic. Physical data for a single crystal of copper is given by

$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2}$ $c_{33} = 17.2 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10}\ Kgm^{-1}s^{-2}$ $C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1}$ $\alpha_1 = 2.98 \times 10^{-5}K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5}K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$, $K_1 = 0.433 \times 10^3 Wm^{-1}K^{-1}$, $K_3 = 0.450 \times 10^3 Wm^{-1}K^{-1}$

Following Dhaliwal and Singh [5], magnesium crystal is chosen for the purpose of numerical calculation(isotropic solid). In case of magnesium crystal like material for numerical calculations, the physical constants used are

$\lambda = 2.17 \times 10^{10} Nm^2$, $\mu = 3.278 \times 10^{10} Nm^2$, $K = 1.7 \times 10^2 Wm^{-1}deg^{-1}$ $\omega_1 = 3.58 \times 10^{11} S^{-1}$ $\beta = 2.68 \times 10^6\ Nm^{-2}deg^{-1}$, $\rho = 1.74 \times 10^3 Kgm^{-3}$ $T_0 = 298$K, $C_E = 1.04 \times 10^3 Jkg^{-1}deg^{-1}$

The values of normal force stress t_{zz} , tangential stress t_{zr} and conductive temperature φ for a transversely isotropic thermoelastic solid with two temperature (TITWT) , isotropic thermoelastic solid with two temperature(ITSWT) and thermoelastic solid without two temperature (TSWT) are presented graphically to show the impact of two temperature and anisotropy.

i). The solid line corresponds to (TITWT) for $a_1 = .02$, $a_3 = .04$

ii) small dashed line corresponds to(TITWT) for $a_1 = .05$, $a_3 = .07$

iii)solid line with centre symbol circle corresponds to (TSWT) for $a_1 = a_3 = 0$

iv) solid line with centre symbol diamond corresponds to (ITSWT) for $a_1 = a_3 = .06$

VIII. NORMAL FORCE ON THE BOUNDARY OF THE HALF-SPACE

Case I: Concentrated normal force

Fig.1 shows the variations of normal stress t_{zz} with distance r. In the initial range there is a sharp decrease in the values of t_{zz} for all the curves i.e. (TITWT) , (ITSWT) and (TSWT) but away from source applied, it follows oscillatory behaviour near the zero value. We can see that the two temperature have significant effect on the normal stress in all the cases as there are more variations in t_{zz} in case of (TITWT) and (ITSWT) as compared to (TSWT). Impact of anisotropy is seen in the range $1 \leq r \leq 3$ where the values of t_{zz} for (TITWT) are more than from (ITSWT). It is evident from fig.2 that near the point of application of source there is increase in the values of t_{zr} for (ITSWT) and have small variation near the zero value in the remaining range. However for (TITWT) and (TSWT) , there is a sharp decrease in the range $0 \leq r \leq 2$ but pattern is oscillatory near the zero value in the rest of the range. In case of (TITWT) oscillations are of greater magnitude than in case of (TSWT), however not much difference in behaviour is noticed in the two cases i.e. i)$a_1 = .02$, $a_3 = .04$ and ii) $a_1 = .05$, $a_3 = .07$. Fig.3 depicts the behaviour of conductive temperature φ. Two temperature and anisotropy effect is more prominent in the range $0 \leq r \leq 5$ for all the curves and curves are close to each other in the remaining range with minor difference in the magnitude.

Case II: Normal force over the circular region

The trend of variations of normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for normal force over the circular region is similar to concentrated normal force with difference in their magnitude. At a first look it seems as mirror image of one another i.e. pattern is similar but the corresponding values are different. These variations are shown in figs. (4-6)

7.2 Thermal source on the boundary of half-space

Case-I: Thermal point source

Fig.7 depicts the variations of normal stress t_{zz} with distance r. In case of (ITSWT), it decreases sharply in the range $0 \leq r \leq 2$ and away from point of application of source the behaviour is oscillatory. Opposite behaviour is exhibited in the remaining cases i.e. in case of (TITWT) and (TSWT). Also for (TITWT) , difference in variations in both cases(case(i) $a_1 = .02$, $a_3 = .04$ and case(ii) $a_1 = .05$, $a_3 = .07$) are not found but follow same pattern for two temperature parameter and are shown in fig.7.

The values of t_{zr} increase sharply in the range $0 \leq r \leq 2$ and afterwards follow oscillatory pattern in case of (TITWT both cases) and (TSWT). In this case difference in variations is shown when temperature parameters are changed. In case of (ITSWT) , there is a decrease in range $0 \leq r \leq 2$ and away from origin ,it has small variations near zero and impact of anisotropy is visible because behaviour is quite different in this case than in transverse isotropy as is depicted in fig.8.

Fig.9 exhibits the behaviour of conductive temperature φ with distance r. In the initial range there is a sharp increase in (TITWT both cases), (TSWT) but away from source behaviour is oscillatory. In case of (ITSWT), there is an increase in the initial range but afterwards there are small variations near zero.

Case-II: Thermal source over the circular region

The trend of variations of normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for thermal source over the circular region is similar to thermal point source with difference in their magnitude. The pattern is similar but the corresponding values are different. These variations are shown in figs. (9-12)

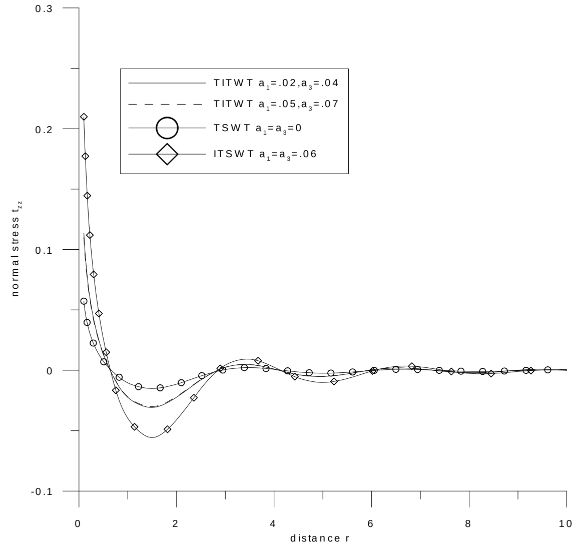

Fig.1 Variation of normal stress t_{ZZ} with distance r (concentrated normal force)

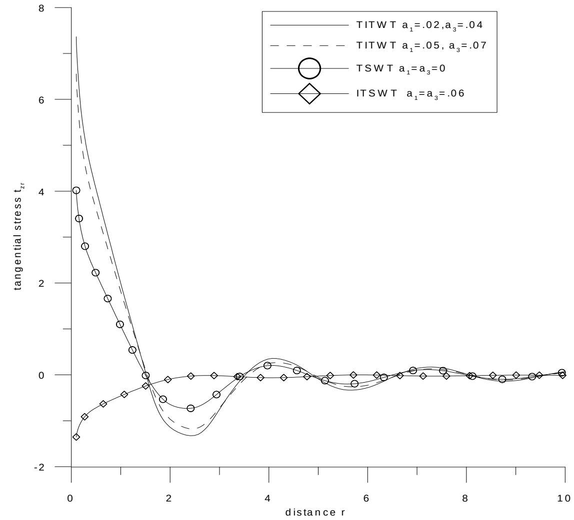

Fig.2 Variation of tangential stress t_{zr} with distance r (concentrated normal force)

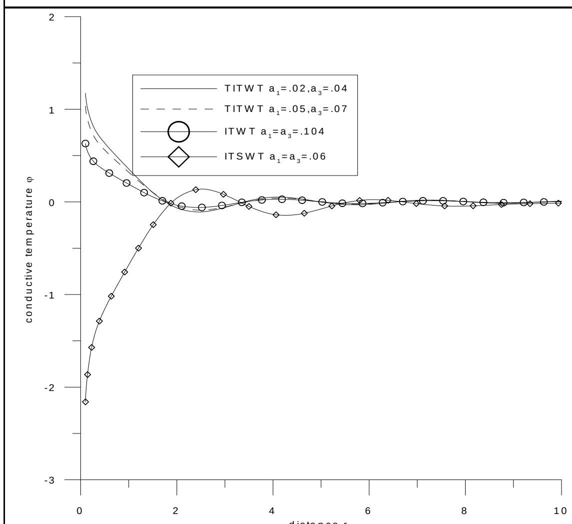

Fig.3 Variation of conductive temperature φ with distance r (concentrated normal force)

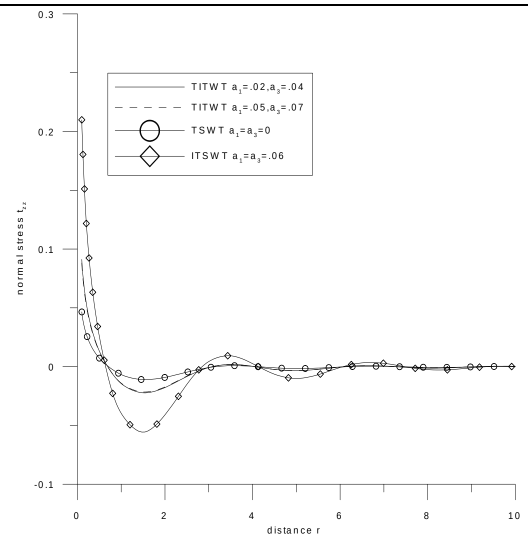

Fig.4 Variation of normal stress t_{zz} with distance r (normal force over the circular region)

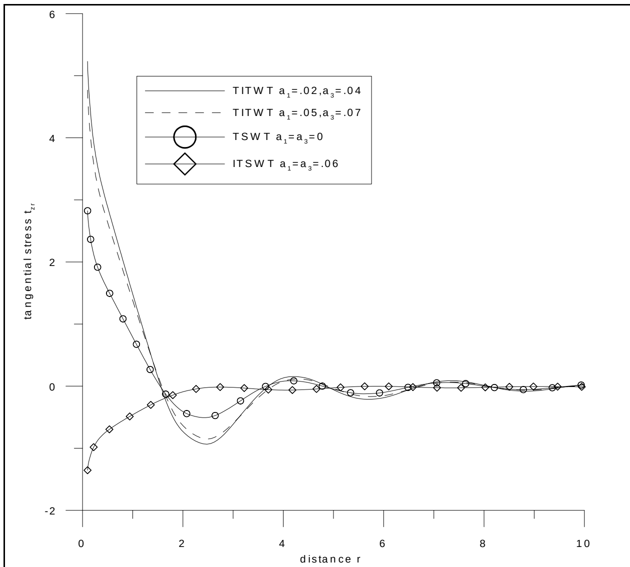

Fig.5 Variation of tangential stress t_{zr} with distance r (normal force over the circular region)

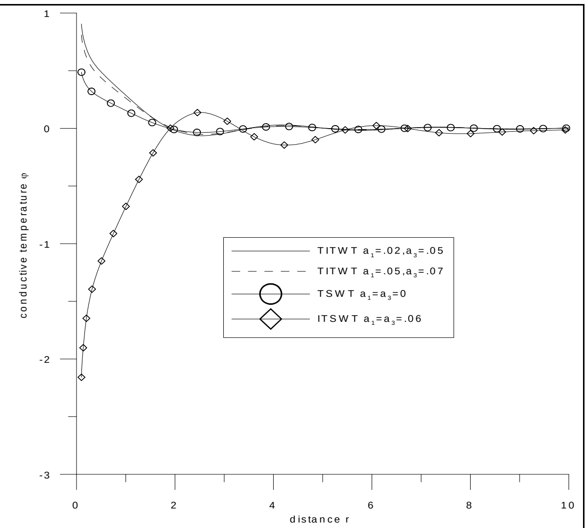

Fig.6 Variation of conductive temperature φ with distance r (normal force over the circular region)

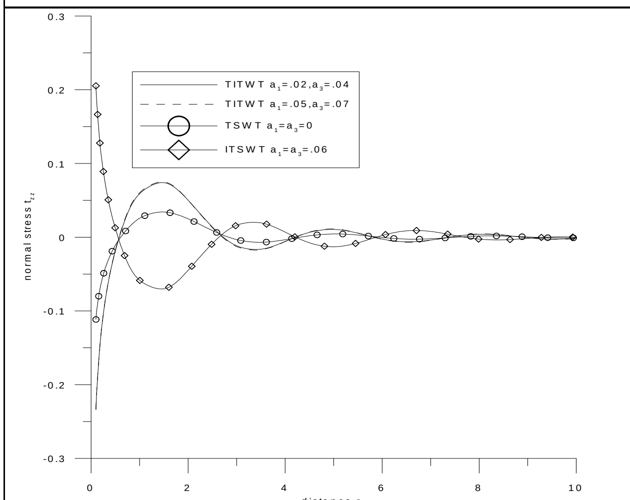

Fig.7 Variation of normal stress t_{zz} with distance r (thermal point source)

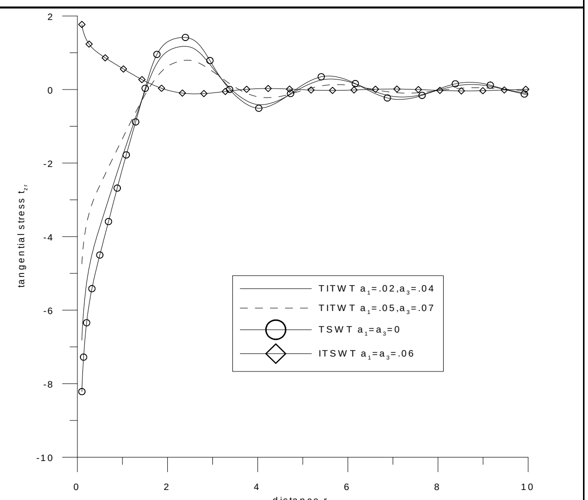

Fig.8 Variation of tangential stress t_{zr} with distance r (thermal point source)

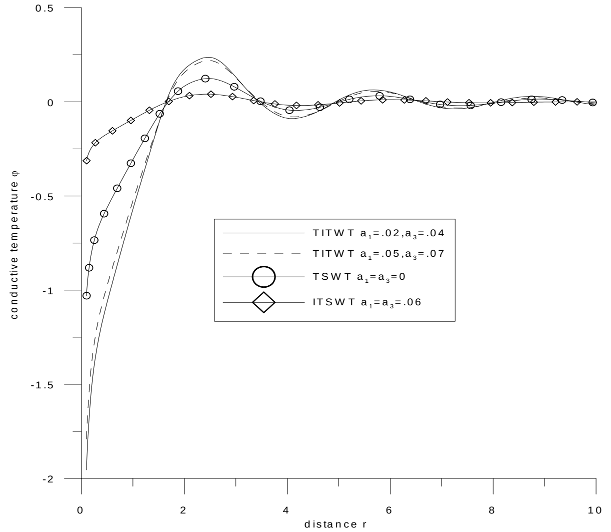

Fig.9 Variation of conductive temperature φ with distance r (thermal point source)

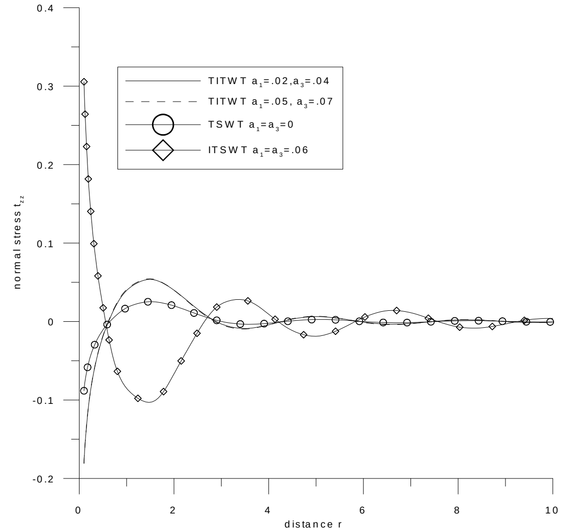

Fig.10 Variation of normal stress t_{zz} with distance r(thermal source over the circular region)

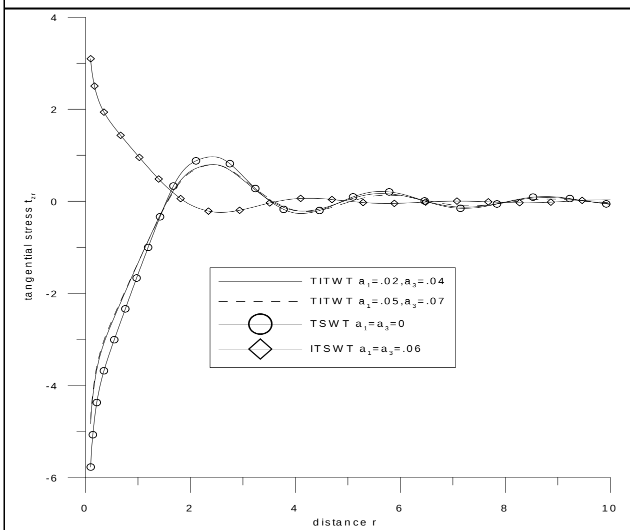

Fig.11 Variation of tangential stress t_{zr} with distance r (thermal source over the circular region)

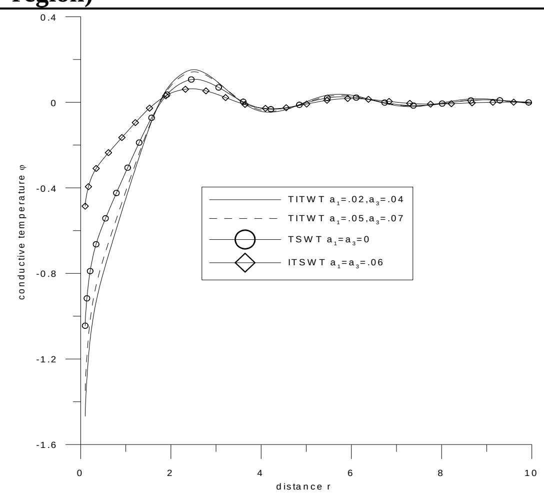

Fig.12 Variation of conductive temperature φ with distance r (thermal source over the circular region)

IX. CONCLUSION

From the graphs it is clear that effect of two temperature plays an important part in the study of the deformation of the body. As r diverse from the point of application of the source the components of normal stress, tangential stress and conductive temperature for all types of sources (concentrated normal force / normal force over the circular region/ thermal point source/ thermal source over the circular region) follow an oscillatory pattern. It is observed that the variations of normal stress t_{zz}, tangential stress t_{zr} and conductive temperature φ for both mechanical forces (concentrated normal force and normal force over the circular region) are same and for both thermal sources(thermal point source and thermal source over the circular region) are same with difference in magnitude. As the disturbances travel through different constituents of the medium , it suffers sudden changes ,resulting in an inconsistent/ non- uniform pattern of curves. The trend of curves exhibits the properties of two temperature of the medium and satisfies the requisite condition of the problem. The results of this problem are very useful in the two dimensional problem of dynamic response due to various sources of the transversely isotropic thermoelastic solid without energy dissipation and with two temperature which has various geophysical and industrial applications.

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Appendix A

$$A = \delta_1^2\zeta_1 - K_1\delta_5\delta_1$$

$$B = -\zeta_2\zeta_1\delta_1 + \delta_5\zeta_2\mathrm{K}_1 - \delta_1{}^2\zeta_4 - \delta_1\zeta_1\zeta_3 + \xi\delta_2{}^2\zeta_1\zeta_5 - \mathrm{K}_1\xi\zeta_7\zeta_6 + \xi\delta_2\delta_5\zeta_5\mathrm{a}_3 + \mathrm{a}_3\zeta_7\xi\delta_3$$

$$C = \zeta_2\delta_1\zeta_4 + \zeta_1\zeta_2\zeta_3 + \zeta_8\delta_5\zeta_2 + \delta_1\zeta_3\zeta_4 - \delta_1\zeta_5\zeta_8 - \xi\delta_2{}^2\zeta_5\zeta_4 + \xi\delta_2\delta_5\zeta_5 - \zeta_6\zeta_5\delta_5\delta_2 - \zeta_6\zeta_7\delta_3 - \mathrm{a}_3\xi\zeta_7\zeta_3$$

$$D = \zeta_6\zeta_3\zeta_7$$

Where

$\zeta_1 = \left(\frac{\delta_6 a_3}{L} + \frac{K_3}{K_1}\right)$, $\zeta_2 = \xi^2 + s^2$ $\quad \zeta_3 = \delta_1\xi^2 + s^2$ $\quad \zeta_4 = \xi^2 + \delta_6 s^2(1+\xi^2)$ $\quad \zeta_5 = \frac{-\xi^2+1}{\xi}$ $\quad \zeta_6 = \xi(1 - a_1\xi)$,
$\zeta_7 = \delta_4\xi s^2$ $\quad \zeta_8 = \frac{\beta_3}{\beta_1}\left(1 + a_1\xi^2\right)s$

Appendix B

$$d_i = \frac{-\lambda_i^3 P^* - \lambda_i Q^*}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3$$

$$l_i = \frac{\lambda_i^2 P^{**} + Q^{**}}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3$$

Where $P^* = \delta_2\zeta_5\zeta_1 - \zeta_7 K_1$

$Q^* = \zeta_7\zeta_8 - \delta_2\zeta_4\zeta_5$

$R^* = \delta_3\zeta_1 - K_1\delta_5$

$S^* = \delta_5\zeta_8 - \zeta_3\zeta_1 - \zeta_4\delta_3$

$T^* = \zeta_4\zeta_3$

$P^{**} = -(\zeta_5\delta_2\delta_5 + \zeta_7\delta_3)$

$Q^{**} = \zeta_7\zeta_3$

Appendix C

$$\tilde{u} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}\left(\Delta_1 e^{-\lambda_1 z} - \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}\right\}\right. \quad \text{(C.1)}$$

$$\tilde{w} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}\left(d_1 \Delta_1 e^{-\lambda_1 z} - d_2\Delta_2 e^{-\lambda_2 z} + d_3\Delta_3 e^{-\lambda_3 z}\right\}\right. \quad \text{(C.2)}$$

$$\widetilde{t_{zz}} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}\left(h_1 \Delta_1 e^{-\lambda_1 z} - h_2\Delta_2 e^{-\lambda_2 z} + h_3\Delta_3 e^{-\lambda_3 z}\right\}\right. \quad \text{(C.3)}$$

$$\widetilde{t_{zr}} = \frac{1}{\Delta}\left\{\frac{P_1}{2\pi}\left(m_1 \Delta_1 e^{-\lambda_1 z} - m_2\Delta_2 e^{-\lambda_2 z} + m_3\Delta_3 e^{-\lambda_3 z}\right\}\right. \quad \text{(C.4)}$$

$$\tilde{\varphi} = \frac{1}{\Delta}\left\{-\frac{P_1}{2\pi}\left(l_1 \Delta_1 e^{-\lambda_1 z} - l_2\Delta_2 e^{-\lambda_2 z} + l_3\Delta_3 e^{-\lambda_3 z}\right\}\right. \quad \text{(C.5)}$$

Where

$\Delta_1 = (l_3\lambda_3)(\lambda_2 + \xi d_2) - (l_2\lambda_2)(\lambda_3 + \xi d_3)$

$\Delta_2 = (l_3\lambda_3)(\lambda_1 + \xi d_1) - (l_1\lambda_1)(\lambda_3 + \xi d_3)$

$\Delta_3 = (l_2\lambda_2)(\lambda_1 + \xi d_1) - (l_1\lambda_1)(\lambda_2 + \xi d_2)$

$\Delta = h_1\Delta_1 - h_2\Delta_2 + h_3\Delta_3$

$h_j = -\xi \frac{c_{31}}{\rho c_1^2} - \frac{c_{33}}{\rho c_1^2} d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j + \frac{\beta_3}{\beta_1} l_j \lambda_j^2 a_3 - \frac{\beta_3}{\beta_1} l_j a_1 \xi^2$, $j = 1,2,3.$

$m_j = c_{44} \frac{\beta_1 T_0}{\rho C_1^2} (\lambda_j + \xi d_j)$ $j = 1,2,3$

Appendix D

$$\tilde{u} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(\Delta_1^* e^{-\lambda_1 z} - \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z}\right\} \quad \text{(D.1)}$$

$$\tilde{w} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(d_1 \Delta_1^* e^{-\lambda_1 z} - d_2 \Delta_2^* e^{-\lambda_2 z} + d_3 \Delta_3^* e^{-\lambda_3 z}\right\} \quad \text{(D.2)}$$

$$\widetilde{t_{zz}} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(h_1 \Delta_1^* e^{-\lambda_1 z} - h_2 \Delta_2^* e^{-\lambda_2 z} + h_3 \Delta_3^* e^{-\lambda_3 z}\right\} \quad \text{(D.3)}$$

$$\widetilde{t_{zr}} = \frac{1}{\Delta}\left\{\frac{P_2}{2\pi}(m_1 \Delta_1^* e^{-\lambda_1 z} - m_2 \Delta_2^* e^{-\lambda_2 z} + m_3 \Delta_3^* e^{-\lambda_3 z}\right\} \quad \text{(D.4)}$$

$$\tilde{\varphi} = \frac{1}{\Delta}\left\{-\frac{P_2}{2\pi}(l_1 \Delta_1^* e^{-\lambda_1 z} - l_2 \Delta_2^* e^{-\lambda_2 z} + l_3 \Delta_3^* e^{-\lambda_3 z}\right\} \quad \text{(D.5)}$$

$\Delta_1^* = -(h_2)(\lambda_3 + \xi d_3) + (h_3)(\lambda_2 + \xi d_2)$

$\Delta_2^* = -(h_1)(\lambda_3 + \xi d_3) + (h_3)(\lambda_1 + \xi d_1)$

$\Delta_3^* = -(h_1)(\lambda_2 + \xi d_2) + (h_2)(\lambda_1 + \xi d_1)$