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An order level EOQ model for deteriorating items in a single warehouse system with price depended demand and shortages.

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Abstract: - In this paper a deterministic inventory model is developed deterioration rate is time proportional. Demand rate is a function of selling price. Deterioration rate, inventory holding cost and ordering cost are all of function of time. The planning horizon is infinite. The optimum replacement policy and decision rule, which minimizes the total cost, is developed. In this study an order level inventory model for deteriorating items with single warehouse is developed where shortages are taken into consideration and it is completely backlogged. The results are illustrated with the help of numerical example. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.

Keywords: - *Deteriorating items; shortages; price dependent demand; time varying holding cost.*

I. INTRODUCTION

In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. One of the most important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergo decay or deterioration. Deterioration is defined as change, damage, decay, spoilage obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one. It is well known that certain products such as vegetable, medicine, gasoline, blood and radioactive chemicals decrease under deterioration during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored.

In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time dependent, stock dependent and price dependent. Selling price plays an important role in inventory system Burwell, [1] developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by Mondal, et. al [2]. You, [3] developed an inventory model with price and time dependent demand.

In most models, holding cost is known and constant. But holding cost may not always be constant. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like Naddor, [4], Van der Veen, [5], Muhlemann and Valtis Spanopoulos, [6], Weiss, [7], and Goh, [8]. In this present paper, I have developed a generalized EOQ model for deteriorating items in a single Warehouse system and demand rate is a function of selling price. Shortages are allowed here and are completely backlogged.

The aim of the paper is to develop an EOQ (Economic Order Quantity) model for a single-item inventory having a price-varying demand. Inventory modelers have so far considered only two types of price dependent demands, linear and exponential. Linear price-dependence of demand implies a uniform change in the demand rate of the product per unit price. This is rarely seen to occur in the real market. On the other hand, an exponentially price-varying demand also seems to be unrealistic because an exponential rate of change is very high and it is doubtful whether the market demand of any product may undergo such a high rate of change as exponential. In the opinion of the authors, an alternative (and perhaps more realistic) approach is to consider selling price dependence of demand. A brief review of the literature dealing with price-varying demands is made in the following paragraphs.

In formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate with time. Silver and Meal [9] developed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. The classical no-shortage inventory problem for a linear trend in demand over a finite time-horizon was analytically solved by Donaldson [10]. However, Donaldson's solution procedure was computationally complicated. Silver [11] derived a heuristic for the special case of a positive, linear trend in demand and applied it to the problem of Donaldson. Ritchie [12] obtained an exact solution, having the simplicity of the EOQ formula, for Donaldson's problem for a linear, increasing demand. Mitra et al [13] presented a simple procedure for adjusting the economic order quantity model for the case of increasing or decreasing linear trend in demand. The possibilities of shortage and deterioration in inventory were left out of consideration in all the models. Dave and Patel [14] developed an inventory model for deteriorating items with time- proportional demand. This model was extended by Sachan [15] to cover the backlogging option. Bahari-Kashani [16] discussed a heuristic model for obtaining order quantities when demand is time-proportional and inventory deteriorates at a constant rate over time. Deb and Chaudhuri [17] studied the inventory replenishment policy for items having a deterministic demand pattern with a linear (positive) trend and shortages; they developed a heuristic to determine the decision rule for selecting the times and sizes of replenishment over a finite time-horizon so as to keep the total costs minimum. This work was extended by Murdeshwar [18]. Subsequent contributions in this direction came from researchers like Goyal ([19],[20]), Dave [21], Hariga [22], Goswami and Chaudhuri [23], Xu and Wang [24], Chung and Ting ([25],[26]), Kim [27], Hariga ([28],[29]), Jalan, Giri and Chaudhuri [30], Jalan and Chaudhuri [31], Giri and Chaudhuri [32], Lin, Tan and Lee [33], etc. The assumption of the constant deterioration rate was relaxed by Covert and Philip [34] who used a two-parameter Weibull distribution to represent the distribution of time to deterioration. This model was further generalized by Philip [35] by taking a three-parameter Weibull distribution. Misra [36] also adopted a two-parameter Weibull distribution deterioration to develop an inventory model with a finite rate of replenishment.
These investigations were followed by several researchers like Shah and Jaiswal [37], Aggarwal [38], Roy-Chowdhury and Chaudhuri [39], etc. Recently Wee [40] and Jalan and Chaudhuri [41] and Chakrabarti and Choudhury [42] worked with an exponentially time-varying demand. In the present paper, we assume a generalized EOQ model for weibull deteriorating items in a single Warehouse system and demand rate is a function of selling price. Shortages are allowed here and are completely backlogged. An analytical solution of the model is discussed and it is illustrated with the help of a numerical example. Sensitivity of the optimal solution with respect to changes in different parameter values is also examined.

II. ASSUMPTIONS & NOTATIONS

The mathematical model in this paper was developed based on the following

Assumptions:

(i) The scheduling period was prescribed constant and no-supply lead time.

(ii) Demand rate is price depending and is of the form $D(p) = ap^{-b}$, a, b >0, p-- is the selling price.

(iii) Shortages are allowed and backlogging.

(iv) The rate of deterioration at any time t>0 follow the two-parameter Weibull distribution as $\theta=\alpha\beta t^{(\beta-1)}$, where α ($0<\alpha<1$) is the scale Parameter and β (>0) is the shape parameter.

(v) Deterioration of the units is considered only after they have been received Into the inventory.

(vi) At the beginning of every period the initial stock level is raised to order level.

Notations:

The following **notation** was used throughout the paper.

(i) T: Scheduling Period.

(ii) D(p) = Demand rate.

(iii) θ: Deteriorating rate.

(iv) Q(t): Inventory level at time t.

(v) S_1: Maximum Shortage level

(vi) S: Initial stock level at the beginning of every inventory.

(vii) Cs: Shortage cost.

(viii) C_2: Shortage cost per unit.

(ix) C_H: Holding Cost.

(x) C_1: Holding cost per unit.

(xi) D_θ : Total-deteriorating items.

(xii) C_d : Deteriorating cost per unit.

(xiii) TIC: Total Inventory Cost.

The Model Description and Analysis:

The initial stock was S at time t = 0, then inventory level decreases mainly due to meet up demands and partly from deterioration. By this process the stock reaches zero level at $t = t_1$. Now shortages occur and accumulate to the level S_1, at t = T.

The differential equations describing the state of Q(t) in the interval (0,T) are given by,

$$\frac{dQ(t)}{dt} + \alpha\,\beta t^{(\beta-1)}\,Q(t) = -a\,p^{-b} \qquad , 0 \le t \le t_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{dQ(t)}{dt} = -a\,p^{-b} \qquad , t_1 \le t \le T \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

With the boundary condition $Q(0)=S$, $Q(t_1)=0$ and $Q(T)= -S_1$
Solving above differential equations using boundary conditions we get,

$$Q(t) = \left[S - ap^{-b}\left(t + \frac{\alpha}{\beta+1}\, t^{\beta+1} \right)\right] e^{-\alpha\, t^{\beta}} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

And $\quad Q(t) = a\,(t_1 - t)\,p^{-b} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$

At $t=t_1$,

$$S = a\,p^{-b}\left(t_1 + \frac{\alpha}{\beta+1}\, t^{\beta+1} \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

At $t=T$,

$S_1 = a\,(T-t_1)\,p^{-b}$, since $Q(t)=-S_1$ …………. (6)

Total deteriorating units during the (0,T) are following,

$$D_\theta = \int_0^{t_1} \theta\, Q(t)\, dt$$

$$= S\left(\alpha t_1^{\beta} - \frac{\alpha^2}{2} t_1^{2\beta} \right) - a\,p^{-b}\left[\frac{\alpha\beta}{\beta+1} t_1^{\beta+1} - \frac{\alpha^2\,\beta}{(\beta+1)(2\beta+1)} t_1^{2\beta+1} - \frac{\alpha^3\,\beta}{(\beta+1)(3\beta+1)}\, t_1^{3\beta+1} \right] \ldots\ldots (7)$$

The deteriorating cost is given by,

$C_d = c_d\, D_\theta$

$$= c_d\, S\left(\alpha t_1^{\beta} - \frac{\alpha^2}{2} t_1^{2\beta} \right) - a\,p^{-b}\left[\frac{\alpha\beta}{\beta+1} t_1^{\beta+1} - \frac{\alpha^2\,\beta}{(\beta+1)(2\beta+1)} t_1^{2\beta+1} - \frac{\alpha^3\,\beta}{(\beta+1)(3\beta+1)}\, t_1^{3\beta+1} \right] \ldots\ldots (8)$$

Carrying cost over the period (0,T) is given by,

$$C_H = c_1 \int_0^{t_1} Q(t)\, dt$$

$$= c_1\, S\left(t_1 - \frac{\alpha}{\beta+1} t_1^{\beta+1} \right) - a\,c_1\, p^{-b}\left[\frac{t_1}{2} - \frac{\alpha\ \beta}{(\beta+1)(\beta+2)} t_1^{\beta+2} - \frac{\alpha^2}{(\beta+1)(2\beta+2)} t_1^{2\beta+2} \right] \ldots. (9)$$

Shortage cost is given by

$$C_s = c_2\left[-\int_{t_1}^{T} Q(t)\, dt \right]$$

$$= a\,c_2\, p^{-b}\left(\frac{T^2 + t_1^2}{2} - T\, t_1 \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Hence the Total Inventory Cost is given by,

$TIC = C_d + C_H + C_s$

$$= c_d\, S\left(\alpha t_1^{\beta} - \frac{\alpha^2}{2} t_1^{2\beta} \right) - a\,p^{-b}\left[\frac{\alpha\beta}{\beta+1} t_1^{\beta+1} - \frac{\alpha^2\,\beta}{(\beta+1)(2\beta+1)} t_1^{2\beta+1} - \frac{\alpha^3\,\beta}{(\beta+1)(3\beta+1)}\, t_1^{3\beta+1} \right]$$

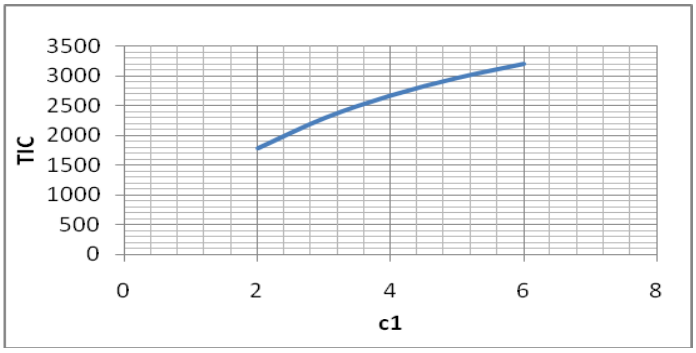
$$+ \quad c_1\, S\left(t_1 - \frac{\alpha}{\beta+1} t_1^{\beta+1} \right) - a\,c_1\, p^{-b}\left[\frac{t_1}{2} - \frac{\alpha\ \beta}{(\beta+1)(\beta+2)} t_1^{\beta+2} - \frac{\alpha^2}{(\beta+1)(2\beta+2)} t_1^{2\beta+2} \right]$$

$$+ \ a\,c_2\, p^{-b}\left(\frac{T^2 + t_1^2}{2} - T\, t_1 \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

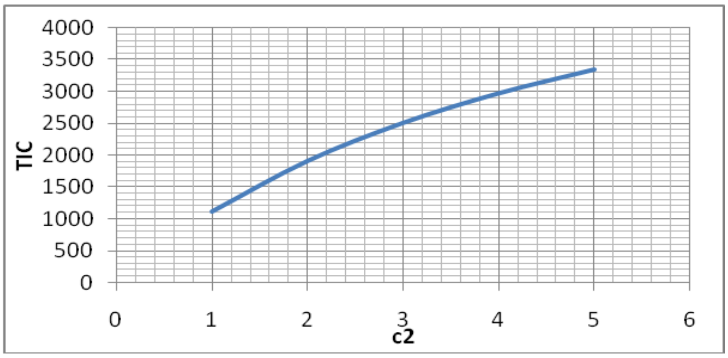
Numerical Examples:
To illustrate the order level model developed here, we consider an inventory system with the following hypothetical values.
We Consider a=10, b=1, α=0.005, β=0.4, p=6.00, C_1=5.00, C_2=4.00, C_d=2.00
Then we obtained S=29.88595~30units
And TIC=2968.41

Sensitivity Analysis:

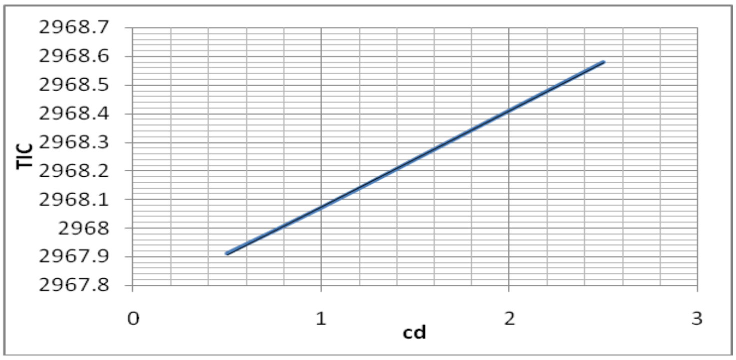
Change Value		S	t_1	TIC
C_1	2	44.95	26.62	1783.99
	3	38.48	22.81	2291.89
	4	33.64	19.95	2672.52
	5	29.89	17.73	2968.41
	6	26.88	15.96	3205.04
C_2	1	11.16	6.65	1111.74
	2	19.17	11.39	1906.79
	3	25.19	14.96	2503.71
	4	29.88	17.73	2968.41
	5	33.64	19.95	3340.45
C_d	0.5	29.88	17.73	2967.91
	1	29.88	17.73	2968.07
	1.5	29.88	17.73	2968.24
	2	29.88	17.73	2968.41
	2.5	29.88	17.73	2968.58
T	25	18.65	11.09	1159.31
	30	22.39	13.30	1669.52
	35	26.14	15.52	2272.55
	40	29.88	17.73	2968.41
	45	33.64	19.95	3757.12
p	3	59.77	17.73	5936.82
	4	44.83	17.73	4452.62
	5	35.86	17.73	3562.09
	6	29.88	17.73	2968.41
	7	25.62	17.73	2544.35



Graphically change of C_1 - TIC



Graphically change of C_2 - TIC



Graphically change of C_d - TIC

Observations:

From the above table we observe the following observations:

(1) Increase in Shortage cost (C_2) per unit, Total Inventory Cost also increases.

(2) Increase in Holding cost (C_1) per unit, Total Inventory Cost also increases

(3) Increase in Deteriorating Cost (C_d) per unit, Total Inventory Cost and decreasing rate. Here also in this case there is a minor increase in Total cost, which is negligible. From 0.91 to 1.58 there is slide change in total cost.

(4) Increase in time Period (T), Total Inventory Cost also increases.

(5) If there is a change in Price (P), Total Inventory Cost decreases.

III. CONCLUSION

In this present paper, I have developed a generalized EOQ model for deteriorating items. The principle features of the model are as follows:

The deterministic demand rate is assumed to be a function of selling price. Selling price is the main criterion of the consumer when he/she goes to the market to buy a particular item.

Shortages are allowed and are completely backlogged in the present model. In many practical situations, stock out is unavoidable due to various uncertainties. There are many situations in which the profit of the stored item is high than its backorder cost. Consideration of shortages is economically desirable in these cases.

The deterioration factor has been taken into consideration in the present model as almost all items undergo either direct spoilage (like fruits, vegetables etc.) or physical decay (in case of radioactive substances, volatile liquids etc.) in the course of time, deterioration is a natural feature in inventory system. There are many items like perfumes, photographic films etc. which incur a gradual loss of potential or quality over time.

In future, researchers can do more work about several types of demand, variable costs etc.

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