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Theoretical Study Of The Interaction Of Cavitation Bubbles With The Interface "Liquid-Gas" Determining Optimum Modes Of Ultrasonic Effect To Increase The Surface Of The Phase Contact

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ABSTRACT: *The article describes the model of the interaction of the cavitation area formed upon influence of ultrasonic vibrations with the interface of gas and liquid phases. In the system "liquid-gas" studied in the frameworks of the model liquid spreads on some solid surface in the form of the film and it is in contact with gas medium. It is shown, that the interaction of cavitation bubbles with the interface of liquid and gas leads to the generation capillary waves and consequently to the increase of the surface of phase contact. The model analysis allows determining the modes of ultrasonic effect, which are necessary for maximum enlargement of interphase boundary area. It leads in turn to the increase of the rate of physic-chemical processes based on the surface interaction of dissimilar substances (absorption of gas mixtures both for cleaning and for separation of specific-purpose components, drying, wet cleaning of gases from dispersed admixtures, etc.). As a result of model analysis we determined threshold vibration amplitudes of solid surface covered with the film of liquid phase, which excess resulted in stability failure of capillary waves and their decomposition to liquid drops. It was shown, that the most efficient frequency of ultrasonic effect was 60 kHz, at which phase contact surface increased in more than 3 times.*

KEYWORDS : *Absorption, capillary waves, cavitation, liquid, ultrasonic*

I. INTRODUCTION

The rate of the most of physicochemical processes is limited by the interface of interacted substances or phases and also by the rate of agent introducing to this boundary. The most part of such processes occurs in two-phase system "liquid-gas". For instance in the systems "liquid-gas" following process such as wet gas cleaning from different dispersed admixtures, absorption of gas mixtures both for their cleaning and for separation of specific-purpose components, drying of the materials and others can be realized. It is evident, that for maximum efficiency of mentioned above processes first of all it is necessary to provide large area of contact surface of liquid and gas phases. In existing chemical engineering apparatuses (absorbers, wet dust collectors, dryers) specific interface area (for the mass unit) required for the industrial realization of physic-chemical processes at the interface can be achieved by the following ways:

a. liquid is sprayed during gas phase in the form of small drops;
b. liquid spreads on the surface of the solids as a film (the thickness is no more than 5 mm) and contacts gas medium.

The first variant has limited application, as for its realization there is a need in good reciprocal solubility of interacted phases (for instance, solubility of gas in the absorbent). In this paper we mainly consider the second variant.However the second method is characterized by insufficient interface area for industrial realization of physic-chemical processes, which is required higher power inputs. One of the promising method of the increase of phase contact surface is the influence by microscopic shock waves leading to the generation of the profile disturbance of interface "liquid-gas" (capillary waves) of small length (no more than 200 µm). The appearance of shock waves can be provided due to the generation of periodically expanding and collapsing cavitation bubbles in liquid phase.

It is known, that the most advantageous method [1, 2] of creation of cavitation bubbles is the introduction ultrasonic vibrations into liquid phase with the frequency of 20…60 kHz. Ultrasonic influence can be carried out by excitation of mechanical vibrations of the solid surface, on which liquid film spreads. It is necessary to develop theoretical model allowing to determine optimum modes of ultra-sonic effect (amplitude and vibration frequency of the solid surface), which provides maximum interface area “liquid-gas”. There is a need to study in details the interaction of shock waves of cavitation bubbles with the surface of phase contact.

For a long time the development of the theories of the interaction of shock waves of cavitation bubbles with the interface had some mathematical difficulties, as there was no correct solution of the equations of the hydrodynamics of supersonic liquid flow streaming cavitation bubble. At the beginning of 20th century foreign scientists (B.E. Nolting, E.A. Neppiras, H.G. Flynn, J.G. Kirkwood, H.A. Bethe) [3-5] gave basic theoretical descriptions of the growth and pulsation of the cavitation cavity (bubble). These descriptions are equations of radial vibrations of the bubble, which take into account possible factors influencing the dynamics of the cavitation cavity including compressibility of liquid and change of its wave properties at the supersonic flow. These equations are non-linear differential equations of second order relative to the radius of the bubble, which is a function on time. It was stated, that the bubble retained its spherical form during the cycle of expanding and collapsing, and it was assumed, that shock wave had spherically divergent character. Such assumption does not allow explaining experimentally observed the generation of capillary waves of small length (no more than 200 μm) at the interface “liquid-gas”.

However, as it was mentioned above in these processes, liquid spread on solid surface as a film. The thickness of the film does not exceed 5 mm [6], solid surface reflects shock waves. Reflecting phenomena break the sphericity of cavitation bubbles at their collapse [7]. This sphericity failure narrows the diagram of shock wave directivity, and this fact explains the generation of capillary waves of small length (no more than 200 μm). Stated factor should be taken into consideration at theoretical studies of the interaction of cavitation bubbles with the interface “liquid-gas”. Thus, the aim of the paper is to develop the mathematical model of the interaction of cavitation bubbles with the interface “liquid-gas” for the determination of the modes of ultrasonic effect providing maximum surface area of phase contact. The model includes following stages of the generation of capillary waves on the interface “liquid-gas” under the action of ultrasonic cavitation:

[1] expansion of cavitation bubble up to maximum radius, which is spherically symmetric due to the low speed of its walls (no more than 15 m/s);
[2] asymmetric collapse of the cavitation bubble from maximum radius to minimum size;
[3] generation and propagation of narrow directional shock wave in the thin liquid film at the collapse of the cavitation bubble;
[4] formation of capillary waves on interface “liquid-gas”. In this stage capillary waves profile is determined and square of the interface “liquid-gas” is calculated.

Further proposed model is described.

II. MODEL OF INTERACTION BETWEEN CAVITATION BUBBLES

Theoretical study of the process is carried out according to the scheme shown in Fig. 1.

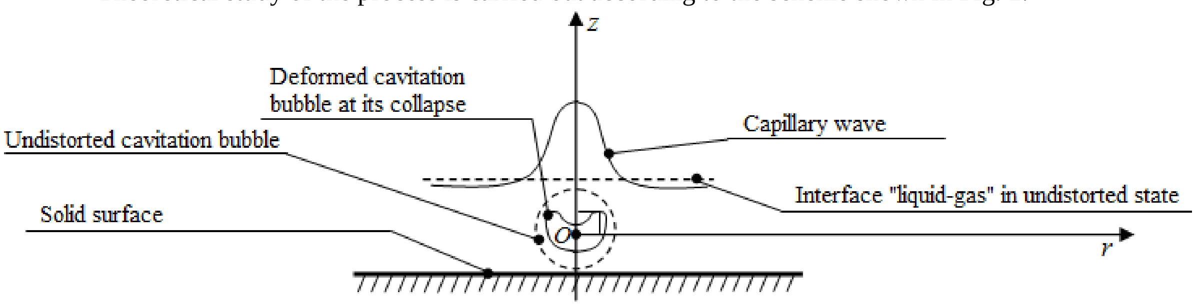


Fig. 1. Scheme of theoretical study of the interaction between cavitation bubbles and interface “liquid-gas”

At the *stage of cavitation bubble expansion* its maximum radius R_{MAX} and center z location relative to the solid surface are determined. At this stage it is assumed that:

[1] expansion of the bubble is spherically symmetric, which is caused by low speed of walls motion, however the bubble center vertically moves relative to the solid surface in the course of time;
[2] in initial time the center of the cavitation bubble is located near the solid surface, as such bubbles mostly influence on the formation of capillary wave.

Maximum radius of the bubble R_{MAX} is defined on the base of Nolting-Neppiras equation [3]:

$$\rho\left(\frac{3}{2}\left(\frac{\partial R}{\partial t}\right)^2+R\frac{\partial^2 R}{\partial t^2}\right)=-4\mu\frac{\frac{\partial R}{\partial t}}{R}+p_V+\left(p_0+\frac{2\sigma}{R_0}\right)\left(\frac{R_0}{R}\right)^{3\gamma}-p_0+4\pi^2 f^2\rho Ah\sin(2\pi ft)$$

where R is the instantaneous radius of the cavitation bubble, m; R_0 is the radius of cavitation nucleus, m; σ is the liquid surface tension, N/m; ρ is the density of liquid, kg/m^3; p_0 is the static pressure in liquid, Pa; f is the frequency of ultrasonic action, Hz; h is the thickness of liquid film, m; A is the amplitude of ultrasonic action, m; p_V is the pressure of saturated vapor of liquid, Pa; t is the time, s; μ is the dynamic viscosity of liquid, Pa·s.

The distance between the center of the cavitation bubble (at the moment of maximum expansion) and the solid surface is defined from the equation given in Rozhdestvenskiy's paper [8]:

$$6b^2\frac{\partial b}{\partial t}\frac{\partial R}{\partial t}+2b^2R\frac{\partial^2 b}{\partial t^2}+3R^2\left(\frac{\partial R}{\partial t}\right)^2=0$$

where b is the distance between the center of the cavitation bubble and the solid surface, m.

Obtained values of maximum bubble radius and the distance between its center and the solid surface are used for theoretical studies of further stages of the capillary wave formation.
During the study of the stage of cavitation bubble collapse its form in the moment of the minimum size is determined.
The form of the cavitation bubble is defined from the integral equation (1) with boundary conditions (2, 3) on the wall of the cavitation bubble for liquid velocity potential and entry conditions (4, 5) on cavitation bubble wall:

$$\frac{\varphi(\mathbf{r}_0)}{2}=\int\limits_{S_A\cup S_B}\left(E_{\mathbf{r}_0}V_{\mathbf{n}}-\frac{\partial E_{\mathbf{r}_0}}{\partial \mathbf{n}}\varphi\right)\partial S \qquad (1)$$

$$\frac{\partial\varphi}{\partial t}+\frac{|V_{\mathbf{n}}|^2+|V_{\boldsymbol{\tau}}|^2}{2}=\frac{2\sigma K}{\rho}-\frac{p_{\mathbf{n}}}{\rho}\left(\frac{3V}{4\pi R_{MAX}{}^3}\right)^{\gamma} \qquad (2)$$

$$\nabla\varphi=\frac{\partial\mathbf{r}}{\partial t} \qquad (3)$$

$$\varphi\big|_{t=0}=0 \qquad (4)$$

$$\|\mathbf{r}\|\big|_{t=0}=R_{MAX} \qquad (5)$$

where $\mathbf{r}_0$, $\mathbf{r}$ are the vectors of the coordinates of the points of the wall of the cavitation bubble or solid surface, m; φ is the fluid velocity potential on the wall of the cavitation bubble or solid surface, m^2/s; $V_{\mathbf{n}}$ and $V_{\boldsymbol{\tau}}$ are the normal and tangential components of fluid velocity, m/s; $E_{r_0}(\mathbf{r})$ is the fundamental solution of Laplace's equation; V is the volume of the cavitation bubble, m^3; p_n is the pressure of saturated vapor of fluid, ρ and σ are the density (kg/m^3) and surface tension (N/m) of fluid, respectively; K is the mean curvature of the walls of the cavitation bubble, m^{-1}; S_A is the wall of the cavitation bubble; S_B is the solid surface on which $V_{\boldsymbol{n}}$ is equal 0.

With the help of system of equations (1-5) we calculate deformation of the walls of the cavitation bubble in the course of time. Entry conditions (3-5) being a part of the system (1-5) is determined by the bubble radius and the position of its center at the moment of maximum expansion, which were found at the previous stage of the model study. Integral equation (1) aimed at the determination of distribution of fluid velocity potential on the walls of the cavitation bubble is solved by the boundary element method. For this purpose the discretization of the cavitation bubble wall into ring elements is carried out, as it is shown in Fig. 2.

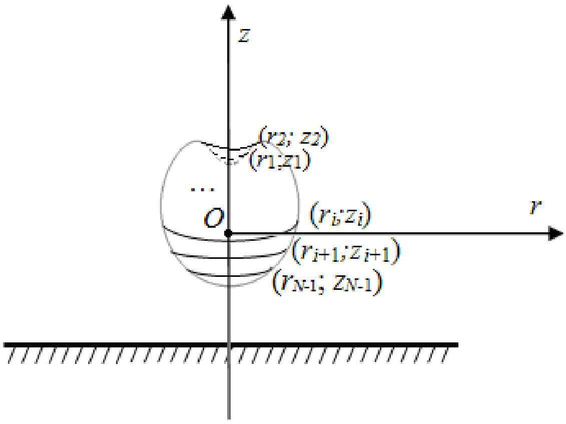


Fig. 2. Discretization of the cavitation bubble wall into ring boundary elements

It is assumed, that in the frameworks of each ring element the velocity potential is constant. It allows solving boundary integral equation (1) as a system of linear equations (6). This system is obtained by using method of “images” (replacing solid surface by symmetrically placed cavitation bubble).

$$\{A_{ij}\}_{i,j=1\ldots 2N}\begin{Bmatrix} V_n^{(1)} \\ V_n^{(2)} \\ \ldots \\ V_n^{(N-1)} \\ V_n^{(N)} \\ V_n^{(1)} \\ V_n^{(2)} \\ \ldots \\ V_n^{(N-1)} \\ V_n^{(N)} \end{Bmatrix} = \{b_i\}_{i=1\ldots 2N} \tag{6}$$

where $\{A_{ij}\}$ is the matrix of linear system; $V_n^{(i)}$ is normal velocity on i-th bubble wall boundary item with coordinates $(r_i;\ z_i)$ and $(r_{i+1};\ z_{i+1})$; $\{b_i\}$ is right part of system; N is count of boundary items;

The coefficients of the system of linear equations (A_{ij} and b_i) are defined by the following obtained expressions:

$$\begin{aligned} b_{i+pN} &= \frac{\varphi_{\max(i-1;1)}+\varphi_{\min(i,N-1)}}{4}+ \\ &+\sum_{q=0}^{1}\sum_{j=1}^{N}\frac{\varphi_{\max(j-1;1)}+\varphi_{\min(j,N-1)}}{2} J\left(\left(\begin{matrix} r_{j-1} \\ (1-2q)z_{j-1}-2qb_0\end{matrix}\right),\left(\begin{matrix} r_j \\ (1-2q)z_j-2qb_0\end{matrix}\right),\right. \\ &\left.\frac{\left(\begin{matrix} r_i+r_{i-1} \\ (1-2p)(z_i+z_{i-1})-2pb_0\end{matrix}\right)}{2},\frac{\left(\begin{matrix} z_j-z_{j-1} \\ (1-2q)(r_j-r_{j-1})\end{matrix}\right)}{\sqrt{(r_j-r_{j-1})^2+(z_j-z_{j-1})^2}}\right) \end{aligned} \tag{7}$$

$$\begin{aligned} \{A_{(i+pN)(j+qN)}\} &= \delta_{ij}\delta_{pq}I_0\left(\left(\begin{matrix} r_{j-1} \\ (1-2q)z_{j-1}-2qb_0\end{matrix}\right),\left(\begin{matrix} r_j \\ (1-2q)z_j-2qb_0\end{matrix}\right)\right)+(1-\delta_{ij}\delta_{pq})\times \\ &\times I\left(\left(\begin{matrix} r_{j-1} \\ (1-2q)z_{j-1}-2qb_0\end{matrix}\right),\left(\begin{matrix} r_j \\ (1-2q)z_j-2qb_0\end{matrix}\right),\frac{1}{2}\left(\begin{matrix} r_i+r_{i-1} \\ (1-2p)(z_i+z_{i-1})-2pb_0\end{matrix}\right)\right) \end{aligned} \tag{8}$$

where I_0, J, I are integrals over on each boundary item; b_0 is starting distance (at maximum bubble expansion) between bubble center and solid surface, m.

In expressions (7-8) it is mentioned that following equalities are true:

$$\begin{pmatrix} r_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} r_1 \\ z_1 \end{pmatrix}$$

$$\begin{pmatrix} r_N \\ z_N \end{pmatrix} = \begin{pmatrix} r_{N-1} \\ z_{N-1} \end{pmatrix}$$

Integrals I_0, J, I are defined as follows (9-11):

$$\begin{aligned} J(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_0,\mathbf{n}) &= v.p.\int_0^{2\pi}\left(\left[r_1 - r_0\cos\varphi\right]n_r + (z_1 - z_0)n_z\right)\times \\ &\times\left(-\frac{r_2-r_1}{4\pi l^2}\frac{1}{\sqrt{t^2 - \frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})^2}{l^4} + \frac{l_1^2 + l_0^2 - 2(\mathbf{r}_1,\mathbf{l}_\varphi)}{l^2}}}\Bigg|_{t=\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}}^{t=1+\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}} + \right. \\ &+\frac{r_1 - \frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}(r_2-r_1)}{4\pi l^2\left(-\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})^2}{l^4} + \frac{l_1^2+l_0^2-2(\mathbf{r}_1,\mathbf{l}_\varphi)}{l^2}\right)}\times \\ &\left.\times\frac{t}{\sqrt{t^2 - \frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})^2}{l^4} + \frac{l_1^2+l_0^2-2(\mathbf{r}_1,\mathbf{l}_\varphi)}{l^2}}}\Bigg|_{t=\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}}^{t=1+\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}}\right)\partial\varphi \end{aligned} \qquad (9)$$

$$\begin{aligned} I(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_0) &= -\frac{1}{4\pi}\int_0^{2\pi}\left(\ln\left|\frac{\sqrt{l^2 + 2(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l}) + l_1^2 + l_0^2 - 2(\mathbf{r}_1,\mathbf{l}_\varphi)} + l + \left(\mathbf{r}_1-\mathbf{l}_\varphi,\frac{\mathbf{l}}{l}\right)}{\sqrt{l_1^2 + l_0^2 - 2(\mathbf{r}_1,\mathbf{l}_\varphi)} + \left(\mathbf{r}_1-\mathbf{l}_\varphi,\frac{\mathbf{l}}{l}\right)}\right|\times\right. \\ &\times\left(r_1 - (r_2-r_1)\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2}\right) + \\ &\left.+(r_2-r_1)\left(\sqrt{1 + 2\frac{(\mathbf{r}_1-\mathbf{l}_\varphi,\mathbf{l})}{l^2} + \frac{l_1^2+l_0^2-2(\mathbf{r}_1,\mathbf{l}_\varphi)}{l^2}} - \sqrt{\frac{l_1^2+l_0^2-2(\mathbf{r}_1,\mathbf{l}_\varphi)}{l^2}}\right)\right)\partial\varphi \end{aligned} \qquad (10)$$

$$\begin{aligned} I_0(\mathbf{r}_1,\mathbf{r}_2) &= -\frac{1}{2\pi}\lim_{\varepsilon\to 0+}\int_\varepsilon^\pi\left(\ln\left|\frac{\sqrt{l^2 + 2(\mathbf{r}_1-\mathbf{l}_{0\varphi},\mathbf{l}) + l_1^2 + \frac{(r_1+r_2)^2+(z_1+z_2)^2}{4} - 2(\mathbf{r}_1,\mathbf{l}_{0\varphi})} + l + \left(\mathbf{r}_1-\mathbf{l}_{0\varphi},\frac{\mathbf{l}}{l}\right)}{\sqrt{l_1^2 + \frac{(r_1+r_2)^2+(z_1+z_2)^2}{4} - 2(\mathbf{r}_1,\mathbf{l}_{0\varphi})} + \left(\mathbf{r}_1-\mathbf{l}_{0\varphi},\frac{\mathbf{l}}{l}\right)}\right|\times\right. \\ &\times\left(r_1 - (r_2-r_1)\frac{(\mathbf{r}_1-\mathbf{l}_{0\varphi},\mathbf{l})}{l^2}\right) + \\ &+(r_2-r_1)\left(\sqrt{2\frac{(\mathbf{r}_1-\mathbf{l}_{0\varphi},\mathbf{l})}{l^2} + \frac{l^2 + l_1^2 + \frac{(r_1+r_2)^2+(z_1+z_2)^2}{4} - 2(\mathbf{r}_1,\mathbf{l}_{0\varphi})}{l^2}} - \right. \\ &\left.\left.-\sqrt{\frac{l_1^2 + \frac{(r_1+r_2)^2+(z_1+z_2)^2}{4} - 2(\mathbf{r}_1,\mathbf{l}_{0\varphi})}{l^2}}\right)\right)\partial\varphi \end{aligned} \qquad (11)$$

where r_1, r_2, r_0 are vectors of coordinates $\mathbf{r}_1 = \begin{pmatrix} r_1 \\ z_1 \end{pmatrix}; \mathbf{r}_2 = \begin{pmatrix} r_2 \\ z_2 \end{pmatrix}; \mathbf{r}_0 = \begin{pmatrix} r_0 \\ z_0 \end{pmatrix}$; $\mathbf{n}$ is vector of normal $\mathbf{n} = \begin{pmatrix} n_r \\ n_z \end{pmatrix}$;

$\mathbf{l}_\varphi = \begin{pmatrix} r_0 \cos\varphi \\ z_0 \end{pmatrix}; \mathbf{l}_{0\varphi} = \frac{1}{2}\begin{pmatrix} (r_1 + r_2)\cos\varphi \\ z_1 + z_2 \end{pmatrix}$; $l = \sqrt{(r_2 - r_1)^2 + (z_2 - z_1)^2}$; $l_1 = \sqrt{r_1^{\ 2} + z_1^{\ 2}}$; $l_0 = \sqrt{r_0^{\ 2} + z_0^{\ 2}}$.

Obtained system of linear equations (6) is solved by iterative Seidel method.

Obtained forms of cavitation bubble walls (by equations (1-5) at the collapse in different moments of time are shown in Fig. 3. The initial moment of time (0 μs) is the moment of the maximum bubble expansion.

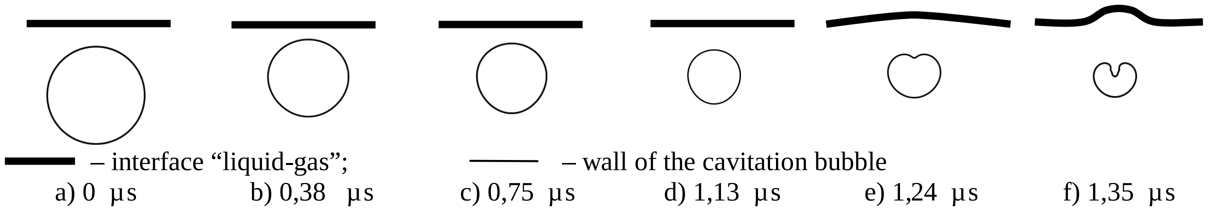


Fig. 3. Evolution of the form of asymmetrically collapsing cavitation bubble in the course of time at different initial distances (at the moment of maximum expansion) between its center and solid surface

As it is shown in Fig. 3, cavitation bubble is a hemispherical radiator of shock wave.

At the study of *the stages of generation and propagation of shock wave* it allows approximating its pressure profile at different distances from the bubble by the following obtained expression (12).

$$p(\mathbf{r},t) = \sum_{n=-\infty}^{\infty} \frac{\omega a^2}{2\pi\sqrt{r^2+z^2}} \mathrm{Re} \int_0^{\frac{\pi}{2}} \int_0^{\frac{2\pi}{\omega}} \Bigg[p_c(t_1) \sin\psi \times \\ \times e^{in\frac{\omega}{c}\left(c(t-t_1)+\sqrt{r^2+z^2}-\frac{\eta n\omega}{2\rho c^2}+\frac{za\cos\psi}{\sqrt{r^2+z^2}}\right)} J_0\left(n\frac{\omega r a \sin\psi}{c\sqrt{r^2+z^2}}\right) \partial t_1 \partial\psi \Bigg]; \quad (12)$$

where (r; z) are the coordinates of the points, m; ω is the circular vibration frequency of solid surface, s^{-1}; t and t_1 are the moments of time, s; η is the viscosity of liquid, Pa·s; ρ and c is the velocity of sound in liquid, m/s; $p_c(t_1)$ is the pressure in the nucleus of the cavitation bubble, Pa; a is the radius of the cavitation bubble at maximum pressure in the nucleus, m.

The function of shock wave pressure in the nucleus of the cavitation bubble $p_c(t_1)$ being a part of the expression (12) is defined as:

$$p_c(t_1) = p_V \left(\frac{4\pi R_{MAX}^{\ 3}}{3V} \right)^{\gamma};$$

p_V is pressure of saturated liquid vapor; R_{MAX} is bubble radius at maximum expansion; V is bubble volume at time t_1; γ is a adiabatic index of gas.

Given profile of shock wave pressure is used further for the definition of capillary wave form and finally interphase boundary area.

The form of the capillary wave is defined from the expression (13):

$$\xi(r,t) = -\frac{1}{\rho} \int_0^{t} \int_0^{t_2} \frac{\partial p}{\partial z} \partial t_1 \partial t_2 \quad (13)$$

where $\xi(r,t)$ is the value of displacement of the interface “liquid-gas” along the axis z.

Thus mathematical description presented above allows determining the profile of single capillary wave generated by separate bubble.

However at the realization of the technological process it is impossible to obtain separate bubble that is why it is necessary to consider the interaction between the aggregate of cavitation bubbles and the interface generating set of capillary waves.

The specific area of the interface "liquid-gas" per unit volume of liquid phase at the generation of the set of capillary waves is defined by the expression:

$$S = 2\pi\langle n\rangle \int_0^{0,5\lambda} r\sqrt{1+\left(\frac{\partial\xi}{\partial r}\right)^2}\,\partial r + \frac{1}{h}$$

where S is the specific area of the interface, m^2/m^3; λ is the length of the capillary wave (m) defined from the condition $\frac{\partial\xi}{\partial r}\left(\frac{\lambda}{2},t\right)=0$.; n is the concentration of cavitation bubbles, m^{-3}; < > is sign of averaging by liquid film thickness; h is thickness of liquid film, m.

The term $2\pi\langle n\rangle \int_0^{0,5\lambda} r\sqrt{1+\left(\frac{\partial\xi}{\partial r}\right)^2}\,\partial r$ characterizes a shock wave energy being generated at bubble collapse.

For the concentration of cavitation bubbles kinetic equation (14) obtained from Smolukhovskiy's equation [9] for the processes of coalescence and breakage of disperse particles (liquid drops, gas and solid particles) is true [10]:

$$\frac{\partial n}{\partial t} = \frac{n(j-1)}{iT_0} - k_B n^2 \tag{14}$$

where n is the calculating concentration of cavitation bubbles depending on time t, m^{-3}, i is the average number of cavitation bubble pulsation before its collapse, k_B is the constant of coalescence rate of the bubbles, m^3/s, T_0 is the period of ultrasonic vibrations, s, j is the mean amount of the nuclei generated at the breakage of the separate bubble.

By solving the equation (14) following analytic expression is obtained:

$$n = \frac{n_\infty n_0}{n_0 + (n_\infty - n_0)e^{-n_\infty k_B t}}; \tag{15}$$

where n_0 is the initial unknown concentration of cavitation bubbles, m^{-3}, n_∞ is the stationary concentration of cavitation bubbles, m^{-3}.

According to the expression (15) the concentration of the bubbles n in time, which equals tens periods of ultrasonic vibrations, achieves stable value and equals to n_∞, which is defined by the expression (16):

$$n_\infty = \frac{j-1}{ik_B T_0}; \tag{16}$$

Variable j being in expression for stable concentration (16) is calculated from experimental data given in Rozenberg's book [3].

The constant of coalescence is defined as follows:

$$k_B = \frac{S_{eff}\langle u\rangle}{2}$$

where S_{eff} is square of effective bubbles collision's cross-section which is proportional to R_{MAX}^2, m^2; <u> is approach velocity of the cavitation bubbles, m/s.

To define approach velocity of the cavitation bubbles <u> the model of bubble interaction caused by the forces of the second order is used.The interaction model is based on the 2nd Newton's Law for the separate cavitation bubble taking into consideration Bjerknes force acting from the neighbor bubbles and caused by radial vibrations of the last ones. According to this model the position of the center of each cavitation bubble making the ensemble can be described by the following equation [10]:

$$\frac{4\pi R_{0i}^{\ 3}}{3}\rho_G\frac{\partial^2\mathbf{r}_i}{\partial t^2}=\frac{4\pi R_i^{\ 3}}{3}\rho_L\frac{\partial\mathbf{v}_L}{\partial t}(\mathbf{r}_i,t)+\sum_{j=1,n,\,j\neq i}\frac{4\pi R_i^{\ 3}}{3\left|\mathbf{d}_{ij}\right|^3}\rho_L\frac{\partial\left(R_j^{\ 2}\frac{\partial R_j}{\partial t}\right)}{\partial t}\mathbf{d}_{ij}+ \qquad ; \qquad (17)$$

$$+\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{4\pi R_i^{\ 3}}{3}\rho_L\left(\mathbf{v}_L(\mathbf{r}_i,t)-\frac{\partial\mathbf{r}_i}{\partial t}\right)\right)+4\pi\eta R_i\left(\mathbf{v}_L(\mathbf{r}_i,t)-\frac{\partial\mathbf{r}_i}{\partial t}\right)$$

where i is the ordinal number of the bubble in zone of liquid phase; R_i is the instantaneous radius of i-th bubble, m; c is the local velocity of sound in liquid phase, m/s; p_{wi} is the gas pressure near the walls of i-th bubble, Pa; p is the instantaneous value of pressure of liquid phase without cavitation bubbles, Pa; ρ_L is the density of liquid phase, kg/m^3; v_L is the instantaneous vibrational speed of liquid phase without cavitation bubbles, m/s; R_{0i} is the radius of i-th bubble nucleus, m; ρ_G is the equilibrium density of gas inside the bubble, kg/m^3; t is the time, s; η is the viscosity of liquid phase, Pa·s; $\mathbf{r}_i$ is the coordinate vector of the center of i- bubble, м; $\mathbf{d}_{ij}=\mathbf{r}_j-\mathbf{r}_i$ is the vector of center line of i-th and j-th bubbles couple, m.

On the base of the results of equation solution (17) approach velocity of cavitation bubbles is defined by the following expression:

$$\langle u\rangle=\frac{\left|\mathbf{d}_{12}(T_0)-\mathbf{d}_{12}(0)\right|}{T_0}.$$

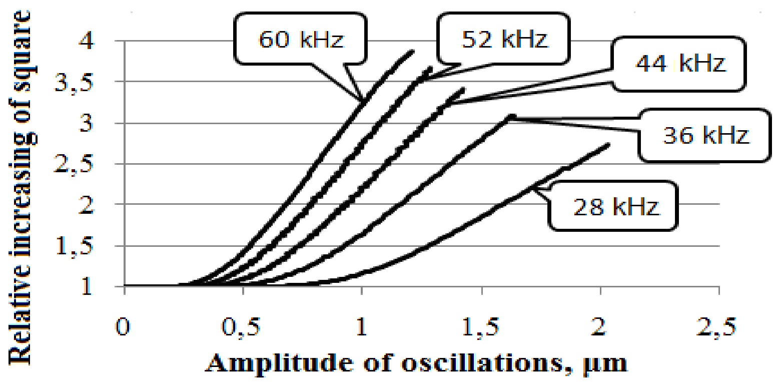
Thus proposed model allows defining dependence surface area of interphase boundary on the modes of ultrasonic action (frequency and vibration amplitude of solid surface covered with liquid film, which borders on gas phase) and liquid properties.

III. RESULTS AND DISCUSSION

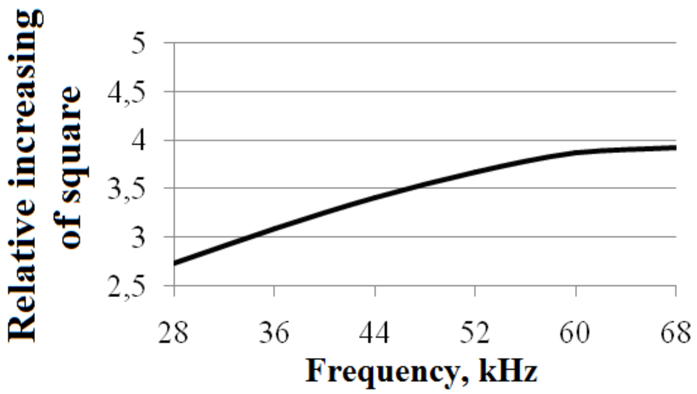
Obtained dependences of relative increase of the interface area on the modes of ultrasonic action are shown in Fig. 4.Relative increase K of interface area is the ratio of the interface area upon ultrasonic action (S_{US}) to the area without ultrasonic action ($S_{\mathrm{without\ US}}$):

$$K=\frac{S_{US}}{S_{Without\ US}}$$

Fig. 4 shows the breakage of the graph corresponds to the fact, that capillary wave loses its stability and breaks into drops [11]. The dependence of frequency (Fig 4b) is built up at threshold amplitudes, when capillary wave remains stable.



a)



b)

Fig. 4. Dependences of specific area of the interface on the modes of ultrasonic action: (a) on amplitude at different frequencies; (b) on frequency at maximum amplitude

From presented dependences it is evident, that with the increase of amplitude interface area grows. If frequency rises, surface area grows (up to more than 3 times) due to the increase of cavitation bubble concentration [3]. However starting with the frequency of 60 kHz the growth of the area essentially becomes slower, and energy loss of the ultrasonic radiator increases quadratically. That is why; the application of frequencies of more than 60 kHz is unpractical. Fig. 5 shows the dependence of threshold vibration amplitude, at which capillary wave remains stable, on the frequency of action.

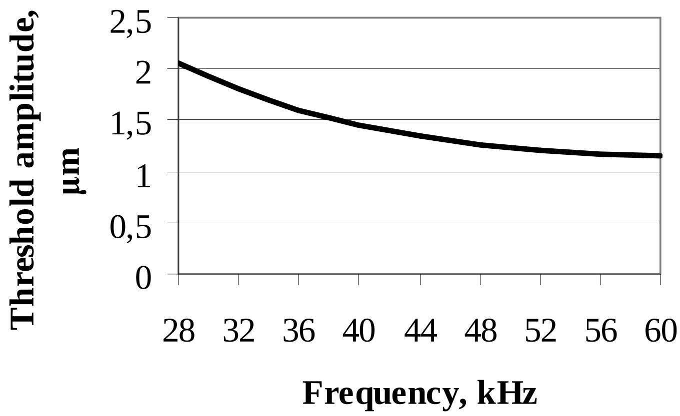


Fig. 5. Dependence of threshold amplitude, at which capillary wave remains stable, on the frequency of action

According to presented dependence the asymptotic amplitude reduces with the rise of frequency. In particular at the frequency of 28 kHz the threshold amplitude exceeds 2 μm, and at the frequency of 60 kHz it is 1…1.2 μm. Fig. 6 shows the dependences of specific interface area on amplitude at the change of physical properties of liquid – viscosity (a) and surface tension (b), which influence on the profile of contact surface together with the modes of ultrasonic action.

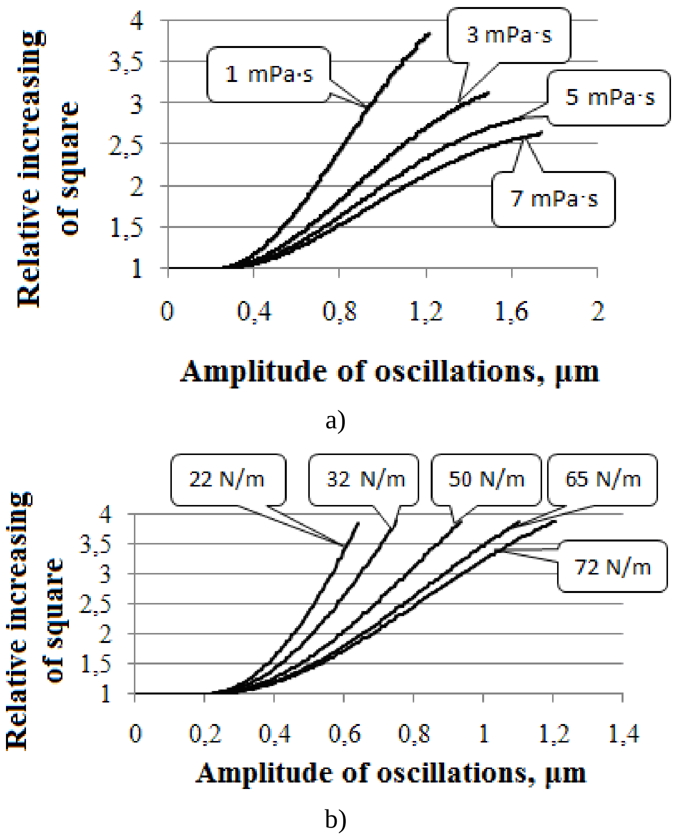


Fig. 6. Dependence of specific area of interphase boundary on amplitude at different properties of liquid (frequency of 60 kHz): viscosity (a) and surface tension (b)

Presented dependences (Fig. 6) can be used for the determination of the area change caused by change of the liquid type and change of its properties. In particular it is stated, that growth of viscosity leads to the decrease specific area of the interface. It is caused by the absorption of energy of shock waves in liquid phase due to forces of viscous friction. At that decrease of surface tension leads to the growth of the area, as surface energy of a liquid directly depends on its surface tension.

IV. CONCLUSION

Thus model of the interaction of cavitation zone generated under the action of ultrasonic vibrations with interface of gas and liquid phases is developed. It is shown, that this interaction leads to the generation of capillary wave and consequently to the growth of surface of phase con-tact. Analysis of the model allows determining the modes of ultrasonic action, which are necessary for maximum increase of interphase boundary area. As a result of the analysis we determine threshold vibration amplitudes of solid surface covered with thin film of liquid phase, which excess leads to stability failure of capillary waves and their breakage into liquid drops. It is shown, that the most appropriate frequency of ultrasonic vibrations is 60 kHz, at which more than 3 times increase of contact surface. Obtained new scientific results have fundamental interest for the understanding of physical mechanism of the interaction of cavitation bubbles with the interface “liquid-gas”. They can be used for the practical realization of physic-chemical processes at the boundary “liquid-gas” (absorption, drying, evaporation, etc.). In particular ultrasonic action in the packed absorbers lets applying in more than 3 times less number of the nozzles at the same productivity of absorption.

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