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Modeling and Optimization of the Rigidity Modulus of Latertic Concrete using Scheffe's Theory

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ABSTRACT: *This investigation is on the modeling and optimization of the rigidity modulus of Lateritic Concrete. The laterite is the reddish soil layer often belying the top soil in many locations and further deeper in some areas, collected from the Vocational Education Building Site of the University of Nigeria, Nsukka. Scheffe's optimization approach was applied to obtain a mathematical model of the form $f(x_{i1},x_{i2},x_{i3})$, where x_i are proportions of the concrete components, viz: cement, laterite and water. Scheffe's experimental design techniques were followed to mould various block samples measuring 220mm x 210mm x 120mm, using the auto-generated components ratios, and tested for 28 days strength to arrive at the mathematical model: $\hat{Y} = 1371.48X_1 + 1459.38X_2 + 1362.68X_3 - 816.84X_1X_2 + 49.80X_1X_3 - 54.00X_2X_3$. To carry out the task, we embarked on experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy was checked using the control factors. Finally a software is prepared to handle the design computation process to select the optimized properties of the mix, and generate the optimal mix ratios for the desired property.*

KEYWORDS: *Model, laterite, pseudo-component, Simplex-lattice, model.*

I. INTRODUCTION

From the beginning of time, the cost of this change has been of major concern to man as the major construction factors are finance and others such as labour, materials and equipment. The construction of structures is a regular operation which creates the opportunity for continued change and improvement on the face of the environment.

Major achievements in the area of environmental development is heavily dependent on the availability of construction materials which take a high proportion of the cost of the structure. This means that the locality of the material and the usability of the available materials directly impact on the achievable development of the area as well as the attainable level of technology in the area.

To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Where the scarcity of one component prevails exceedingly, the cost of the concrete production increases geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the component when it is possible to do so without losing the quality of the concrete. In the present time, concrete is the main material of construction, and the ease or cost of its production accounts for the level of success in the of area environmental upgrading through the construction of new roads, buildings, dams, water structures and the renovation of such structures.

Concept of Optimization

The target of planning is the maximization of the desired outcome of the venture. In order to maximize gains or outputs it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is referred to as optimization [1]. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables.

The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. The constraint at any time is the amount of money available to the entrepreneur at the time of investment.
Hence or otherwise, an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements [2]. The function is called the objective function and the specified requirements are known as the constraints of the problem.

Making structural concrete is not all comers' business. Structural concrete are made with specified materials for specified strength. Concrete is heterogeneous as it comprises sub-materials. Concrete is made up of fine aggregates, coarse aggregates, cement, water, and sometimes admixtures. Researchers [3] report that modern research in concrete seeks to provide greater understanding of its constituent materials and possibilities of improving its qualities. For instance, Portland cement has been partially replaced with ground granulated blast furnace slag (GGBS), a by–product of the steel industry that has valuable cementations properties [4].

Bloom and Bentur [5] reports that optimization of mix designs require detailed knowledge of concrete properties. Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage. Apart from the larger deformations, the acceleration of dehydration and strength gain will cause cracking at early ages.

Modeling
Modeling has to do with formulating equations of the parameters operating in the physical or other systems. Many factors of different effects occur in nature in the world simultaneously dependently or independently. When they interplay they could inter-affect one another differently at equal, direct, combined or partially combined rates variationally, to generate varied natural constants in the form of coefficients and/or exponents [6]. The challenging problem is to understand and asses these distinctive constants by which the interplaying factors underscore some unique natural phenomenon towards which their natures tend, in a single, double or multi phase system.
A model could be constructed for a proper observation of response from the interaction of the factors through controlled experimentation followed by schematic design where such simplex lattice approach [7]. Also entirely different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of mathematics [8].

II. LITERATURE REVIEW

The mineralogical and chemical compositions of laterites are dependent on their parent rocks [9]. Laterite formation is favoured in low topographical reliefs of gentle crests and plateaus which prevent the erosion of the surface cover [10].

Of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete [2]. To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, laterite or concrete are none of these, nevertheless they are popular construction materials [11]. With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio [12].

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other [13] When studying the properties of a q-component mixture, which are dependent on the component ratio only the factor space is a regular (q-1)–simplex [14]. Simplex lattice designs are saturated, that is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used [14].

Background Theory

The Scheffe's involves the application of a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying componental ratios to fix approximately equal mixture properties. The optimization is the selectability, from some criterial (mainly economic) view point, the optimal ratio from the component ratios list that can be automatedly generated. His theory is one of the adaptations to this work in the formulation of response function for compressive rigidity modulus of lateritic concrete.

Simplex Lattice

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture [13], and they are equidistant from each other. Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3,……, and X_i which obey these laws:

$$\left.\begin{array}{l} X_i < 0 \\ X \neq \text{negative} \\ 0 \leq x_i \leq 1 \\ \sum_{i=1} x_i = 1 \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

That is, a lattice is an abstract space.

To achieve the desired strength of concrete, one of the essential factors lies on the adequate proportioning of ingredients needed to make the concrete Henry Scheffe [7] developed a model whereby if the rigidity modulus desired is specified, possible combinations of needed ingredients to achieve the rigidity modulus can easily be predicted by the aid of computer, and if proportions are specified the rigidity modulus can easily be predicted.

Simplex Lattice Method

In designing experiment to attack mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial [14]. When investigating multi-components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a special representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained.

As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has C^n_{q+n} coefficients. If a mixture has a total of q components and x_1 be the proportion of the i^{th} component in the mixture such that,

$$x_i >= 0 \ (i=1,2, \ldots.q), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

then the sum of the component proportion is a whole unity i.e.

$$X_1 + x_2 + x_3 = 1 \text{ or } \sum x_i - 1 = 0 \quad \ldots. \quad .. \quad .. \quad .. \qquad (3)$$

where i = 1, 2, …., q… Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if q = 2, we have 2 points of connectivity. This gives a straight line simplex lattice. If q=3, we have a triangular simplex lattice and for q = 4, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

i. The factor space has uniformly distributed points,

ii. Simplex lattice designs are saturated (Akhnarova and Kafarov, 1982). That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, ½ and 1; for the cubic (m=3) polynomials, the levels are 0, 1/3, 2/3 and 1, etc; Scheffe [7] showed that the number of points in a (q,m) lattice is given by

$$C_{q+m-1} = q(q+1) \ldots (q+m-1)/m! \ \ldots\ldots\ldots\ldots.. \quad .. \quad .. \quad .. \quad .. \qquad (4)$$

The (3,2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable x_1, x_2 ……, x_q, subject to Eqn (3), and will be called a (q,m) polynomial having a general form:

$$\hat{Y} = b_0 + \sum_{i\leq 1\leq q} b_iX_i + \sum_{i\leq 1<j\leq q} b_{ij}X_iX_{ij} + \ldots + \sum_{i\leq 1<j<k\leq q} b_{ijk} + \sum b_{i1i2\ldots in}X_{i1}X_{i2}\ldots X_{in} \quad \ldots\ldots \quad .. \quad .. \qquad (5)$$

where b is a constant coefficient.

The relationship $\sum xi = 1$ enables the q^{th} component to be eliminated and the number of coefficients reduced to $C^m{}_{q+m-1}$, but the very character of the problem dictates that all the q components be introduced into the model.

Substituting into equation Eqn (5), the polynomial has the general usable form:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_{13} + b_{23}X_2X_3 + b_{11}X^2{}_1 + b_{22}X^2{}_2 + b_{33}X^2{}_3 \quad \ldots\ldots \quad \ldots.. \quad \ldots \qquad (6)$$

By applying the normalization condition of Eqn. (3) and to Eqn (6) and jogling [6], we get that

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_{23} \quad . \quad . \qquad (7)$$

where

$$\beta_i = b_0 + b_i + b_{ii} \quad \text{and} \quad \beta_{ij} = b_{ij} - b_{ii} - b_{jj}, \; . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

Thus, the number of coefficients has reduced from 10 in Eqn (6) to 6 in Eqn (7). That is, the reduced second degree polynomial in q variables is

$$\hat{Y} = \sum \beta_iX_i + \sum \beta_{ij}X_i \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (9)$$

Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (refer to fig 3.1), others are zero.
Hence if we substitute in Eqn. (3.11), the coordinates of the first point (X_1=1, X_2=0, and X_3=0, Table 3.1), we get that $Y_1 = \beta_1$.
And doing so in succession for the other two points if the hexahedron, we obtain

$$Y_2 = \beta_2, Y_3 = \beta_3 \quad .. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

\

The substitution of the coordinates of the fourth point yields

$$Y_{12} = ½ X_1 + ½X_2 + ½X_1.X_2$$
$$= ½ \beta_1 + ½ \beta_2 + {}^1/_4 \beta_{12}$$

But as $\beta_i = Y_i$ then

$$Y_{12} = ½ \beta_1 - ½ \beta_2 - {}^1/_4 \beta_{12}$$

Thus

$$\beta_{12} = 4 Y_{12} - 2Y_1 - 2Y_2 \quad . \quad . \quad . \quad . \quad .$$

And similarly,

$$\beta_{13} = 4 Y_{13} - 2Y_1 - 2Y_2$$
$$\beta_{23} = 4 Y_{23} - 2Y_2 - 2Y_3$$

Or generalizing,

$$\beta_i = Y_i \text{and } \beta_{ij} = 4 Y_{ij} - 2Y_i - 2Y_j \quad . \quad . \quad . \quad . \quad . \quad . \qquad .(11)$$

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the three points defining the coefficients β_{ij} lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X_i alone and the property of the mixture is denoted by Y_i. Mixture 4 includes X_1 and X_2, and the property being designated Y_{12}.

Actual and Pseudo Components

The requirements of the simplex that $\sum_{X=1}^{q} Xi = 1$ makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios say $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$ for the i^{th} experimental points are called pseudo components. Since X_1, X_2 and X_3 are subject to $\sum Xi = 1$, the transformation of cement:laterite:water at say 0.60 water/cement ratio cannot easily be computed because X_1, X_2 and X_3 are in pseudo expressions $X_1^{(i)}$, $X_2^{(i)}$, and $X_3^{(i)}$. For the i^{th} experimental point, the transformation computations are to be done.

The arbitrary vertices chosen on the triangle are A_1(1:7.50:0.05), A_2(1:8.20:0.03) and A_3(1:6.90:0.10), based on experience and earlier research reports.

Transformation Matrix

If Z denotes the actual matrix of the i^{th} experimental points, observing from Table 3.2 (points 1 to 3),

$$BZ = X = 1 \qquad (12)$$

where B is the transformed matrix.

Therefore, $B = I.Z^{-1}$

Or $B = Z^{-1}$ (13)

For instance, for the chosen ratios, at the vertices A_1, A_2 and A_3,

$$Z = \begin{bmatrix} 1.00 & 7.50 & 0.50 \\ 1.00 & 8.20 & 0.30 \\ 1.00 & 6.90 & 0.10 \end{bmatrix} \qquad (14)$$

From Eqn (13),

$B = Z^{-1}$

$$Z^{-1} = \begin{bmatrix} 26.65 & -17.60 & -8.04 \\ -30.04 & 2.17 & 0.87 \\ -56.52 & 26.08 & 30.43 \end{bmatrix}$$

Hence,

$$B\,Z^{-1} = Z.\,Z^{-1} = 0 \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, for actual component Z, the pseudo component X is given by

$$X \begin{bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{bmatrix} = B \begin{bmatrix} 26.65 & -17.60 & -8.04 \\ -30.04 & 2.17 & 0.87 \\ -56.52 & 26.08 & 30.43 \end{bmatrix} Z \begin{bmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

which gives the X_i(i=1,2,3) values in Table 1.

The inverse transformation from pseudo component to actual component is expressed as

$$AX = Z \qquad (15)$$

where A = inverse matrix

$A = Z X^{-1}$.

From Eqn 3.16, X = BZ, therefore,

$A = Z.(BZ)^{-1}$

$A = Z.Z^{-1}B^{-1}$

$A = IB^{-1}$

$$B = B^{-1} \qquad (16)$$

This implies that for any pseudo component X, the actual component is given by

$$Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \end{pmatrix} = B \begin{pmatrix} 1 & 7.50 & 0.05 \\ 1 & 8.20 & 0.03 \\ 1 & 6.9 & 0.10 \end{pmatrix} X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{pmatrix} \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

Eqn (17) is used to determine the actual components from points 4 to 6 , and the control values from points 7 to 9 (Table 1).

Table 1 Value for Experiment

N	X_1	X_2	X_3	RESPONSE	Z_1	Z_2	Z_3
1	1	0	0	Y_1	1	1	1
2	0	1	0	Y_2	7.50	8.20	6.90
3	0	0	1	Y_3	0.05	0.03	0.10
4	1/2	1/2	0	Y_{12}	1	7.85	0.04
5	½	0	1/2	Y_{13}	1	7.20	0.075
6	0	1/2	1/2	Y_{23}	1	7.55	0.065
Control Points (6-9)							
7	1/3	1/3	1/3	Y_{123}	1	7.458	0.0594
8	1/3	2/3	0	Y_{122}	1	7.698	0.0366
9	0	1/3	2/3	Y_{233}	1	7.329	0.0769

III. METHODOLOGY

Materials

The disturbed samples of laterite material were collected at the Vocational Education project site at the University of Nigeria, Nsukka, at the depth of 1.5m below the surface.

The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag.

Material Properties

All samples of the laterite material conformed to the engineering properties already determined by a team of engineering consultants from the Civil Engineering Department, U.N.N, who reported on the Sieve Analysis Tests, Natural Moisture Content, etc, carried out according to the British Standard Specification, BS 1377 – "Methods of Testing Soils for Civil Engineering Purposes".

Preparation of Samples

The sourced materials for the experiment were transferred to the laboratory where they were allowed to dry. A samples of the laterite were prepared and tested to obtain the moisture content for use in proportioning the components of the lateritic concrete to be prepared. The laterite was sieved to remove debris and coarse particles. The component materials were mixed at ambient temperature. The materials were mixed by weight according to the specified proportions of the actual components generated in Table 1. In all, two blocks of 220mm x210 x120mm for each of six experimental points and three control points were cast for the rigidity modulus test, cured for 28 days after setting and hardening.

Strength Test

After 28 day of curing, the cubes and blocks were crushed, with dimensions measured before and at the point of shearing, to determine the lateritic concrete block strength, using the compressive testing machine to the requirements of BS 1881:Part 115 of 1986.

IV. RESULT AND ANALYSIS

Replication Error And Variance of Response

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the six design points.

Hence we have below, the table of the results (Tables 5.1a,b and c) which contain the results of two repetitions each of the 6 design points plus three Control Points of the (3,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$\ddot{Y} = \sum(Y_r)/r$ . . . . (18)

where $\hat{Y}$ is the mean of the response values and r =1,2.

$S_Y^2 = \sum[(Y_i - \ddot{Y}_i)^2]/(n-1)$ . . . . . . (19)

where n = 9.

5.1.2 Results and Analysis for the Modulus of Rigidity Property

The Modulus of Elasticity, E_c, and the Modulus of Rigidity, G, of the lateritic concrete block were computed from the relation

$E_c = 1.486 f_c^{1/3} \rho^2 x 10^{-3}$ . . . . . (20)

where E_c and fc are measured in MPa and ρ in Kg/m^3 [15], and ρ = density of block.

f_c = Failure load/Cross-sectional Area contact area of block,

$G = E_c/2(1+ v)$ . . . . (21)

where v = Poisson's Ratio = lateral strain/longitudinal strain

Table 2: 220x210x120 Block Sample Results

No	Replication	Failure Load (kN)	Dx (10^{-2}xmm)	dy (10^{-2}xmm)	Wet Weight (kg)	Dry Weight (kg)
1	A	55.60	300	120	8.68	8.63
	B	54.90	324	74	8.69	8.33
2	A	60.40	355	94	8.75	8.33
	B	71.00	360	89	8.89	8.28
3	A	69.40	301	132	8.87	8.66
	B	68.20	245	103	8.73	8.62
4	A	57.80	230	84	8.69	8.42
	B	62.50	240	109	8.77	8.46
5	A	65.30	295	112	8.79	8.67
	B	69.30	302	110	8.98	8.63
6	A	68.90	344	138	8.82	8.91
	B	66.10	307	146	8.71	8.22
CONTROL POINTS						
7	A	81.30	282	97	8.91	8.45
	B	82.60	327	93	8.43	8.33
8	A	71.30	280	112	8.83	8.63
	B	65.30	330	113	8.68	8.55
9	A	80.00	312	120	8.64	8.42
	B	74.20	344	92	8.67	8.79

Table 3 Compressive Strength, Poisson's Ratio, Young's Modulus and Modulus of Rigidity Results

No	Replication	f_c (MPa)	Average f_c (MPa)	ρ (kg/m^3)	V	E_c (N/mm^2)	G (N/mm^2)
1	A	2.10	2.08	1552.31	0.33	4569.01	1371.48
	B	2.07					
2	A	2.28	2.48	1498.10	0.27	4501.31	1459.38
	B	2.68					
3	A	2.62	2.60	1558.44	0.45	4950.84	1362.68
	B	2.58					
4	A	2.18	2.27	1522.36	0.43	4518.88	1208.72
	B	2.36					
5	A	2.47	2.54	1560.24	0.39	4925.61	1379.53
	B	2.62					

6	A	2.60	2.55	1526.87	0.36	4728.05	1397.53
	B	2.50					
Control Points							
7	A	3.07	3.09	1513.34	0.33	4945.85	1476.20
	B	3.12					
8	A	2.70	3.59	1551.22	0.39	4899.21	1453.30
	B	2.49					
9	A	3.03	2.77	1552.12	0.34	5097.61	1531.64
	B	2.81					

Table 4 Result of the Replication Variance of the G Response for 220x210x120 mm Block

Experiment No (n)	Repetition	Response G (N/mm^2)	Response Symbol	$\sum Y_r$	$\ddot{Y}_r$	$\sum(Y_r - \ddot{Y}_r)^2$	S_i^2
1	1A 1B	1356.02 1386.95	Y_1	2742.97	1371.48	478.35	239.17
2	2A 2B	1439.51 1479.25	Y_2	2918.77	1459.38	789.48	394.74
3	3A 3B	1437.98 1297.38	Y_3	2735.36	1362.68	9883.84	4941.92
4	4A 4B	1189.85 1227.58	Y_{12}	2417.44	1208.72	711.72	355.86
5	5A 5B	1397.73 1361.20	Y_{13}	2758.93	1379.53	667.20	333.60
6	6A 6B	1597.42 1197.42	Y23	2795.06	1397.53	79916.67	39958.53
Control Points							
7	7A 7B	1430.13 1522.28	C_1	2952.41	1476.20	4245.77	2122.88
8	8A 8B	1500.19 1406.42	C_2	2906.61	1453.30	4396.18	2198.09
9	9A 9B	1493.21 1570.07	C_3	3063.28	1531.64	2953.65	1476.82

$\sum$104042.89

Replication Variance

$S_{Yc}^2 = (\sum S_i^2)/(n-1) = 104042.89/8 = 13005.36$

Replication Error

$S_{Yc} = (S_{\hat{Y}}^2)^{1/2} = 13005^{1/2} = 114.04$

$\beta_1=1371.48$
$\beta_2=1459.38$
$\beta_3=1362.68$
$\beta_{12}=4(1208.72)-2(1371.48)-2(1459.38)=-816.84$
$\beta_{13}=4(1379.53)-2(1371.48)-2(1362.68)=49.80$
$\beta_{23}=4(1397.53)-2(1459.38)-2(1362.68)=-54.00$

Determination of Regression Equation for the G

From Eqns 3.15 and Table 5.1 the coefficients of the reduced second degree polynomial is determined as follows:
Thus, from Eqn (7),

$$\hat{Y} = 1371.48X_1 + 1459.38X_2 + 1362.68X_3 - 816.84X_1X_2 + 49.80X_1X_3 - 54.00X_2X_3 \quad (22)$$

Eqn (22) is the mathematical model of the G of the lateritic concrete based on the 28-day strength.

Test of Adequacy of the G Model
Eqn (22), the equation model, will be tested for adequacy against the controlled experimental results.

We apply the statistical hypothesis as follows:
1. Null Hypothesis (H_0): There is no significant difference between the experimental values and the theoretical expected results of the Modulus of Rigidity.

2. Alternative Hypothesis (H_1): There is a significant difference between the experimental values and the theoretical expected results of the Modulus of Rigidity.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, S_Y^2, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a quaternary mixture, the following points are assumed:

X_i can be observed without errors.

The replication variance, S_Y^2, is similar at all design points, and

Response values are the average of n_i and n_{ij} replicate observations at appropriate points of the simplex

Then the variance $S_{\hat{Y}i}$ and $S_{\hat{Y}ij}$ will be

$$(S_{\hat{Y}}^2)_i = S_Y^2/n_i \qquad (23)$$

$$S_{\hat{Y}}^2)_{ij} = S_Y^2/n_{ij}. \qquad (24)$$

In the reduced polynomial,

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{23}X_2X_{23} + \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \qquad (25)$$

If we replace coefficients by their expressions in terms of responses,

$\beta_i = Y_i$ and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$

$$\hat{Y} = Y_1X_1 + Y_2X_2 + Y_3X_3 + + Y_4X_4 + (4Y_{12} - 2Y_1 - 2Y_2)X_1X_2 + (4Y_{13} - 2Y_1 - 2Y_3)X_1X_3 + (4Y_{14} - 2Y_1 - 2Y_4)X_1X_4 + (4Y_{23} - 2Y_2 - 2Y_3)X_2X_3 + (4Y_{24} - 2Y_2 - 2Y_4)X_2X_4 + (4Y_{34} - 2Y_3 - 2Y_4)X_3X_4$$

$$= Y_1(X_1 - 2X_1X_2 - 2X_1X_3 - 2X_1X_4) + Y_2(X_2 - 2X_1X_2 - 2X_2X_3 - 2X_2X_4) + Y_3(X_3 - 2X_1X_3 + 2X_2X_3 + 2X_3X_4) + Y_4(X_4 - 2X_1X_4 + 2X_2X_4 + 2X_3X_4) + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{34}X_3X_4 \qquad (26)$$

Using the condition $X_1+X_2+X_3+X_4=1$, we transform the coefficients at Y_i

$$X_1 - 2X_1X_2 - 2X_1X_3 - 2X_1X_4 = X_1 - 2X_1(X_2 + X_3 + X_4)$$

$$= X_1 - 2X_1(1 - X_1) = X_1(2X_1 - 1) \text{ and so on.} \qquad (27)$$

Thus

$$\hat{Y} = X_1(2X_1 - 1)Y_1 + X_2(2X_2 - 1)Y_2 + X_3(2X_3 - 1)Y_3 + X_4(2X_4 - 1)Y_4 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{34}X_3X_4 \qquad (28)$$

Introducing the designation

$$a_i = X_i(2X_1 - 1) \text{ and } a_{ij} = 4X_iX_j \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

and using Eqns (23) and (24) give the expression for the variance S_Y^2.

$$S_{\hat{Y}}^2 = S_Y^2 (\sum_{1\le i\le q} a_{ii}/n_i + \sum_{1\le i<j\le q} a_{ij}/n_{ij}) \quad . \quad . \quad . \quad . \quad .. \qquad (30)$$

If the number of replicate observations at all the points of the design are equal, i.e. $n_i=n_{ij}= n$, then all the relations for $S_{\hat{Y}}^2$ will take the form

$$S_{\hat{Y}}^2 = S_Y^2\xi/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

where, for the second degree polynomial,

$$\xi = \sum_{1\le i\le q} a_i^2 + \sum_{1\le i<j\le q} a_{ij}^2 \quad . \quad . \qquad (32)$$

As in Eqn (32), ξ is only dependent on the mixture composition. Given the replication variance and the number of parallel observations n, the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$$t = \Delta_Y/(S_{\hat{Y}}^2 + S_Y^2) = \Delta_Y n^{1/2} /(S_Y(1 + \xi)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

where $\Delta_Y = Y_{exp} - Y_{theory}$ . . . . (34)

and n = number of parallel observations at every point.

The t-statistic has the student distribution, and it is compared with the tabulated value of $t_{\alpha/L}(V)$ at a level of significance α, where L = the number of control points, and V = the number for the degrees of freedom for the replication variance.

The null hypothesis is that the equation is adequate is accepted if $t_{cal}< t_{Table}$ for all the control points.

The confidence interval for the response value is

$$\hat{Y} - \Delta \le Y \le \hat{Y} + \Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (35)$$

$$\Delta = t_{\alpha/L,k} S_{\hat{Y}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

where k is the number of polynomial coefficients determined.

Using Eqn (31) in Eqn (36)

$$\Delta = t_{\alpha/L,k} S_Y(\xi/n)^{1/2} \quad . \quad . \quad . \quad . \quad . \qquad (37)$$

t-Test for the G Model

If we substitute for X_i in Eqn (20) from Tables 1, the theoretical predictions of the response ($\hat{Y}$) can be obtained. These values can be compared with the experimental results (Table 3 and 4). a, ξ, t and Δ_y are evaluated using Eqns 29, 32, 33 and 37 respectively.

Table 5 t-Test for the Test Control Points

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	$\ddot{Y}$	$\hat{Y}_a$	Δ_y	t
1	C_1	1	2	-0.333	0.444	0.011	0.197		1476.20	1301.18	175.02	0.011
		1	3	-0.333	0.444	0.011	0.197					
		2	3	-0.333	0.444	0.011	0.197					
					$\sum$	0.033	0.591	0.624				
2	C_2	1	2	-0.333	0.887	0.011	0.787		1453.30	1258.74	194.56	0.012
		1	3	-0.333	0.000	0.011	0.000					
		2	3	0.333	0.000	0.011	0.000					
					$\sum$	0.033	0.787	0.820				
3	C_3	1	2	0.000	0.000	0.000	0.000		1531.64	1382.91	148.73	0.008
		1	3	0.000	0.000	0.000	0.000					
		2	3	-0.333	0.887	0.011	0.787					
					$\sum$	0.011	0.787	0.798				

Significance level $\alpha = 0.05$,

i.e. $t_{\alpha/L}(V_c) = t_{0.05/3}(9)$, where L=number of control point.

From the Student t-Table, the tabulated value of $t_{0.05/3}(9)$ is found to be 5.722 which is greater than any of the calculated t-values in Table 5. Hence we can accept the Null Hypothesis.

From Eqn 37, with k=6 and $t_{\alpha/k}(V) = t_{0.05/6}(9) = 5.722$,

$\Delta = 325.10$ which satisfies the confidence interval equation of Eqn (35) when viewed against the response values in Table 5.

V. COMPUTER PROGRAM

In the developed computer program any desired Rigidity Modulus can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the G tolerance of 0.50 N/mm^2.

Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

Choosing a Combination

It can be observed that the strength of 1209 N/sq mm yielded 6combinations. To accept any particular proportions depends on the factors such as workability, cost and honeycombing of the resultant lateritic concrete.

VI. CONCLUSION AND RECOMMENDATION

Conclusion

Henry Scheffe's simplex design was applied successfully to prove that the modulus of rigidity of lateritic concrete is a function of the proportion of the ingredients (cement, laterite and water), but not the quantities of the materials.

The maximum elastic modulus obtainable with the Rigidity Modulus model is 1459.38 N/sq mm. See the computer run outs which show all the possible lateritic concrete mix options for the desired rigidity modulus property, and the choice of any of the mixes is the user's. One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using sand as aggregate. This is due to the predominantly high silt content of laterite.

It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if workability and other demands of the resulting lateritic concrete have to be satisfied. This is an important area for further research work.

The project work is a great advancement in the search for the applicability of laterite in concrete mortar production in regions where sand is extremely scarce with the ubiquity of laterite.

Recommendations
From the foregoing study, the following could be recommended:

i) The model can be used for the optimization of the strength of concrete made from cement, laterite and water.
ii) Laterite aggregates cannot adequately substitute sharp sand aggregates for heavy construction.
iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete.
iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

REFERENCE

[1] Orie, O.U., Osadebe, N.N., "Optimization of the Compressive Strength of Five Component Concrete Mix Using Scheffe's Theory – A case of Mound Soil Concrete", Journal of the Nigerian Association of Mathematical Physics, Vol. 14, pp. 81-92, May, 2009.
[2] Majid, K.I., Optimum Design of Structure, Butterworths and Co., Ltd, London, pp. 16, 1874.
[3] David, J., Galliford, N., Bridge Construction at Hudersfield Canal, Concrete, Number 6, 2000.
[4] Ecocem Island Ltd, Ecocem GBBS Cement. "The Technically Superior and Environmentally Friendly Cement", 56 Tritoville Road Sand Dublin 4 Island, 19.
[5] Bloom, R. and Benture, A. "Free and restrained Shrinkage of Normal and High Strength Concrete", int. ACI Material Journal, Vol. 92, No. 2, pp. 211-217.
[6] Onuamah, P.N. and Okpube G.C., "Mechanical Strength Modeling and Optimization Lateritic Solid Block, with 4% Mound Soil Inclusion", *American Journal of Engineering Research (AJER), 2015, Vol' 4, No.5, pp . 178 - 192.*
[7] Schefe, H., (1958), Experiments with Mixtures, Royal Statistical Society journal. Ser B., Vol. 20, 1958. pp 344-360, Rusia.
[8] Erwin, K., Advanced Engineering Mathematics, 8th Edition, John Wiley and Sons, (Asia) Pte. Ltd, Singapore, pp. 118-121 and 240 – 262.
[9] Tardy Yves, (1997), Petrology of Laterites and Tropical Soils. ISBN 90-5410-678-6, http://www.books.google.com/books. Retrieved April 17, 2010.
[10] Dalvi, Ashok, D., Bacon, W. Gordon, Osborne, Robert, C. (March 7-10, 2004), PDAC 2004 International Convention, Trade Show & Investors Exchange.
[11] Wilby, C.B., Structural Concrete, Butterworth, London, UK, 1983.
[12] Reynolds, C. and Steedman, J.C., Reinforced Concrete Designers Handbook, View point Publications, 9th Edition, 1981, Great Britain.
[13] Jackson, N., Civil Engineering Materials, RDC Arter Ltd, Hong Kong, 1983.
[14] Akhanarova, S. and Kafarov, V., Experiment and Optimization in Chemistry and Chemical Engineering, MIR Publishers, Mosco, 1982, pp. 213 - 219 and 240 – 280.
[15] Takafumi, N., Fminori, T., Kamran, M.N. Bernardino, M.C. and Alessandro P.F., (2009), "Practical Equations for the Elatic Modulus of Concrete", ACI Structural Journal, September-October, 2009, Vol. 106, No. 5.

APPENDIX 1

QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF LATERITIC CONCRETE MIXES

```
'USING THE SCHEFFE'S MODEL FOR CONCRETE RIGIDITY MODULUS
Cls
C1$ = "(ONUAMAH.HP) RESULT OUTPUT ": C2$ = "A COMPUTER PROGRAM "
C3$ = "ON THE OPTIMIZATION OF THE RIGIDITY MODULUS OF A 3-COMPONENT LATERITIC CONCRETE
MIX"
Print C2$ + C1$ + C3$
Print
'VARIABLES USED ARE
'X1, X2, X3, Z1, Z2, Z3, YT, YTMAX, DS

'INITIALISE I AND YTMAX

I = 0: YTMAX = 0

For MX1 = 0 To 1 Step 0.01
    For MX2 = 0 To 1 - MX1 Step 0.01
     MX3 = 1 - MX1 - MX2
     YTM = 1371.48 * MX1 + 1459.38 * MX2 + 1362.68 * MX3 - 816.84 * MX1 * MX2 + 49.8 * MX1 * MX3 - 54! *
MX2 * MX3
     If YTM >= YTMAX Then YTMAX = YTM
```

```
       Next MX2
      Next MX1
INPUT "ENTER DESIRED RIGIDITY MODULUS, DS = "; DS

    'PRINT OUTPUT HEADING
    Print
    Print Tab(1); "No"; Tab(10); "X1"; Tab(18); "X2"; Tab(26); "X3"; Tab(32); "YTHEORY"; Tab(45); "Z1"; Tab(53);
"Z2"; Tab(61); "Z3"
    Print
    'COMPUTE THEORETICAL RIGIDITY MODULUS, YT
     For X1 = 0 To 1 Step 0.01
       For X2 = 0 To 1 - X1 Step 0.01
         X3 = 1 - X1 - X2
YT = 1371.48 * X1 + 1459.38 * X2 + 1362.68 * X3 - 816.84 * X1 * X2 + 49.8 * X1 * X3 - 54! * X2 * X3

If Abs(YT - DS) <= 0.5 Then

'PRINT MIX PROPORTION RESULTS
          Z1 = X1 + X2 + X3: Z2 = 7.5 * X1 + 8.2 * X2 + 6.9 * X3: Z3 = 0.05 * X1 + 0.03 * X2 + 0.1 * X3
          I = I + 1
          Print Tab(1); I; USING; "##.###"; Tab(7); X1; Tab(15); X2; Tab(23); X3; Tab(32); YT; Tab(42); Z1; Tab(50); Z2;
Tab(58); Z3
          Print
          Print
          If (X1 = 1) Then GoTo 550
        Else
          If (X1 < 1) Then GoTo 150
        End If

150    Next X2
     Next X1
   If I > 0 Then GoTo 550
   Print
   Print "SORRY, THE DESIRED RIGIDITY MODULUS IS OUT OF RANGE OF MODEL"
   GoTo 600

550 Print Tab(5); "THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS "; YTMAX; "N / Sq mm; "
 600 End
```

A COMPUTER PROGRAM (ONUAMAH.HP) RESULT OUTPUT ON THE OPTIMIZATION OF THE RIGIDIT Y MODULUS OF A 3-COMPONENT LATERITIC CONCRETE MIX

ENTER DESIRED RIGIDITY MODULUS, DS = ? 1205

No X1 X2 X3 YTHEORY Z1 Z2 Z3

SORRY, THE DESIRED RIGIDITY MODULUS IS OUT OF RANGE OF MODEL

Press any key to continue

ENTER DESIRED RIGIDITY MODULUS, DS = ? 1209

No	X1	X2	X3	YTHEORY	Z1	Z2	Z3
1	0.530	0.470	0.000	%1209.318	1.000	7.829	0.041
2	0.540	0.460	0.000	%1209.011	1.000	7.822	0.041
3	0.550	0.450	0.000	%1208.867	1.000	7.815	0.041
4	0.560	0.440	0.000	%1208.887	1.000	7.808	0.041
5	0.570	0.430	0.000	%1209.070	1.000	7.801	0.041
6	0.580	0.420	0.000	%1209.416	1.000	7.794	0.042