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Some Common Fixed Point Results for a Rational inequality in Complex Valued Metric Spaces

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ABSTRACT : *We prove some common fixed point theorems for two pairs of weakly compatible mappings satisfying a rational type contractive condition using (E.A.) and (CLR)-property in Complex Valued metric spaces. Our results generalize and extend some of the existing results in the literature.*

KEYWORDS: *Complex valued metric space, weakly compatible mappings, (E.A.) - property, (CLR)-property.*
Mathematics subject classification*: 47H10, 54H25.*

I. INTRODUCTION

Azam et al. [2] introduced the concept of complex valued metric spaces and obtained sufficient conditions for the existence of common fixed points of a pair of contractive type mappings involving rational expressions. Subsequently many authors have studied the existence and uniqueness of the fixed points and common fixed points of self mapping in view of contrasting contractive conditions. Aamri and Moutawakil [1] introduced the notion of (E.A.) - property. Sintunavrat and P. Kumam [8] introduced the notion of (CLR) - property. Then many authors proved several fixed point theorems using the concept of weakly compatible maps with (E.A.) and (CLR)-property. The main purpose of this paper is to present fixed point results for two pair of weakly compatible mappings satisfying a generalize contractive condition by using the concept of (E.A.) and (CLR)-property in complex valued metric space. The proved results generalize and extend some of the existing results in the literature.

II. PRELIMINARIES

Let C be the set of complex numbers and let z_1 , $z_2 \in C$. Define a partial order $\precsim$ on C as follows:
$z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2)$, $Im(z_1) \leq Im(z_2)$. It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

[1] $Re(z_1) = Re(z_2)$, $Im(z_1) < Im(z_2)$,
[2] $Re(z_1) < Re(z_2)$, $Im(z_1) = Im(z_2)$,
[3] $Re(z_1) < Re(z_2)$, $Im(z_1) < Im(z_2)$,
[4] $Re(z_1) = Re(z_2)$, $Im(z_1) = Im(z_2)$.

In particular, we will write $z_1 \lesssim z_2$ if one of (i), (ii) and (iii) is satisfied and we will write $z_1 < z_2$ if only (iii) is satisfied.

Definition2.1. Let X be a non-empty set. Suppose that the mapping $d: X \times X \to C$ satisfies:
[1] $0 \precsim d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;
[2] $d(x,y) = d(y,x)$ for all $x, y \in X$;
[3] $d(x,y) \precsim d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.
A point $x \in X$ is called an interior point of a set $A \subseteq X$ whenever there exists $0 < r \in C$ such that $B(x,r) = \{y \in X : d(x,y) < r\} \subseteq A$. A subset A in X is called open whenever each point of A is an interior

point of A. The family $F = \{B(x,r): x \in X, 0 < r\}$ is a sub-basis for a Hausdorff topology τ on X. A point $x \in X$ is called a limit point of A whenever for every $0 < r \in C$, $B(x,r) \cap (A \backslash X) \neq \phi$.
A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.
Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in C$, with $0 < c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) < c$, then x is called the limit point of $\{x_n\}$ and we write $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.
If for every $c \in C$, with $0 < c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x_{n+m}) < c$, then $\{x_n\}$ is called a Cauchy sequence in (X, d) is called a complete complex valued metric space.
Lemma2.2. Let (X, d) be a complex valued metric space and $\{x_n\}$ is a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma2.3. Let (X, d) be a complex valued metric space and $\{x_n\}$ is a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.

Definition2.4. Let f and g be self-maps on a set X, if $w = fx = gx$ for some x in X, then x is called coincidence point of f and g, w is called a point of coincidence of f and g, w is called a point of coincidence of f and g.

Definition2.5. Let f and g be two self-maps defined on a set X, then f and g are said to be weakly compatible if they commute at coincidence points.

Definition 2.6. Let f and g be two self-mappings of a complex valued metric space (X, d). We say that f and g satisfy the (E.A.)-property if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = t$, for some $t \in X$.

Definition 2.7. Let f and g be two self-mappings of a complex valued metric space (X, d). We say that f and g satisfy the (CLR_f) property if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = fx$.

III. MAIN RESULTS

Theorem: 3.1 Let (X, d) be a Complex valued metric space and $A, B, S, T: X \to X$ four self-mappings satisfying the following conditions:

(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$;

(ii) for all $x, y \in X$,

$$d(Ax, By) \leq a_1 \frac{d(Ty,By)[d(Ax,Ty)+d(Sx,Ax)]}{[1+d(Sx,Ty)+d(Ax,Ty)]} + a_2 \frac{d(Ax,Ty)d(Sx,By)[d(Sx,Ax)+d(Ty,By)]}{[1+d(Sx,Ty)+d(Ax,Ty)]}$$

$$+a_3[d(Ax,Ty) + d(Sx,By)] + a_4[d(Sx,Ax) + d(Ty,By)] + a_5 d(Sx,Ty) \quad (3.1)$$

where $2a_3 + a_4 + a_5 < 1$ and $a_1, a_2, a_3, a_4, a_5 > 0$;

(iii) the pairs (A, S) and $(B, T)$are weakly compatible;

(iv) One of the pairs (A, S) or (B, T)satisfy (E.A.)-property.

If the range of one of the mapping $S(X)$ or $h(X)$ is closed subspace of X, then the mappings A, B, S and T have a unique common fixed point in X.

Proof: First suppose that the pair (B, T) satisfies (E.A.) property then there exists a sequence $\{x_n\}$ in X, such that
$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.
Further, since $B(X) \subseteq S(X)$, there exists a sequence $\{y_n\}$ in X, such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = t$.

Now, we claim that $\lim_{n\to\infty} Ay_n = t$. Let $\lim_{n\to\infty} Ay_n = t_1 \neq t$ then putting $x = y_n, y = x_n$ in (3.1), and we have

$$d(Ay_n, Bx_n) \le a_1 \frac{d(Tx_n,Bx_n)[d(Ay_n,Tx_n)+d(Sy_n,Ay_n)]}{[1+d(Sy_n,Tx_n)+d(Ay_n,Tx_n)]}$$

$$+a_2 \frac{d(Ay_n,Tx_n)d(Sy_n,Bx_n)[d(Sy_n,Ay_n)+d(Tx_n,Bx_n)]}{[1+d(Sy_n,Tx_n)+d(Ay_n,Tx_n)]}$$

$$+a_3[d(Ay_n,Tx_n)+d(Sy_n,Bx_n)]+a_4[d(Sy_n,Ay_n)+d(Tx_n,Bx_n)]$$
$$+a_5 d(Sy_n,Tx_n)$$

Letting $n\to\infty$, we have

$$d(t_1,t) \le a_1 \frac{d(t,t)[d(t_1,t)+d(t,t_1)]}{[1+d(t,t)+d(t_1,t)]} + a_2 \frac{d(t_1,t)d(t,t)[d(t,t_1)+d(t,t)]}{[1+d(t,t)+d(t_1,t)]}$$

$$+a_3[d(t_1,t)+d(t,t)]+a_4[d(t,t_1)+d(t,t)]+a_5 d(t,t)$$
$$\Rightarrow [1-(a_3+a_4)]\, d(t,t_1) \le 0$$

as $a_3 + a_4 < 1$

$\Rightarrow |d(t,t_1)| \le 0$. Hence $t_1 = t$ and that is, $\lim\limits_{n\to\infty} Ay_n = \lim\limits_{n\to\infty} Bx_n = t$.

Now suppose that $S(X)$ is a closed subspace of X, then $t = Su$ for some $u \in X$, subsequently we have $\lim\limits_{n\to\infty} Ay_n = \lim\limits_{n\to\infty} Bx_n = \lim\limits_{n\to\infty} Tx_n = \lim\limits_{n\to\infty} Sy_n = t = Su$.

We claim that $Au = Su$. For this put $x = u$, $y = x_n$ in (3.1), and we have

$$d(Au,Bx_n) \le a_1 \frac{d(Tx_n,Bx_n)[d(Au,Tx_n)+d(Su,Au)]}{[1+d(Su,Tx_n)+d(Au,Tx_n)]} + a_2 \frac{d(Au,Tx_n)d(Su,Bx_n)[d(Su,Au)+d(Tx_n,Bx_n)]}{[1+d(Su,Tx_n)+d(Au,Tx_n)]}$$

$$+a_3[d(Au,Tx_n)+d(Su,Bx_n)]+a_4[d(Su,Au)+d(Tx_n,Bx_n)]+a_5 d(Su,Tx_n)$$

Letting $n\to\infty$, we have

$$d(Au,t) \le a_1 \frac{d(t,t)[d(Au,t)+d(t,Au)]}{[1+d(t,t)+d(Au,t)]} + a_2 \frac{d(Au,t)d(t,t)[d(t,Au)+d(t,t)]}{[1+d(t,t)+d(Au,t)]}$$

$$+a_3[d(Au,t)+d(t,t)]+a_4[d(t,Au)+d(t,t)]+a_5 d(t,t)$$
$$\Rightarrow [1-(a_3+a_4)]\, d(Au,t) \le 0$$

as $a_3 + a_4 < 1$

$\Rightarrow |d(Au,t)| \le 0$, which is a contradiction. Hence u is a coincidence point of (A,S).

Now the weak compatibility of pair (A,S) implies that $ASu = SAu$ or $At = St$.

On the other hand, Since $A(X) \subseteq T(X)$, there exists v in X such that $Au = Tv$.

Thus, $Au = Su = Tv = t$. Now, we show that v is a coincidence point of (B,T); that is $Bv = Tv = t$.

Put $x = u$, $y = v$ in (3.1), and we have

$$d(Au,Bv) \le a_1 \frac{d(Tv,Bv)[d(Au,Tv)+d(Su,Au)]}{[1+d(Su,Tv)+d(Au,Tv)]} + a_2 \frac{d(Au,Tv)d(Su,Bv)[d(Su,Au)+d(Tv,Bv)]}{[1+d(Su,Tv)+d(Au,Tv)]}$$

$$+a_3[d(Au,Tv)+d(Su,Bv)]+a_4[d(Su,Au)+d(Tv,Bv)]+a_5 d(Su,Tv)$$

or
$$d(t,Bv) \le a_1 \frac{d(t,Bv)[d(t,t)+d(t,t)]}{[1+d(t,t)+d(t,t)]} + a_2 \frac{d(t,t)d(t,Bv)[d(t,t)+d(t,Bv)]}{[1+d(t,t)+d(t,t)]}$$

$$+a_3[d(t,t)+d(t,Bv)]+a_4[d(t,t)+d(t,Bv)]+a_5 d(t,t)$$

$$\Rightarrow [1-(a_3+a_4)]\, d(t,Bv) \le 0,$$

as $a_3 + a_4 < 1$

$\Rightarrow |d(t,Bv)| \le 0$, which is a contradiction. Thus $Bv = t$.

Hence, $Bv = Tv = t$, and v is the coincidence point of B and T. Further, the weak compatibility of pair (B,T) implies that $BTv = TBv$, or $Bt = Tt$. Therefore, t is a common coincidence point of A, B, S and T.

Now, we show that t is a common fixed point. Put $x = u$, $y = t$ in (3.1), and we have

$$d(t,Bt) = d(Au,Bt) \leq a_1 \frac{d(Tt,Bt)[d(Au,Tt)+d(Su,Au)]}{[1+d(Su,Tt)+d(Au,Tt)]} + a_2 \frac{d(Au,Tt)d(Su,Bt)[d(Su,Au)+d(Tt,Bt)]}{[1+d(Su,Tt)+d(Au,Tt)]}$$
$$+a_3[d(Au,Tt)+d(Su,Bt)] + a_4[d(Su,Au)+d(Tt,Bt)] + a_5 d(Su,Tt)$$

$$\Rightarrow (1-2a_3+a_5)\, d(t,Bt) \leq 0,$$

as $2a_3 + a_5 < 1$

$\Rightarrow |d(t,Bt)| \leq 0$, which is a contradiction. Thus $Bt = t$.

Hence, $At = Bt = St = Tt = t$.

Similar arguments arises if we assume that $T(X)$ is closed subspace of X. Similarly, the (E.A.)- property of the pair (A,S) will give a similar result.

For uniqueness of the common fixed point, let us assume that w is another common fixed point of A, B, S and T. Therefore $Aw = Bw = Sw = Tw = w$. Then, Put $x = w$ and $y = t$ in (3.1), and we have

$$d(w,t) = d(Aw,Bt) \leq a_1 \frac{d(Tt,Bt)[d(Aw,Tt)+d(Sw,Aw)]}{[1+d(Sw,Tt)+d(Aw,Tt]} + a_2 \frac{d(Aw,Tt)d(Sw,Bt)[d(Sw,Aw)+d(Tt,Bt)]}{[1+d(Sw,Tt)+d(Aw,Tt)]}$$
$$+a_3[d(Aw,Tt)+d(Sw,Bt)] + a_4[d(Sw,Aw)+d(Tt,Bt)] + a_5 d(Sw,Tt)$$

or
$$d(w,t) \leq a_1 \frac{d(t,t)[d(w,t)+d(w,w)]}{[1+d(w,t)+d(w,t]} + a_2 \frac{d(w,t)d(w,t)[d(w,w)+d(t,t)]}{[1+d(w,t)+d(w,t)]}$$
$$+a_3[d(w,t)+d(w,t)] + a_4[d(w,w)+d(t,t)] + a_5 d(w,t)$$

$$\Rightarrow (1-2a_3+a_5)\, d(w,t) \leq 0$$

as $2a_3 + a_5 < 1$

$\Rightarrow |d(w,t)| \leq 0$, which is a contradiction. Thus, $w = t$. Hence $At = Bt = St = Tt = t$, and t is the unique common fixed point of A, B, S and T.

Corollary: 3.2 Let (X,d) be a Complex valued metric space and $A, T: X \to X$ self-mappings satisfying the following conditions:

(i) $A(X) \subseteq T(X)$;

(ii) for all $x, y \in X$,

$$d(Ax,Ay) \leq a_1 \frac{d(Ty,Ay)[d(Ax,Ty)+d(Tx,Ax)]}{[1+d(Tx,Ty)+d(Ax,Ty]} + a_2 \frac{d(Ax,Ty)d(Tx,Ay)[d(Tx,Ax)+d(Ty,Ay)]}{[1+d(Tx,Ty)+d(Ax,Ty)]}$$
$$+a_3[d(Ax,Ty)+d(Sx,By)] + a_4[d(Sx,Ax)+d(Ty,By)] + a_5 d(Sx,Ty), \quad (3.2)$$

where $2a_3 + a_4 + a_5 < 1$ and $a_1, a_2, a_3, a_4, a_5 > 0$;

(iii) the pairs (A,T) is weakly compatible;

(iv) the pair (A,T) satisfies (E.A.)-property.

If the range of the mapping $T(X)$ is a closed subspace of X.Then the mappings A and T have a unique common fixed point in X.

Theorem: 3.3 Let (X,d) be a Complex valued metric space and $A, B, S, T: X \to X$ four self-mappings satisfying the following conditions:

(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$;

(ii) for all $x, y \in X$,

$$d(Ax,By) \leq a_1 \frac{d(Ty,By)[d(Ax,Ty)+d(Sx,Ax)]}{[1+d(Sx,Ty)+d(Ax,Ty]} + a_2 \frac{d(Ax,Ty)d(Sx,By)[d(Sx,Ax)+d(Ty,By)]}{[1+d(Sx,Ty)+d(Ax,Ty)]}$$

$$+a_3[d(Ax,Ty)+d(Sx,By)]+a_4[d(Sx,Ax)+d(Ty,By)]+a_5d(Sx,Ty) \quad (3.3)$$

where $2a_3+a_4+a_5<1$ and $a_1,a_2,a_3,a_4,a_5>0$;

(iii) the pairs (A,S) and $(B,T)$are weakly compatible.

If the pair (A,S) satisfies $(\mathrm{CL}R_A)$ property or (B,T) satisfies $(\mathrm{CL}R_B)$ property, then the mappings A,B,S and T have a unique common fixed point in X.

Proof: First, we suppose that the pair (B,T) satisfies $(\mathrm{CL}R_B)$ property then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = Bx$, for some $x\in X$.

Further, since $B(X)\subseteq S(X)$,we have $Bx=Su$, for some $u\in X$.

we claim that $Au=Su=t$ (say). put $x=u, y=x_n$ in (3.3), and we have

$$d(Au,Bx_n)\le a_1\frac{d(Tx_n,Bx_n)[d(Au,Tx_n)+d(Su,Au)]}{[1+d(Su,Tx_n)+d(Au,Tx_n)]}+a_2\frac{d(Au,Tx_n)d(Su,Bx_n)[d(Su,Au)+d(Tx_n,Bx_n)]}{[1+d(Su,Tx_n)+d(Au,Tx_n)]}$$

$$+a_3[d(Au,Tx_n)+d(Su,Bx_n)]+a_4[d(Su,Au)+d(Tx_n,Bx_n)]+a_5d(Su,Tx_n)$$

Letting $n\to\infty$ we have,

$$d(Au,Bx)\le a_1\frac{d(Bx,Bx)[d(Au,Bx)+d(Su,Au)]}{[1+d(Su,Bx)+d(Au,Bx)]}+a_2\frac{d(Au,Bx)\,d(Bx,Bx)[d(Su,Au)+d(Bx,Bx)]}{[1+d(Su,Bx)+d(Au,Bx)]}$$

$$+a_3[d(Au,Bx)+d(Su,Bx)]+a_4[d(Su,Au)+d(Bx,Bx)]+a_5d(Su,Bx)$$

$\Rightarrow [1-(a_3+a_4)]\,d(Au,Su)\le 0$,

as $a_3+a_4<1$

$\Rightarrow |d(Au,Bx)|\le 0$, which is a contradiction. Thus, $Au=Su$.

Hence, $Au=Su=Bx=t$.

Now the weak compatibility of pair (A,S) implies that $ASu=SAu$ or $At=St$.

Further, since $A(X)\subseteq T(X)$, there exist v in X such that $Au=Tv$. Thus, $Au=Su=Tv=t$. Now, we show that v is a coincidence point of (B,T) that is, $Bv=Tv=t$. Put $x=u, y=v$ in (3.3) and we have

$$d(Au,Bv)\le a_1\frac{d(Tv,Bv)[d(Au,Tv)+d(Su,Au)]}{[1+d(Su,Tv)+d(Au,Tv)]}+a_2\frac{d(Au,Tv)d(Su,Bv)[d(Su,Au)+d(Tv,Bv)]}{[1+d(Su,Tv)+d(Au,Tv)]}$$

$$+a_3[d(Au,Tv)+d(Su,Bv)]+a_4[d(Su,Au)+d(Tv,Bv)]+a_5d(Su,Tv)$$

or $$d(t,Bv)\le a_1\frac{d(t,Bv)[d(t,t)+d(t,t)]}{[1+d(t,t)+d(t,t)]}+a_2\frac{d(t,t)d(t,Bv)[d(t,t)+d(t,Bv)]}{[1+d(t,t)+d(t,t)]}$$

$$+a_3[d(t,t)+d(t,Bv)]+a_4[d(t,t)+d(t,Bv)]+a_5d(t,t)$$

$\Rightarrow [1-(a_3+a_4)]\,d(t,Bv)\le 0$,

as $a_3+a_4<1$

$\Rightarrow |d(t,Bv)|\le 0$, which is a contradiction. Thus $Bv=t$.

Hence, $Bv=Tv=t$ and v is coincidence point of B and T. Further, the weak compatibility of pair (B,T) implies that $BTv=TBv$, or $Bt=Tt$. Therefore, t is a common coincidence point of A,B,S and T.

Now, we show that t is a common fixed point. Put $x=u$ and $y=t$ in (3.3), and we have

$$d(t,Bt)=d(Au,Bt)\le a_1\frac{d(Tt,Bt)[d(Au,Tt)+d(Su,Au)]}{[1+d(Su,Tt)+d(Au,Tt)]}+a_2\frac{d(Au,Tt)d(Su,Bt)[d(Su,Au)+d(Tt,Bt)]}{[1+d(Su,Tt)+d(Au,Tt)]}$$

$$+a_3[d(Au,Tt)+d(Su,Bt)]+a_4[d(Su,Au)+d(Tt,Bt)]+a_5d(Su,Tt)$$

$\Rightarrow (1-2a_3+a_5)\,d(t,Bt)\le 0$,

as $2a_3+a_5<1$

$\Rightarrow |d(t,Bt)|\le 0$, which is a contradiction. Thus $Bt=t$.

Therefore $At=Bt=St=Tt=t$. The uniqueness of the common fixed point follows easily.

In a similar way, the argument that the pair (A,S) satisfies property $(\mathrm{CL}R_A)$ will also give the unique common fixed point of A,B,S and T. Hence the result follows.

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