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Quasi-Static transient Thermal Stresses in a Neumann's thin Solid cylinder with internal moving heat source

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Abstract: - This paper concern with transient non-homogeneous thermoelastic problem with Neumann's boundary condition in thin solid cylinder of isotropic material of radius a and height h, occupying the space $0 \le r \le a, 0 \le z \le h, 0 \le \phi \le 2\pi$ having initial temperature $f(r,\phi,z)$ placed in an ambient temperature zero. The cylinder is subjected to the activity of moving heat source along circular trajectory of radius r_0, where $0 < r_0 < a$, around the centre of the cylinder with constant angular velocity ω. Heat conduction equation containing heat generation term is solved by applying integral transform technique and Green's theorem is adopted in deducing the solution of heat conduction equation. The solution is obtained in a series form of Bessel function and trigonometric function and derived thermal stresses.

Keywards: *- Neumann's solid thin cylinder, moving heat source, thermal stresses, Green's theorem.*

I.　　INTRODUCTION

During the second half of 20th century, non-isothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of aircraft structure.

No one previously studied such type of thermal stresses with moving heat source in thin solid cylinder with Neumann's boundary condition. This is new contribution to the field of thermoelasticity. Recently D. T. solanke ,S. M. Durge have studied the thermal stresses, in thin solid cylinder with Dirichlet's boundary condition [4]. In this present paper we determine temperature, thermal stresses, in a Neumann's thin solid cylinder, determined by $0 \le r \le a, 0 \le z \le h, 0 \le \phi \le 2\pi$ with internal moving heat source. Heat conduction equation with heat generation term is solved by applying integral transform technique and Green's theorem. Solution is obtained in series form of Bessel function and trigonometric function. The direct problems is very important in view of its relevance to various industrial mechanics subjected to heating such as main shaft of lathe, turbines and the role of rolling mill for base of furnace boiler of thermal power plant, gas power plant and measurement of aerodynamic heating.

II.　　FORMULATION OF THE PROBLEM

Consider a thin solid cylinder of isotropic material of radius a and height h, occupying the space $0 \le r \le a, 0 \le z \le h, 0 \le \phi \le 2\pi$ having initial temperature $f(r,\phi,z)$ placed in an ambient temperature zero. The cylinder is subjected to the activity of moving heat source which changes its place along circular trajectory of radius r_0, where $0 < r_0 < a$, around the centre of the cylinder with constant angular velocity ω. The activity of moving heat source and its initial temperature of the cylinder may cause the generation of heat due to nuclear interaction that may be a function of position and time in the form $g(r,\phi,z,t)$. The temperature distribution of thin solid cylinder is described by the differential equation of heat heat conduction with heat generation term as in [3] on page no. 232 is given by

$$\nabla^2 T + \frac{1}{k}g = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where ∇^2 is Laplacian operator in cylinder, $T = T(r,\phi,z)$ is temperature distribution, k is thermal conductivity of the material of the cylinder, $\alpha = \dfrac{k}{\rho c_p}$ is thermal diffusivity, ρ is density, c_p is specific heat of the material and g is volumetric energy generation in the cylinder.

Now consider an instantaneously moving heat source $g_s^{\,i}$ located at a point (r_0,ϕ',ξ) and releasing its energy spontaneously at time τ. Such volumetric heat source in cylindrical coordinates is given by

$$g(r,\phi,z,t) = g_s^{\,i}\frac{1}{2\pi r}\delta(r-r_0)\delta(\phi-\phi')\delta(z-\xi)\delta(t-\tau)$$

Hence above equation reduces to

$$\nabla^2 T + g_s^{\,i}\frac{1}{2\pi r}\delta(r-r_0)\delta(\phi-\phi')\delta(z-\xi)\delta(t-\tau) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(3.1)

$$\phi' = \omega t$$

(3.2)

With initial and boundary condition

$$\frac{\partial T}{\partial r} = 0 \qquad\qquad\qquad \text{at} \qquad\qquad\qquad r = a$$

(3.3)

$$\frac{\partial T}{\partial z} = 0 \qquad \text{at} \qquad z = 0$$

(3.4)

$$\frac{\partial T}{\partial z} = 0 \qquad \text{at} \qquad z = h$$

(3.5)

$$T = f(r,\phi,z) \qquad \text{at} \qquad t = 0$$

(3.6)

4. Formulation of thermoelastic Problem:

Let us introduce a thermal stress function χ related to component of stress in the cylindrical coordinates system as in [3]

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \chi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \chi}{\partial r^2}$$

(4.1)

$$\sigma_{\phi\phi} = \frac{\partial^2 \chi}{\partial r^2}$$

(4.2)

$$\sigma_{r\phi} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \chi}{\partial \phi}\right)$$

(4.3)

The boundary condition for a traction free body are

$$\sigma_{rr} = 0, \sigma_{r\phi} = 0 \quad \text{at} \quad r = a \tag{4.4}$$

Where $\chi = \chi_c + \chi_p$ (4.5)

Where χ_c is complementary solution and χ_p is particular solution and

χ_c satisfies the equation $\nabla^4 \chi_c = 0$ (4.6)

χ_p satisfies the equation $\nabla^4 \chi_p = -\lambda E \nabla^2 \Gamma$ (4.7)

Where Γ is temperature change and $\Gamma = T - T_i$ T_i is initial temperature

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} \qquad \text{since cylinder is thin, z component is negligible}$$

5.Solution:

We define integral transform of T by

$$\overline{T}(\beta_m, v, \alpha_n) = \int\limits_{z=0}^{h} \int\limits_{\phi=0}^{2\pi} \int\limits_{r=0}^{a} T(r, \phi, z, t) r J_v(\beta_m r) \cos v(\phi - \phi') \cos \alpha_n z \, dr \, d\phi \, dz$$

(5.1)

$$T(r, \phi, z) = \frac{4}{\pi h} \sum_{m,v,n=0}^{\infty} \overline{T} \frac{\beta_m^{2} Jv(\beta_m r) \cos v(\phi - \phi') \cos \alpha_n z}{(\beta_m^{2} a^2 - v^2) J_v^{2}(\beta_m a)}$$

(5.2)

Where β_m is roots of transcendental equation $J_{v+1}(\beta_m a) = 0$ (5.3)

$v = 0, 1, 2, 3, 4, ...$

(5.4)

$$\alpha_n = \frac{p\pi}{h}, \qquad p = 0, 1, 2, 3, 4, ...$$

(5.5)

By taking integral transform of equation (3.1) and using following Green's theorem

$$\int\limits_{R} \nabla^2 T \psi_k \, dv = \int\limits_{R} T\nabla^2 \psi_k \, dv + \sum_{i=1}^{N} \int\limits_{s_i} \left[\psi_k \frac{\partial T}{\partial n_i} - T \frac{\partial \psi_k}{\partial n_i} \right] ds_i$$

(5.6)

Which yield as

$$\frac{d\overline{T}}{dt} + \alpha(\beta_m^{2} + \alpha_n^{2})\overline{T} = \frac{\alpha}{2\pi k} g_s^{i} Jv(\beta_m r_0) \cos(\alpha_n \xi) \delta(t - \tau)$$

(5.7)

This is linear differential equation of first order whose solution by applying initial condition (3.6) is

$$\overline{T} = \left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{i} J_v(\beta_m r_0) \sin \alpha_n (\xi + \tfrac{h}{2}) e^{\alpha(\beta_m^{2} + \alpha_n^{2})\tau} \right] e^{-\alpha(\beta_m^{2} + \alpha_n^{2})t}$$

(5.8)

Taking inverse integral Transform we obtain

$$T = \frac{4}{\pi h} \sum_{m,n,v=0}^{\infty} \frac{\beta_m^{2} J_v(\beta_m r) \cos v(\phi - \phi') \cos(\alpha_n z)}{(\beta_m^{2} a^2 - v^2) J_v^{2}(\beta_m a)} \left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{i} J_v(\beta_m r_0) \cos(\alpha_n \xi) e^{\alpha(\beta_m^{2} + \alpha_n^{2})\tau} \right] \cdot$$

$$e^{-\alpha(\beta_m^{2} + \alpha_n^{2})t}$$

(5.9)

$$\Gamma = \frac{4}{\pi h} \sum_{m,n,v=0}^{\infty} \frac{\beta_m^{2} J_v(\beta_m r) \cos v(\phi - \phi') \cos(\alpha_n z)}{(\beta_m^{2} a^2 - v^2) J_v^{2}(\beta_m a)} \left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{i} J_v(\beta_m r_0) \cos(\alpha_n \xi) e^{\alpha(\beta_m^{2} + \alpha_n^{2})\tau} \right] \cdot$$

$$[e^{-\alpha(\beta_m^{2} + \alpha_n^{2})t} - 1]$$

(5.10)

III.   SOLUTION OF THERMOELASTIC PROBLEM

Let suitable form of χ_c satisfying (4.6) is

$$\chi_c = (Ar^{v+2} + Br^{-v+2})\cos v(\phi - \phi') + (Cr^{v+2} + Dr^{-v+2})\sin v(\phi - \phi')$$

(6.1)

Let suitable form of χ_p satisfying (4.7) is

$$\chi_p = \frac{4\lambda E}{\pi h} \sum_{m,n,v=0}^{\infty} \frac{J_v(\beta_m r)\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].$$
$$[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.2)

$$\chi = \chi_c + \chi_p$$

$$\chi = \sum_{v=0}^{\infty}(Ar^{v+2} + Br^{-v+2})\cos v(\phi-\phi') + (Cr^{v+2} + Dr^{-v+2})\sin v(\phi-\phi') +$$
$$\frac{4\lambda E}{\pi h}\sum_{m,n,v=0}^{\infty}\frac{J_v(\beta_m r)\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].$$
$$[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.3)

$$\sigma_{rr} = \sum_{v=0}^{\infty}\left[A(2+v-v^2)r^v + B(2-v-v^2)r^{-v}\right]\cos v(\phi-\phi') + \left[C(2+v-v^2)r^v + D(2-v-v^2)r-v\right]\sin(\phi-\phi') +$$
$$\sum_{m,v,n=0}^{\infty}\frac{4\lambda E}{\pi h}\left[\frac{\beta_m}{r}J_{v-1}(\beta_m r) - \frac{v+v^2}{r^2}J_v(\beta_m r)\right]\frac{\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}.$$
$$\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.4)

$$\sigma_{\phi\phi} = \sum_{v=0}^{\infty}\left[A(v^2+3v+2)r^v + B(v^2-3v+2)r^{-v}\right]\cos v(\phi-\phi') + \left[C(v^2+3v+2)r^v + D(v^2-3v+2)r^{-v}\right]\sin v(\phi-\phi') +$$
$$\sum_{m,v,n=0}^{\infty}\frac{4\lambda E}{\pi h}\left[\frac{\beta_m}{r}J_{v+1}(\beta_m r) - \beta_m^2 J_v(\beta_m r)\right]\frac{\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}.$$
$$\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.5)

$$\sigma_{r\phi} = \sum_{v=0}^{\infty}\left[A(v^2+v)r^v + B(v-v^2)r^{-v}\right]\sin(\phi-\phi') + \left[C(-v^2-v)r^v + D(v^2-v)r^{-v}\right]\cos(\phi-\phi') +$$
$$\sum_{m,v,n=0}^{\infty}\frac{4\lambda Ev}{\pi h r^2}\left[\beta_m rJ_{v-1}(\beta_m r) - (1+v)J_v(\beta_m r)\right]\frac{\sin v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}.$$
$$\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.6)

Applying condition (4.4) to (6.4) and (6.6) we obtain

$$A = \frac{4\lambda E}{\pi h}\frac{a^{-v-2}}{2v}\left[2J_v(\beta_m a) - \beta_m aJ_{v-1}(\beta_m a)\right]\frac{\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}$$
$$\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s^{\ i} J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m^2+\alpha_n^2)\tau}\right].[e^{-\alpha(\beta_m^2+\alpha_n^2)t} - 1]$$

(6.7)

$$B = \frac{4\lambda E}{\pi h}a^{v-2}\left[\left(\frac{1+v-v^2-v^3}{v^3-v}\right)J_v(\beta_m a) + \frac{1}{2v}\beta_m aJ_{v-1}(\beta_m a)\right]\frac{\cos(\alpha_n z)}{(\beta_m^2 a^2 - v^2)J_v^2(\beta_m a)}$$

$$\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1]$$

(6.8)

$$C = D = 0$$

(6.9)

Substituting above value in (6.4), (6.5) and (6.6) we obtain

$$\sigma_{rr} = \sum_{m,v,n=0}^{\infty} \frac{4\lambda E}{\pi h}\frac{\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m{}^2 a^2 - v^2)J_v{}^2(\beta_m a)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1].$$

$$\left\{\begin{array}{l}(2+v-v^2)\dfrac{a^{-v-2}r^v}{2v}\left[2J_v(\beta_m a)-\beta_m a J_{v-1}(\beta_m a)\right]+(2-v-v^2)a^{v-2}r^{-v}\left[\left(\dfrac{1+v-v^2-v^3}{v^3-v}\right)J_v(\beta_m a)+\dfrac{1}{2v}\beta_m a J_{v-1}(\beta_m a)\right]+ \\[2mm] +\left[\dfrac{\beta m}{r}J_{v-1}(\beta_m r)-\dfrac{v+v^2}{r^2}J_v(\beta_m r)\right]\end{array}\right\}$$

(6.10)

$$\sigma_{\phi\phi} = \sum_{m,v,n=0}^{\infty} \frac{4\lambda E}{\pi h}\frac{\cos v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m{}^2 a^2 - v^2)J_v{}^2(\beta_m a)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1].$$

$$\left\{\begin{array}{l}(v^2+3v+2)\dfrac{a^{-v-2}r^v}{2v}\left[2J_v(\beta_m a)-\beta_m a J_{v-1}(\beta_m a)\right]+(v^2-3v+2)a^{v-2}r^{-v}\left[\left(\dfrac{1+v-v^2-v^3}{v^3-v}\right)J_v(\beta_m a)+\dfrac{1}{2v}\beta_m a J_{v-1}(\beta_m a)\right]+ \\[2mm] +\left[\dfrac{\beta m}{r}J_{v+1}(\beta_m r)-\dfrac{v+v^2}{r^2}J_v(\beta_m r)\right]\end{array}\right\}$$

(6.11)

$$\sigma_{r\phi} = \sum_{m,v,n=0}^{\infty} \frac{4\lambda E}{\pi h}\frac{\sin v(\phi-\phi')\cos(\alpha_n z)}{(\beta_m{}^2 a^2 - v^2)J_v{}^2(\beta_m a)}\left[\overline{f}(\beta_m, v, \alpha_n) + \frac{\alpha}{2\pi k} g_s{}^i J_v(\beta_m r_0)\cos(\alpha_n \xi)e^{\alpha(\beta_m{}^2+\alpha_n{}^2)\tau}\right].[e^{-\alpha(\beta_m{}^2+\alpha_n{}^2)t}-1].$$

$$\left\{\begin{array}{l}(v+1)\dfrac{a^{-v-2}r^v}{2}\left[2J_v(\beta_m a)-\beta_m a J_{v-1}(\beta_m a)\right]+(1-v)a^{v-2}r^{-v}\left[\left(\dfrac{1+v-v^2-v^3}{v^2-1}\right)J_v(\beta_m a)+\dfrac{1}{2}\beta_m a J_{v-1}(\beta_m a)\right]+ \\[2mm] +\left[v\beta_m r J_{v-1}(\beta_m r)-(v+v^2)J_v(\beta_m r)\right]\end{array}\right\}$$

(6.12)

IV. CONCLUSION

In this paper we determined the temperature distribution and thermal stresses in a Neumann's thin solid cylinder with moving heat source with analytical approach on the surface is established. By giving particular values to the parameter one can obtain their desired results by putting values of the parameters in the equations (5.9), (6.10), (6.11), (6.12)

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