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Numerical Solution of Higher Order Linear Fredholm – Integro – Differential Equations.

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ABSTRCT : *Power series and Chebyshev series approximation methods were used to solve higher order linear Fredholm integro – differential equations via two collocation points: Standard collocation point and Chebyshev – Guass – Lobatto collocation point. Numerical examples were given to demonstrate the effectiveness of the methods.*

KEY WORDS: *Fredholm integro differential equation, Power series, Chebyshev series, Standard collocation point, Chebyshev – Guass – Lobatto collocation point.*

I. INTRODUCTION:

In recent years considerable work has been done both in the development of the technique, its theoretical analysis and numerical application in the treatment of Integro – Differential equations, because of it wide range of applications in scientific field such as fluid dynamics, solid state physics, plasma physics and mathematical biology [2]. Integro – differential equations are classified into various types among which Fredholm – integro – differential equation, the focus of this paper.
Generally, Fredholm – integro – differential equation is of the form

$$u^{(j)}(x) = F(x) + \int_a^b k(x, t)\,u(t)\,dt \qquad (1)$$

$$u^{(k)}(0) = b_k, \quad 0 \le k \le n-1 \qquad (2)$$

Where $u^{(j)}(x)$ are the n[th] derivatives, $F(x)$, $k(x, t)$ are given continuous smooth functions, $u(x)$ is the unknown function to be determined and a, b, b_k are constants. Because the result of (1) combine the differential and integral operators, then it is necessary to define initial conditions as in (2). The Fredholm – integro – differential equation of the second kind appear in a variety of scientific application such as the theory of signal processing and neural networks [1]. Because of the importance of Fredholm – integro – differential equation in scientific research, several numerical methods were used to solve both linear and non – linear Fredholm – integro – differential equation such as Tau operational method [8], Haar wavelets method [5], Lagrange interpolation method [11] and Differential transformation method [3], just to mention but a few.[11] focused on the use of Chebyshev interpolation to solve mixed linear integro – differential equation with piecewise interval. Also in [12], Lagrange and Chebyshev interpolation was applied on functional integral equation. The use of inverse Fuzzy transforms based on fuzzy partition with combination in collocation techniques has been investigated (see [4]). Research has been conducted on the use of Legendre multi-wavelets to solve weakly singular Fredholm – integro – differential equations [7]. Power series method was use by [9] to solve system of linear and non-linear integro – differential equations and obtain a close form solution if the exact solutions are polynomial otherwise produces their Taylor series solution. Chebyshev series has been used to solve Fredholm integral equations at three different collocation points [6].In this paper we consider the use of power series and Chebyshev series approximation methods to solve higher order Fredholm – integro – differential equations using two collocation points.

II. POWER SERIES APPROXIMATION METHOD

In this section we consider the use of power series approximation solution of the form

$$u(x) = \sum_{i=0}^{N} a_i x^i, \quad a_i \, (i \geq 0) \tag{3}$$

Where the coefficients a_i are unknown to be determined.
Putting (3) in (1) gives,

$$\left(\sum_{i=0}^{N} a_i x^i \right)^{(j)} = F(x) + \int_a^b k(x, t) \left(\sum_{i=0}^{N} a_i x^i \right) dt \tag{4}$$

(4) can be written in a simpler form as

$$\left(a_0 + a_1 x + a_2 x^2 + \ldots \right)^{(j)} = F(x) + \int_a^b k(x, t) \left(a_0 + a_1 t + a_2 t^2 + \ldots \right) dt \tag{5}$$

We integrate the right hand side of (5) and after simplification, the resulting equation is then collocated using the following collocation points
(1)Standard collocation point defined as

$$x_p = a + \frac{(b - a)}{N} * p, \quad p = 1, 2, \ldots, N - 1. \tag{6}$$

(2)Chebyshev – Guass – Lobatto collocation point defined as

$$x_q = \cos \left(\frac{\pi}{N} \right) * q, \quad q = 1, 2, \ldots, N - 1. \tag{7}$$

Each of the two collocation points describe above together with the initial condition given in (2) resulted in (N + 1) linear algebraic equations in (N + 1) unknown constants which are then solved to obtain the unknown constants that are substituted in (3) to get the numerical solution of (1).

III. CHEBYSHEV SERIES APPROXIMATION SOLUTION:

In this section we consider the use of Chebyshev series approximation solution of the form

$$u(x) = \sum_{j=0}^{N} a_n T_n(x), \quad a_n \, (n \geq 0) \tag{8}$$

Where $T_n(x)$ is Chebyshev polynomial defined as

$$T_n(x) = \cos\left(n \cos^{-1} x\right), \quad x \, \varepsilon \, [-1, 1] \tag{9}$$

and it satisfied the recurrence relation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \qquad n \geq 1 \tag{10}$$

Putting (10) in (1), we obtain

$$\left(\sum_{j=0}^{N} a_n T_n(x) \right)^{(j)} = F(x) + \int_a^b k(x, t) \sum_{j=0}^{N} a_n T_n(t) \, dt \tag{11}$$

(11) can be written in a simpler form as

$$\left(a_0 + a_1 T_1(x) + a_2 T_2(x) + \ldots \right)^{(j)} = F(x) + \int_a^b k(x, t) \left(a_0 + a_1 T_1(t) + a_2 T_2(x) + \ldots \right) dt \tag{12}$$

Using the same procedure as in 2.0 above and using shift Chebyshev polynomial where applicable, (12) together with (2) gives (N + 1) linear algebraic equations in (N + 1) unknown constants. These equations are solved using maple 13 to obtain the unknown constants a_n's which are then substituted into (8) to get the numerical solution of (1).

IV. NUMERICAL EXAMPLES AND RESULTS:

In this section we consider the following examples on linear Fredholm - integro – differential equations. These examples have been chosen from [1].

Example 1:

$$u''(x) = 32x + \int_{-1}^{1} (1 - xt)\, u(t)\, dt$$

Subject to the conditions

$$u(0) = 1, \quad u'(0) = 1$$

The analytical solution is given as

$$u(x) = 1 + \frac{3}{2} * x^2 + 5 * x^3$$

Table 1: Numerical solution of example 1 for N = 10

x	Exact solution	Power series solution		Chebyshev series solution	
		Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000
0.1	1.020000000	1.019987500	1.019987508	1.019855160	1.019853296
0.2	1.100000000	1.099500000	1.099500027	1.097441281	1.097438523
0.3	1.270000000	1.269562500	1.269562557	1.267989324	1.267963120
0.4	1.560000000	1.559963000	1.550000098	1.558530249	1.558512781
0.5	2.000000000	1.999737500	1.995937649	1.944095018	1.984063738
0.6	2.620000000	2.619800000	2.617600207	2.595914590	2.606907820
0.7	3.450000000	3.449812500	3.447812774	3.437419929	3.347742003
0.8	4.520000000	4.518700000	4.509700350	4.509241993	4.418242167
0.9	5.860000000	5.858787500	5.849787937	5.678611744	5.759611462
1.0	7.500000000	7.497500000	7.489800353	7.395160143	7.399962085

Example 2:

$$u'''(x) = 1 - e + e^{x} + \int_{0}^{1} u(t)\, dt$$

Subject to the conditions

$$u(0) = u'(0) = u''(0) = 1$$

The analytical solution is give as

$$u(x) = e^{x}$$

Table 2: Numerical solution of example 2 for N = 10

x	Exact solution	Power series solution		Chebyshev series solution	
		Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	1.000000000	1.000000000	1.000000000	0.999999998	0.999999973
0.1	1.105170918	1.105170918	1.105170317	1.105170906	1.105170586
0.2	1.221402758	1.221402759	1.221393616	1.221402664	1.221402328
0.3	1.349858808	1.349858808	1.349796664	1.349858494	1.349853557
0.4	1.491824698	1.491824698	1.491779456	1.491824493	1.491822318
0.5	1.648721271	1.648721270	1.648687634	1.648720056	1.648681450
0.6	1.822118800	1.822118800	1.822091428	1.822117485	1.822097068
0.7	2.013752707	2.013752706	2.013736257	2.013751484	2.013735387
0.8	2.225540928	2.225540929	2.225110041	2.225542106	2.225528215
0.9	2.459603111	2.459603109	2.459401851	2.459602032	2.459591944
1.0	2.718281828	2.718281828	2.718165764	2.718304575	2.717964004

Example 3:

$$u^{(iv)}(x) = \frac{1}{4} + (1 - 2\ \text{in}\ (2)) * x - \frac{6}{(1+x)^4} + \int_0^1 (x - t)\, u(t)\, dt$$

Subject to the conditions

$u(0) = 0,\ u'(0) = 1,\ u''(0) = -1,\ u'''(0) = 2$

The analytical solution is given as

$u(x) = \text{In}\ (1 + x).$

Table 3: Numerical solution of example 3 for N = 10

x	Exact solution	Power series solution		Chebyshev series solution	
		Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	0	0	0	2.538281×10^{-11}	2.538237×10^{-11}
0.1	0.095310180	0.095309970	0.095308282	0.094929653	0.094900490
0.2	0.182321557	0.182320192	0.182285697	0.181952718	0.181969997
0.3	0.262364265	0.262341612	0.262329925	0.262019395	0.262013595
0.4	0.336472237	0.336448896	0.336437637	0.336129984	0.336131787
0.5	0.405465108	0.405437993	0.405430058	0.401196354	0.401908328
0.6	0.470003629	0.469975523	0.469967299	0.467508962	0.466601689
0.7	0.530628251	0.530595414	0.530091041	0.528493721	0.528138621
0.8	0.587786665	0.587608614	0.587374025	0.586233239	0.585401225
0.9	0.641853886	0.641684342	0.641502316	0.640367171	0.639716176
1.0	0.693147181	0.692912617	0.692874871	0.691938947	0.691122621

Tables of Errors:

Table 4: Errors for example 1

	Power series		Chebyshev series	
X	Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	0	0	0	0
0.1	1.250×10^{-5}	1.2492×10^{-5}	1.4484×10^{-4}	1.46704×10^{-4}
0.2	5.000×10^{-4}	4.99973×10^{-4}	2.558719×10^{-3}	2.561477×10^{-3}
0.3	4.375×10^{-4}	4.37443×10^{-4}	2.010676×10^{-3}	2.036880×10^{-3}
0.4	3.700×10^{-4}	9.999902×10^{-3}	1.469751×10^{-3}	1.487219×10^{-3}
0.5	2.625×10^{-4}	4.062351×10^{-3}	5.5904982×10^{-2}	1.5936262×10^{-2}
0.6	2.000×10^{-4}	2.399793×10^{-3}	2.4085410×10^{-2}	1.3092180×10^{-2}
0.7	1.875×10^{-4}	2.187226×10^{-3}	1.2580071×10^{-2}	1.02257997×10^{-1}
0.8	1.300×10^{-3}	1.0299650×10^{-2}	1.0758007×10^{-2}	1.01757833×10^{-1}
0.9	1.2125×10^{-3}	1.0212063×10^{-2}	1.81388256×10^{-1}	1.00388538×10^{-1}
1.0	2.5000×10^{-3}	1.01996468×10^{-2}	1.04839857×10^{-1}	1.00037915×10^{-1}

Table 5: Errors for example 2

	Power series		Chebyshev series	
X	Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	0	0	2.0×10^{-9}	2.7×10^{-8}
0.1	0	6.01×10^{-7}	1.2×10^{-8}	3.32×10^{-7}
0.2	1.0×10^{-9}	9.142×10^{-6}	9.4×10^{-8}	4.23×10^{-7}
0.3	0	6.2144×10^{-5}	3.14×10^{-7}	5.251×10^{-6}
0.4	0	4.5242×10^{-5}	2.05×10^{-7}	2.378×10^{-6}
0.5	1.0×10^{-9}	3.3637×10^{-5}	1.315×10^{-6}	3.9821×10^{-5}
0.6	0	2.7372×10^{-5}	1.297×10^{-6}	2.1732×10^{-5}
0.7	1.0×10^{-9}	1.6450×10^{-5}	1.223×10^{-6}	1.7320×10^{-5}
0.8	1.0×10^{-9}	4.30887×10^{-4}	1.178×10^{-6}	1.2713×10^{-5}
0.9	2.0×10^{-9}	2.01267×10^{-4}	1.079×10^{-6}	1.1167×10^{-5}
1.0	0	1.16064×10^{-4}	2.2747×10^{-5}	3.1782×10^{-4}

Table 6: Errors for example 3

	Power series		Chebyshev series	
X	Standard Collocation	C.G.L Collocation	Standard Collocation	C.G.L Collocation
0	0	0	2.58281×10^{-11}	2.538237×10^{-11}
0.1	2.1×10^{-7}	1.898×10^{-6}	3.80527×10^{-4}	4.0969×10^{-4}
0.2	1.365×10^{-6}	3.586×10^{-5}	3.68839×10^{-4}	3.5156×10^{-4}
0.3	2.2653×10^{-5}	3.434×10^{-5}	3.44870×10^{-4}	3.5067×10^{-4}
0.4	2.3341×10^{-5}	3.460×10^{-5}	3.42253×10^{-4}	3.4045×10^{-4}
0.5	2.7115×10^{-5}	3.505×10^{-5}	4.268754×10^{-3}	3.55678×10^{-3}
0.6	2.8371×10^{-5}	3.633×10^{-5}	2.494667×10^{-3}	3.40194×10^{-3}
0.7	3.2837×10^{-5}	5.3721×10^{-4}	2.134530×10^{-3}	2.48963×10^{-3}
0.8	1.78051×10^{-4}	4.1264×10^{-4}	1.553426×10^{-3}	2.38544×10^{-3}
0.9	1.69544×10^{-4}	3.5157×10^{-4}	1.486715×10^{-3}	2.13771×10^{-3}
1.0	2.34564×10^{-4}	2.7231×10^{-4}	1.208234×10^{-3}	2.02456×10^{-3}

V. CONCLUSION:

Most integro – differential equations are difficult to solve analytically, in many cases it require to obtain the approximate solutions, for this purpose we present the solution of higher order linear Fredholm integro – differential equations. Our methods are based on Power series and Chebyshev series which reduces a linear Fredholm integro – differential equation to a set of linear algebraic equations that can be easily solved by computer. The result obtained show that the two methods used can handle those problems effectively as can be seen in the tables of errors.

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