

Effects of Radiation and Chemical Reaction on MHD Free Convection Flow past a Vertical Plate in the Porous Medium

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ABSTRACT: The objective is to study the effects of thermal radiation and chemical reaction on mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate through porous medium in the presence of magnetic field. The fluid is considered here as absorbing/emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and the concentration level near the plate is raised to C'_∞ . We use proper transformations to make the governing equations dimensionless. The dimensionless governing equations are reduced to a set of ordinary differential equations. Then we solve these equations with the help of transformed boundary conditions. The effect of various parameters such as Grashof number, Modified Grashof number, Schmidt number, Prandtl number, Magnetic parameter, time, accelerating parameter, Dimensionless porous medium factor and Dimensionless chemical reaction parameter on velocity profiles, temperature profiles, concentration profiles, skin friction profiles, rate of heat transfer profiles and rate of mass transfer profiles are shown graphically.

KEYWORDS: Radiation, MHD, Free Convection, Grashof number and Prandtl number.

I. INTRODUCTION

In many engineering applications, combined heat and mass transfer play an important role in fluids condensing and boiling at a solid surface. Natural convection induced by the simultaneous action of buoyancy forces from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. The effect of the magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gases. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. When free convection flows occur at high temperature, radiation effects on the flow become significant. Many processes in engineering areas occur at high temperatures and knowledge of radioactive heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles are examples of such engineering areas.

Murali [1] examined the thermal radiation effect on unsteady magneto hydrodynamic flow past a vertical porous plate with variable suction. Damala [2] make a study on the effect of the steady two dimensional free convection heat and mass transfer flow electrically conducting and chemically reacting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. The effects of chemical reaction and radiation absorption have been discussed on unsteady MHD free convection heat and mass transfer flow on a viscous, incompressible, electrically conducting fluid past a semi-infinite inclined porous plate, moving with a uniform velocity are discussed in Sudersan [3]. Gupta [4] studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Rapits [5] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar [6]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [7]. Jha [8] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy et al. [9] studied mass transfer effects on exponentially accelerated isothermal vertical plate. Soundalgerkar and Takhar [10] have considered the radiative free convective flow of an optically thin gray gas past a semi- infinite vertical plate.

Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar [11]. Raptis and Perdakis [12] studied the effects of thermal radiation and free convection flow past a moving vertical plate and solve the governing equations analytically. Das et al. [13] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Muthucumaraswamy and Janakiraman [14] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. An exact solution to one dimensional unsteady natural convection flow past an infinite vertical accelerated plate, immersed in a viscous thermally stratified fluid is investigated by Rudra and Bhaben [15]. Tasawar et al. [16] investigated the influence of radiation on magneto hydrodynamic (MHD) and mass transfer flow over a porous stretching sheet. Recently the thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion has been studied by Muthucumaraswamy and Visalakshi [17]. An analytical study is performed to examine the effects of temperature dependent heat source on the unsteady free convection and mass transfer flow of an elasto-viscous fluid past an exponentially accelerated infinite vertical plate in the presence of magnetic field through porous medium by Rajesh [18]. Suneetha et al. [19] investigated thermal radiation effects on MHD flow past an impulsively started vertical plate in the presence of heat source/ sink by taking into account the heat due to viscous dissipation. In S.F Ahmed et al. [20] have analyzed the numerical study on MHD free convection and mass transfer flow past a vertical flat plate.

We study the effects of thermal radiation and chemical reaction on mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate with variable temperature and concentration in the presence of magnetic field in the porous medium. We use proper transformations to make the governing equations dimensionless. The dimensionless governing equations are reduced to a set of ordinary differential equation. Then we solve these equations with the help of transformed boundary conditions.

II. GOVERNING EQUATIONS

$$\frac{\partial u'}{\partial t'} = g \beta (T' - T'_\infty) + g \beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu}{K_p} u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_1 (C' - C'_\infty) \tag{3}$$

with the boundary conditions

$$t' > 0, u' = u_0 \exp(a't'), T' = T'_\infty + (T'_w - T'_\infty) A t', C' = C'_\infty + (C'_w - C'_\infty) A t' \text{ at } y' = 0$$

$$u' = 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \tag{4}$$

MATHEMATICAL FORMULATION

We use the following transformations to make the equations (1) to (4) dimensionless.

$$u = \frac{u'}{u_0}, t = \frac{t' u_0^2}{\nu}, y = \frac{y' u_0}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, Gr = \frac{g \beta \nu (T'_w - T'_\infty)}{u_0^3}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Gm = \frac{g \beta^* \nu (C'_w - C'_\infty)}{u_0^3}, Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, R = \frac{16 a^* \nu^2 \sigma T'_\infty}{k u_0^2}, \tag{5}$$

$$a = \frac{a' \nu}{u_0^2}, K = \frac{K_p u_0^2}{\nu^2}, \gamma = \frac{K_1 \nu}{u_0^2}$$

We know, Gr, Gm, Pr, Sc, M, R, θ , ϕ , K and γ are Thermal Grashof number, Modified Grashof number, Prandtl number, Schmidt number, Magnetic field parameter, Radiation parameter, Dimensionless temperature, Dimensionless concentration, Dimensionless porous medium factor and Dimensionless chemical reaction parameter respectively, where $A = e^{a't'}/t'$. In this case the local radiant of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4 a^* \sigma (T_\infty'^4 - T'^4) \tag{6}$$

By using the parameter our governing equation becomes as follows

$$\frac{\partial u}{\partial t} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K} \quad (7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{Pr} \theta \quad (8)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \phi\gamma \quad (9)$$

Thus the boundary conditions becomes as follows

$$t > 0 : u = \exp(at), \theta = \exp(at), \phi = \exp(at) \text{ at } y = 0 \quad (10)$$

$$u = 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty$$

Let us consider the solution of the equations (7) to (9) be of the form $u = u_0 e^{at}$, $\theta = \theta_0 e^{at}$ and $\phi = \phi_0 e^{at}$ respectively. Using the above u , θ and ϕ equations (7) to (9) become

$$u_0'' - (M + \frac{1}{K} + a)u_0 + Gr\theta_0 + Gm\phi_0 = 0 \quad (11)$$

$$\theta_0'' - Pr(\frac{R}{Pr} + a)\theta_0 = 0 \quad (12)$$

$$\frac{1}{Sc}\phi_0'' - (a + \gamma)\phi_0 = 0 \quad (13)$$

The boundary conditions for the equations are reduced to the following form

$$t > 0 : u_0 = 1, \theta_0 = 1, \phi_0 = 1 \text{ at } y = 0 \quad (14)$$

$$u_0 = 0, \theta_0 = 0, \phi_0 = 0 \text{ as } y \rightarrow \infty$$

Finally the velocity field, temperature field and concentration field can be written as

$$u = (1 + \frac{Gr}{m_1 - m_3} + \frac{Gm}{m_2 - m_3})e^{at - \sqrt{m_3}y} - Gr \frac{1}{m_1 - m_3} e^{at - \sqrt{m_1}y} - Gm \frac{1}{m_2 - m_3} e^{at - \sqrt{m_2}y} \quad (15)$$

$$\theta = e^{at - \sqrt{m_1}y} \quad (16)$$

$$\phi = e^{at - \sqrt{m_2}y} \quad (17)$$

Now we want to calculate the skin friction, the rate of heat transfer (Nusselt number) and the rate of mass transfer (Sherwood number). For this purpose we differentiate u , θ and ϕ with respect to y and get

$$\tau = \frac{du}{dy} = -\sqrt{m_3} (1 + \frac{Gr}{m_1 - m_3} + \frac{Gm}{m_2 - m_3})e^{at - \sqrt{m_3}y} + \frac{Gr \cdot \sqrt{m_1}}{m_1 - m_3} e^{at - \sqrt{m_1}y} + \frac{Gm \cdot \sqrt{m_2}}{m_2 - m_3} e^{at - \sqrt{m_2}y} \quad (18)$$

$$Nu = \frac{d\theta}{dy} = -\sqrt{m_1} e^{at - \sqrt{m_1}y} \quad (19)$$

$$Sh = \frac{d\phi}{dy} = -\sqrt{m_2} e^{at - \sqrt{m_2}y} \quad (20)$$

Thus for $y=0$ we can write the equation (18), (19) and (20)

$$\text{The skin friction } \tau = -\sqrt{m_3} (1 + \frac{Gr}{m_1 - m_3} + \frac{Gm}{m_2 - m_3})e^{at} + \frac{Gr \sqrt{m_1}}{m_1 - m_3} e^{at} + \frac{Gm \sqrt{m_2}}{m_2 - m_3} e^{at}$$

$$\text{The rate of heat transfer } Nu = -\sqrt{m_1} e^{at}$$

$$\text{The rate of mass transfer } Sh = -\sqrt{m_2} e^{at}$$

III. RESULTS AND DISCUSSION

The numerical values for the velocity profiles, temperature distribution, concentration profiles are computed for different physical parameters like Grashof number (Gr), Magnetic parameter (M), Modified Grashof number (Gm), Radiation parameter (R), time (t), Prandtl number (Pr), Schmidt number (Sc) and accelerating parameter (a). The purpose of the calculations is to study the effects of the parameters upon the nature of the velocity profiles, temperature profiles and concentration profiles.

Fig. 1 represents variation in the velocity field for different values of Magnetic parameter (M) in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plates with Pr = 0.71, R = 4, a = 0.5, Sc = 2.01, $\gamma = 2$, K = 4 and t = 0.2. It is observed that for an externally cooled plate an increase in Magnetic parameter (M), the velocity field decreases. For an externally heated plate the results are observed in reverse order. The variation of velocity field for various values of Schmidt number (Sc) in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plate with M = 1, Pr = 0.71, a = 0.5, R = 4, $\gamma = 2$, K = 4 and t = 0.2 are given in Fig. 2. Here we choose Sc = 0.22 (Hydrogen), Sc = 0.30 (Helium), Sc = 0.60 (water vapor) and Sc = 0.78 (Ammonia). It is noted from the figure that velocity field is decreasing with the increasing values of Schmidt number (Sc) in the cooling plate. But in the heating plate the velocity profile is increasing for the increasing values of Sc. Fig. 3 depicts the variation of velocity field with respect to the increasing values of Grashof number (Gr) and Modified Grashof number (Gm) both in the cooled and heated plate with M = 1, Pr = 0.71, Sc = 3.01, a = 0.5, R = 4, t = 0.2, $\gamma = 2$ and K = 4. Here we see that velocity is increasing with an increase in Gr and Gm for the cooling plate. The reverse effects are seen in the heated plate. It is clear in the Fig. 4 that in case of cooling (Gr = 10, Gm = 5) and heating (Gr = -10, Gm = -5) of the plates M = 1, Pr = 0.71, Sc = 2.01, a = 0.5, $\gamma = 2$, K = 4 and R = 4 the velocity field is increasing and decreasing respectively with respect to the increasing values of t. The influence of Gr on the velocity field is represented in Fig. 5. The velocity fields are increasing with the increasing values of Gr in the cooled plate. It is noted that the reverse effects are shown in the heated plate.

Fig. 6 depicts the temperature profiles against y for different values of the parameters. The magnitude of the temperature is maximum near the plate. But when we increase the values of t the magnitude of temperature is maximum at the plate and then decays to zero. Temperature profiles increase with an increase in t. In Fig. 7 we observe that an increase in R decreases the temperature field and then goes to zero for large values of y. In the Fig. 8 we observe that temperature field is increasing at the plate with an increase in a and finally goes to zero.

The numerical values of the concentration are computed for different physical parameters like Schmidt number (Sc), time (t), accelerating parameter (a) and Dimensionless chemical reaction parameter (γ). The purpose of the calculations given here is to study the effects of the parameters Sc, a and t upon the nature of the concentration. In the Fig. 9 we see that concentration field is decreasing with an increasing value of Sc. The maximum values of concentration are found at the plate. The influence of t on the concentration field is presented in the Fig. 10. It is clear from the behavior of t that the concentration field increases for increasing values of t whose effect is significant at the plate but negligible far away from the plate i.e. $y \rightarrow \infty$.

The numerical values of the skin frictions are computed for different physical parameters like Magnetic field parameter (M), Radiation parameter (R), Schmidt number (Sc), Dimensionless porous medium factor (K), Dimensionless chemical reaction parameter (γ) and accelerating parameter (a). The purpose of the calculations given here is to study the effects of the parameters M, R, Sc, Gr, Gm, K, γ and a upon the nature of the rate of flow and transport. The heating and cooling take place by setting up free convection current due to temperature and concentration gradient. Fig. 11 shows the influence of accelerating parameter (a) on the skin friction. Skin frictions are increased in the case of cooling (Gr = 10, Gm = 5) of the plate with the increasing values of a. The reverse effects are shown in the heating plate (Gr = -10, Gm = -5). In Fig. 12 it is shown that skin friction is decreasing with the increasing values of M. These effects are shown in the cooling plate (Gr = 10, Gm = 5) with the increasing values of M. But no effects are shown in the heating plate (Gr = -10, Gm = -5) with the increasing values of M.

The numerical values of the rate of heat transfer are computed for different physical parameters like Radiation parameter (R), Prandtl number (Pr) and accelerating parameter (a). The purpose of the calculations given here is to study the effects of the parameters R, Pr and a upon the nature of the rate of heat transfer. In the Fig. 13 we see that for increasing values of R, Pr and a the rate of heat transfer profiles are decreased and gradually goes to infinity as $t \rightarrow \infty$. Also in the Fig. 14 represents that an increase in a decreases the rate of heat transfer profiles.

IV. FIGURES

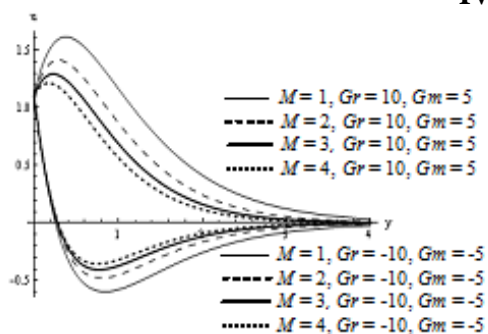


Fig. 1- Velocity profile u for different values of M With $R = 4, Pr = 0.71, a = 0.5, Sc = 2.01, K = 4, \gamma = 2$ and $t = 0.2$ against y .

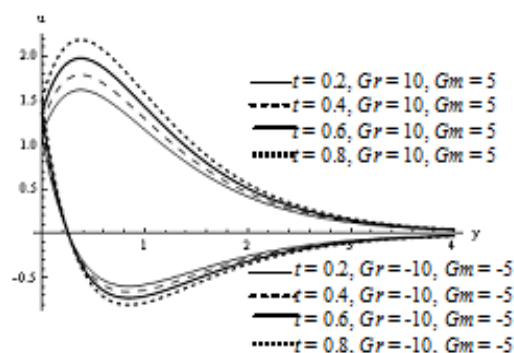


Fig. 4- Velocity profile u for different values of t with $M = 1, Pr = 0.71, Sc = 2.01, a = 0.5, K = 4, \gamma = 2$ and $R = 4$ against y .

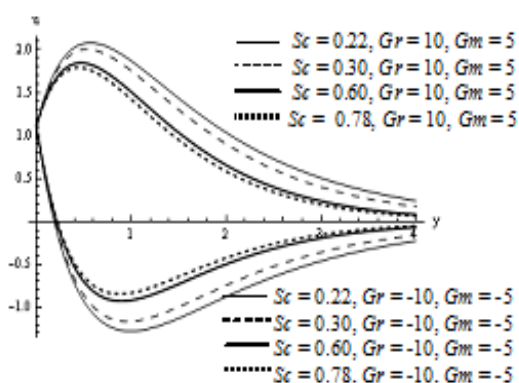


Fig. 2- Velocity profile u for different values of Sc with $M = 1, Pr = 0.71, a = 0.5, R = 4, K = 4, \gamma = 2$ and $t = 0.2$ against y .

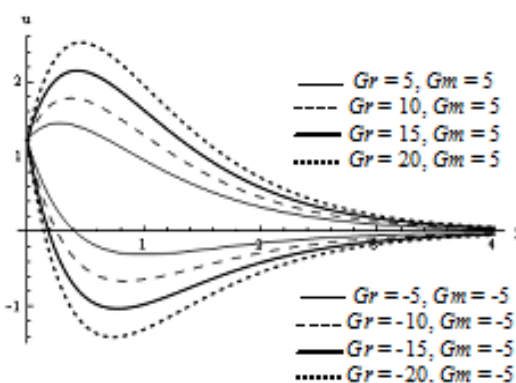


Fig. 5- Velocity profile u for different values of Gr with $M = 1, Pr = 0.71, Sc = 2.01, a = 0.5$ and $R = 4$ against y .

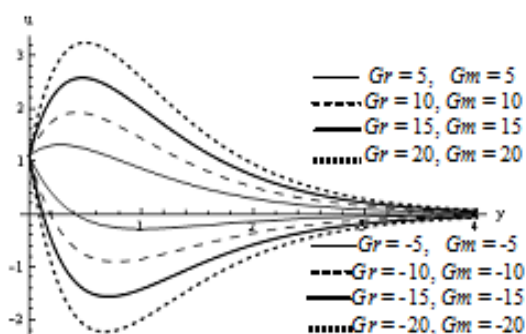


Fig. 3- Velocity profile u for different values of Gr and Gm with $M = 1, Pr = 0.71, Sc = 2.01, a = 0.5, R = 4, K = 4, \gamma = 2$ and $t = 0.2$ against y .

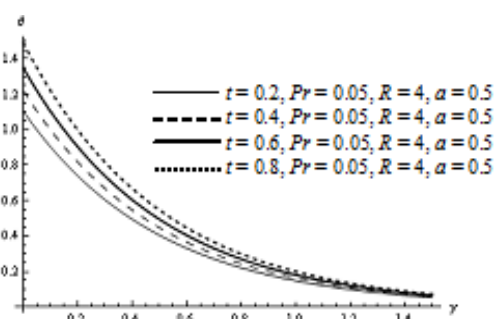


Fig. 6- Dimensionless temperature profiles θ for different values of t against y .

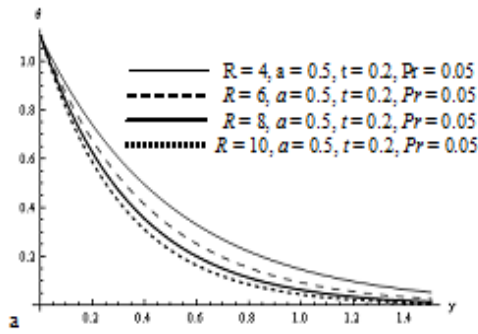


Fig. 7-Dimensionless temperature profiles θ for different values of R against y .

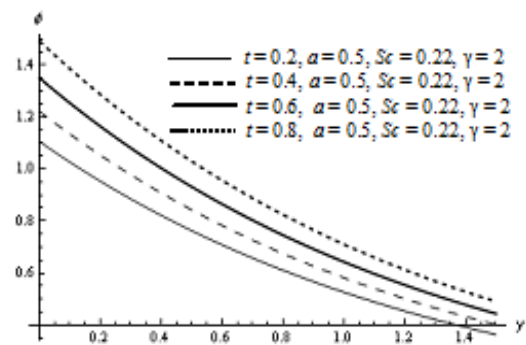


Fig. 10- Dimensionless concentration profiles ϕ for different values of t against y .

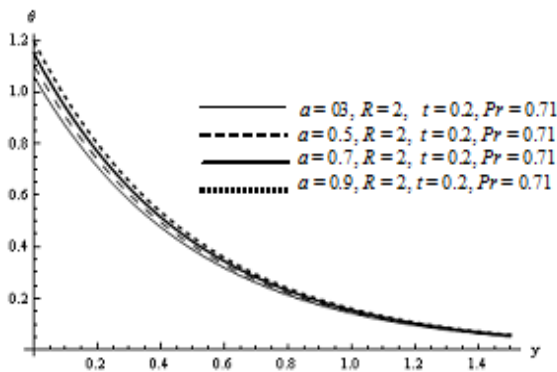


Fig. 8- Dimensionless temperature profiles θ for different values of a against y

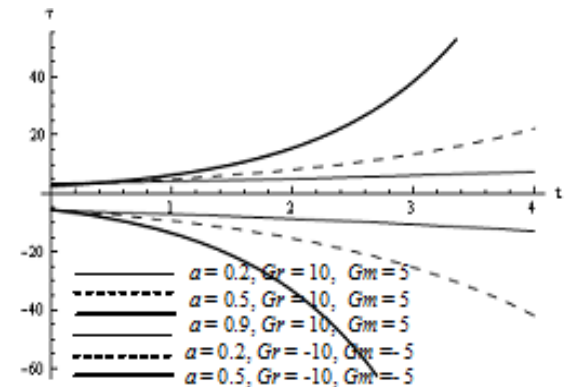


Fig. 11- The skin friction profile τ for different values of a with $R = 4, M = 1, Sc = 0.22, \gamma = 2, K = 4$ and $Pr = 0.71$ against t .

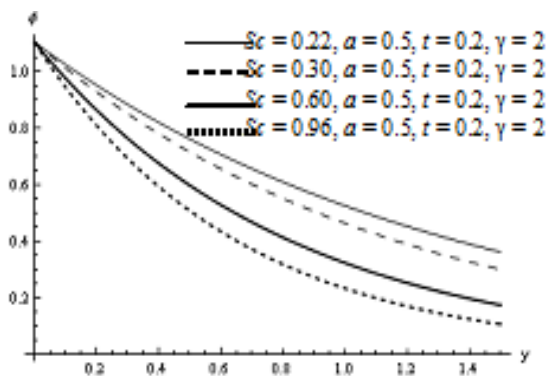


Fig. 9- Dimensionless concentration profiles ϕ for different values of Sc against y .

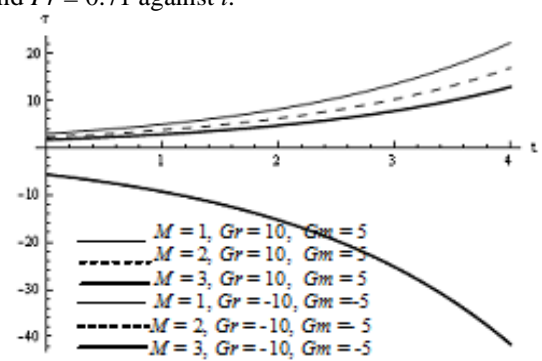


Fig. 12- The skin friction profile τ for different values of M with $R = 4, a = 0.5, Sc = 0.22, \gamma = 2, K = 4$ and $Pr = 0.71$ against t .

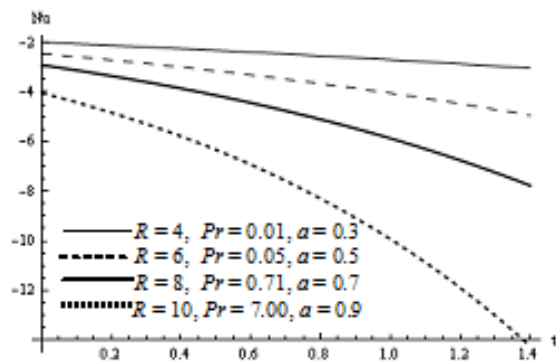


Fig. 13- Rate of heat transfer profiles Nu for different values of R , Pr and α against t .

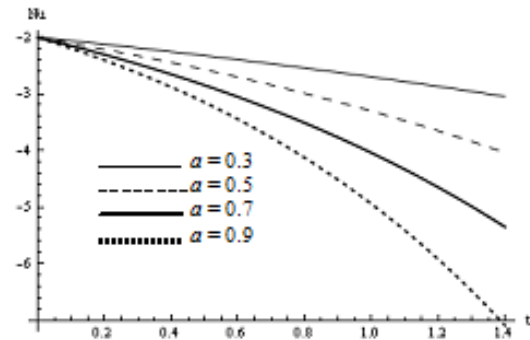


Fig. 14- Rate of heat transfer profiles Nu for different values of α with $Pr = 0.05$ and $R = 4$ against t .

V. CONCLUSION

From the study of the paper we concluded that the velocity is increasing with the increasing value of Grashof number (Gr), Soret number (S_0) and modified Grashof number (Gm) on the other hand it is decreasing with the increasing value of magnetic parameter (M), Prandtl number (Pr), Schmidt number (Sc). We see that if the time is increasing then the velocity is increasing. The temperature increase with increase of heat source parameter (S) and decrease with increase of Prandtl number (Pr). Also we see that the concentration is increase with the increase of Soret number (S_0) and decrease with the increase of Schmidt number (Sc).

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