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Numerical solutions of second-orderdifferential equationsby Adam Bashforth method

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Abstract:- So far, many methods have been presented to solve the first-order differential equations. Not many studies have been conducted for numerical solution of high-order differential equations. In this research, we have applied Adam Bashforthmulti-step methods to approximate higher-order differential equations. To solve it, first we convert the equation to the first-order differential equation by order reduction method. Then we use a single-step method such as Euler, Taylor or Runge-Kutta for approximation of initially orders which are required to start Adam Bashforth method. Now we can use the proposed method to approximate rest of the points. Finally, we examine the accuracy of method by presenting examples.

Keywords*:-* high-order differential equations, Adams Bashforth multi-step methods, order reduction

I. INTRODUCTION

Differentialequations arevery useful indifferent sciencessuch asphysics, chemistry, biology and economy. To learn more aboutthe use of theseequationsin mentioned sciences, you can see theapplication of theseequationsinphysics[1,8,6,7,12], chemistry in [16,2,15], biology in [5,4] and economy in [10]. Considering that most of the time analyticsolution of such equations and finding an exact solution has either high complexity or cannot be solved, we applied numerical methods for the solution. Due to our subject which is solving the second-order differential equations, we will refer to some solution methods which have been proposed in recent years by other researchers to solve the equations. In 1993, Zhang presented a solution method for second-order boundary value problems [20]. In 2000, Yang introduced quasi-approximate periodic solutions for second-order neutral delay differential equations[17]. In 2003, Yang presented a method for solving second-order differential equations with almost periodic coefficients [18]. In 2005, Liu et alobtained periodic solutions for high-order delayedequations [9]. In 2005, Nieto and Lopez used Green's function to solve second-order differential equations which boundary value is periodic [11]. In 2006, Yang et al also applied Green's function to solve second-order differential equations [19]. In 2008, Pan obtained periodic solutions for high-order differential equations with deviated argument [13]. In 2011, Lopez used non-local boundary value problems for solving second-order functional differential equations [14]. It should be noted that most of these equations have piecewisearguments. Other parts of this paper are organized as follows. In the second part, we willreview therequired definitionsandbasic concepts. In the third section, we will propose the basic idea for solve insecond orderdifferential equations. In thefourth section, we will provideexamplesfor further explanation ofthe method and in fifth section; it will end with results of discussion.

II. REQUIRED DEFINITIONS AND BASIC CONCEPTS

Definition 2.1. [3]the general form of a second-order differential equation is as follows:

$$\frac{d^2y}{dt^2}+p(\mathrm{t})\frac{\mathrm{dy}}{\mathrm{dt}}+\mathrm{q(t)y}=\mathrm{R(t)} \qquad (3-1)$$

Or it can be more simply stated as below:

$$y''+p(t)y_0'+q(t)y=R(t)$$

In which, p (t), q (t) and R (t) are functions of t and without any reduction in totality, the coefficient of y'' is equal to 1 because also in another way, this coefficient will already convert to one for y'' with dividing by coefficient of y''.

Note: Ifine theequation (3.1) the value ofR (t) is 0, then the equationiscalledahomogeneousequation.

Definition 2.2. [3]we say that the function of $f(t,y)$ with variable of y on series of $D \subset R^2$is true in Lipshitz condition. If a fix such $L > 0$exists with this property that when

$(t,y_1),(t,y_2) \in D$,

(3-2) $$|f(t,y_1) - f(t,y_2)| \leq L|y_1 - y_2|$$

We consider the fix L as a Lipschitz fix for$f$.

Definition 2.3. [3]we say that the set of$D \subset R^2$is convex. If, whenever $(t_1,y_1),(t_2,y_2)$belongs to$D$, the point of $((1-\lambda)t_1 + \lambda t_2\ ,\ (1-\lambda)y_1 + \lambda y_2)$which $0 \leq \lambda \leq 1$belongs to$D$ per eachλ.

Theorem2.1.[3] Suppose that $f(t,y)$is described on a convex set of$D \subset R^2$. If a fix such$L > 0$ exists that per each$(t,y) \in D$,

$$|\frac{\partial f}{\partial y}(t,y)| \leq L$$

Then f according to the variable of y on$D$ in lipschitz condition is true with L fix lipschitz.

Theorem3.2. [3]suppose that $D = \{(t,y)a \leq t \leq b, -\infty < y < \infty\}$and $f(t,y)$ are contiguous on D. when f is true in lipschitz condition according to the variable of y on$D$, then problem of initial value of $y(a) = \alpha$, $a \leq t \leq b$ and $y' = f(t,y)$ have unique solution of $y(t)$ per $a \leq t \leq b$.

Definition 3.4. [3]a multi-steps technique to solve the problem of initial value.

(3-3) $$y' = f(t,y), \qquad a \leq t \leq b, \qquad y(a) = \alpha$$

It is a technique that its' differential equation is to find the approximation of w_{i+1} in the network point of t_{i+1} which can be shown by below equation in which m is an integer greater

$$w_{i+1} = a_{n-1}w_i + a_{m-2}w_{i-1} + \cdots + a_0 w_{i+1-m} + h[b_m f(t_{i+1}, w_{i+1}) + b_{m-1} f(t_i, w_i)$$
(3-4) $$+ \cdots + b_0 f(t_{i+1-m}, w_{i+1-m})]$$

Per each $i = m-1, m, \dots, N-1$ in which the initial values of

(3-5) $$w_0 = \alpha_0, w_1 = \alpha_1, w_2 = \alpha_2, \dots, w_{m-1} = \alpha_{m-1}$$

$$h = {(b-a)}/{N}$$

Are determined and as typical

When $b_m = 0$, we call it explicit or open method and in the equation (3-2) it gives the value of w_{i+1} explicitly based on predetermined values. When $b_m \neq 0$, we call it implicit or close method, because w_{i+1} appears in both sides of (3-4) and it can be determined only with an implicit method.

III. THE MAIN IDEAFOR SOLVINGSECOND-ORDERDIFFERENTIAL EQUATIONS

We consider the differential equations as following in which t is an independent variable and y is a dependant variable.

(4-1) $$\frac{d^2y}{dt^2} = f(t,y,\frac{dy}{dt})$$

Now to solve such equations, we act as follows:

(4.2) $$\frac{dy}{dt} = p$$

According to the equation (2-3), we convert the relation (1-3) to two first-order differential equation as follows:

(4.3) $$\frac{dy}{dt} = p = f_1(t,y,p)$$

(4.4) $$\frac{dp}{dt} = p = f_2(t,y,p)$$

Then we use Adam Bashforth's two-steps, three-steps and … method to solve the equations (4-3) and (4.4).

Suppose that the initial condition for (4-3) and (4-4) are given as below:

(4.5) $$y(t_0) = y_0,\ y'(t_0) = p(t_0) = p_0$$

Now, if we want to use n-step Adam Bashforth method for n= 2, 3, 4… in this case we should use n-1 initial step with a single-step method such as Euler, Taylor or Runge-Kutta. In this research, for example we describe the Runge-Kutta single-step methodto approximate the n-1 initial step as following:

(4.6) $y(t_m) = y_m,\ y'(t_m) = p(t_m) = p_m \qquad 0 < m < n$

Then we describe

(4.7) $$\begin{cases} k_1 = hf_1(t_m, w_m, v_m) \\ . \\ k_2 = hf_1(t_m + h, w_m + k_1, v_m + l_1) \end{cases}$$

(4.8) $$\begin{cases} k_2 = hf_2(t_m, w_m, v_m) \\ . \\ k_2 = hf_2(t_m + h, w_m + k_1, v_m + l_1) \end{cases}$$

Now suppose that we want to obtain the value of$v_{m+1}, w_{m+1}$by v_m, w_m, we have:

(4.9) $$\begin{cases} w_{m+1} = w_m + \frac{1}{2}(k_1 + k_2) \\ . \\ v_{m+1} = v_m + \frac{1}{2}(l_1 + l_2) \end{cases}$$

Now suppose that we are in n step that after this step we want to use Adam Bashforth multi-step method; general form of these methods is as below:

(4.10)

$$\begin{cases} w_{i+1} = a_{n-1}w_i + a_{n-2}w_{i-1} + \cdots + a_0 w_{i-(n-1)} + h(b_{n-1}f_1(t_i, w_i, v_i) \\ \qquad + \cdots + b_0 f_1\left(t_{i-(n-1)}, w_{i-(n-1)}, v_{i-(n-1)}\right) \\ . \\ w_0 = \alpha_0, w_1 = \alpha_1, \ldots, w_{n-1} = \alpha_{n-1} \end{cases}$$

That the relation (10-3) is determined per$i = n, n+1, \ldots, N$

And also

(4.11) $$\begin{cases} v_{i+1} = c_{n-1}v_i + c_{n-2}v_{i-1} + \cdots + c_0 v_{i-(n-1)} + h(b_{n-1}f_2(t_i, w_i v_i) \\ \qquad + \cdots + b_0 f_2\left(t_{i-(n-1)}, w_{i-(n-1)}, v_{i-(n-1)}\right) \\ . \\ v_0 = \beta_0, v_1 = \beta_1, \ldots, v_{n-1} = \beta_{n-1} \end{cases}$$

Now in continue we will discuss about the process of obtaining multi-step methods mentioned above briefly.

5. Numerical examplesand tables

In this section, the approximate solutions obtained from Adam Bashforth multi-step methods are compared with exact solution by using numerical example and we obtain their proximity rate to the exact solution.

Example: find the solution of the following second-order linear equation

$$y'' - 6y' + 9y = 0 \quad y(0) = 0,\ y'(0) = 2,\ t \in [0,1]$$

The exact solution is$y = 2te^{3t}$.

Note 1:we have shown m-step Adam Bashforth methods and their derivatives with AB'_m و AB_m respectively in below tables.

Note 2:in below tables, which we have used m-step Adam Bashforth methods for approximation, we have obtained previous m-1 step with a single-step method; here Euler and Runge-Kutta methods are applied to obtain previous m-1 step.

Table 1: in this table, the approximate solutions obtained from AB_4AB_2,, are compared with each other to approximate the true answer.

t	*y*	*AB_2*	*AB_4*	$\lvert y - AB_2\rvert$	$\lvert y - AB_4\rvert$
0	*0*	-	-	-	-
0.1	*0.2700*	-	-	-	-
0.2	*0.7288*	*0.5800*	-	*0.1488*	-
0.3	1*.4758*	*1.2015*	-	*0.2743*	-
0.4	*2.6561*	*2.1779*	*1.9920*	*0.4782*	*0.7341*
0.5	*4.4817*	*6.677*	*3.4089*	*0.8046*	*1.0728*
0.6	*7.2596*	*5.9408*	*5.6759*	*1.3181*	*1.5836*
0.7	*11.4326*	*9.3145*	*9.0902*	*2.1182*	*2.3424*
0.8	*17.6371*	*14.2897*	*14.2047*	*3.3474*	*3.4324*
0.9	*26.7835*	*21.5636*	*21.7309*	*5.2199*	*4.9934*
1	*40.1711*	*32.1216*	*32.9390*	*8.0495*	*7.2321*

Table 2:in this table, the approximate solutions obtained from AB'_4 و AB'_2 methods are compared witheach other to approximate true answer derivation which means y'.

t	Y	AB'_2	AB'_4	$\lvert y - AB'_2\rvert$	$\lvert y - AB'_4\rvert$
0	2	-	-	-	-
0.1	3.5096	-	-	-	-
0.2	5.8308	5.2100	-	0.6208	-
0.3	9.3465	8.2460	-	1.005	-
0.4	14.6085	12.7434	11.6656	1.865	2.9429
0.5	22.4084	19.3392	18.2125	3.0693	4.1959
0.6	33.8870	28.9374	27.8049	4.9406	6.0732
0.7	50.6303	42.8139	41.8019	7.8163	8.8284
0.8	74.9576	62.7640	62.2173	12.1936	12.7403
0.9	110.1100	91.3079	91.8201	18.8022	18.2899
1	160.6843	131.9753	134.5014	28.7090	26.1828

Again we solve the previous example with two-step and three-step methods except that in previous examples we selected Euler driver single-step method and tables 3 and 4 are Runge-Kuttadriver single-step method.

Table 3: in this table, the approximate solutions obtained from AB_4 و AB_2 methods are compared with eachother to approximate the true solution (Runge-Kutta second-order single step method is used).

T	Y	AB_2	AB_4	$\lvert y - AB_2\rvert$	$\lvert y - AB_4\rvert$
0	0	-	-	-	-
0.1	0.2700	-	-	-	-
0.2	0.7288	0.6805	-	0.0483	-
0.3	1.4758	1.3533	-	0.1225	-
0.4	2.6561	2.4028	2.4247	0.2533	**0.2314**
0.5	4.4817	4.0069	4.1485	0.4748	**0.3332**
0.6	7.2596	6.4207	6.7695	0.8389	**0.4900**
0.7	11.4326	10.0083	10.7052	1.4244	**0.7275**
0.8	17.6371	15.2872	16.5673	2.3499	**1.0698**
0.9	26.7835	22.9908	25.2212	3.7927	**1.5623**
1	40.1711	34.1549	37.9010	6.0162	**2.2701**

Table 4: in this table, the approximate solutions obtained from AB'_4 و AB'_2 are compared with each other for derived approximation of true solution (Runge-Kutta second-order single step method is used).

T	y	AB'_2	AB'_4	$\lvert y - AB'_2\rvert$	$\lvert y - AB'_4\rvert$
0	2	-	-	-	-
0.1	3.5096	-	-	-	-
0.2	5.8308	5.6420	-	0.1888	-
0.3	9.3465	8.8771	-	0.4694	-
0.4	14.6085	13.6532	13.7373	0.9554	**0.8712**
0.5	22.4084	20.6432	21.1749	0.7653	**0.2336**
0.6	33.8780	30.7981	32.0877	3.0800	**1.7904**
0.7	50.6303	45.4586	48.0042	5.1717	**2.6260**
0.8	74.9576	66.5100	71.1368	8.4476	**3.8208**
0.9	110.1100	96.5974	104.5831	13.5126	**5.5269**
1	160.6843	139.4237	152.7439	21.2606	**7.9404**

IV. CONCLUSION

In this research, we used Adams Bashforth multi-step methods to solve high-order differential equations. In the example solved, tables 1 and 2 respectively are showing the comparison of approximate solutions obtained from multi-step methods and their derivatives with true answer and derivatives of true answer. Tables 3 and 4 are as well as tables 1 and 2 except that in tables 1 and 2, Euler driver single-step method is used and in tables 3 and 4, Runge-Kutta deriver single-step method is used. It should be noted that the accuracy of Runge-Kutta driver method is more than the Euler driver method. However, in Adam Bashforth method, as the number of steps increase, the greater accuracy will be obtained.

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