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Research Paper

Study on MHD Free Convection Heat and Mass Transfer Flow past a Vertical Plate in the Presence of Hall Current

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ABSTRACT: A two-dimensional MHD free convection heat and mass transfer flow of viscous, incompressible and electrically conducting fluid past a vertical flat plate embedded in porous medium in the presence of hall current under the influence of uniform magnetic field applied normal to the flow is studied analytically. In this research work, we make the governing equations dimensionless by usual non-dimensional variables and we obtained a set of ordinary differential equations. Then these obtained ordinary differential equations are solved analytically by using perturbation technique. The expressions for velocity field, temperature distribution, concentration field, skin friction, the rate of heat transfer and the rate of mass transfer are derived. Finally the results are discussed in detailed with the help of graphs and tables to observe the effect of different parameters like Magnetic parameter (M), radiation parameter (F), Grashof number (Gr), modified Grashof number (Gm), Prandtl number (Pr), permeability parameter (k), Eckert number (Ec) and the chemical reaction parameter (Kc).

KEY WORDS: Hall current, Chemical reaction, MHD, Radiation, Porous medium.

I. INTRODUCTION

Magneto-hydrodynamics (MHD) is the science which deals with the motion of a highly conduction fluid in the presence of magnetic field. It is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Many natural phenomena and engineering problems are worth being subjected to an MHD analysis. Furthermore, Magneto-hydrodynamic (MHD) has attracted the attention of a large number of scholars due to its diverse applications. In engineering it finds its application in MHD pumps, MHD bearings etc. Workers likes Hossain and Mandal [1] have investigated the effects of magnetic field on natural convection flow past a vertical surface. Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. As a branch of plasma physics the field of Magneto-hydrodynamics consists of the study of a continuous, electrically conducting fluid under the influence of electromagnetic fields. Originally, MHD included only the study of strictly incompressible fluid but today the terminology is applied to studies of partially ionized gases as well. Soundalgekar and Takhar [2] first, studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly-Vincentine-Gilles equilibrium model. For the same gas Takhar et al. [3] investigated the effects of radiation on the MHD free convection flow past a semi-infinite vertical plate. Later Hossain et al. [4] studied the effect of radiation on free convection from a porous vertical plate. Poonia and Chaudhary [5] studied about the flows through porous media. Recently researchers like Alam and Rahman [6], Sharma and Singh [7] Chaudhary and Arpita [8] studied about MHD free convection heat and mass transfer in a vertical plate or sometimes oscillating plate. Muthucumaraswamy and Janakiraman [9] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion.

Therefore several authors, viz. Raptis and Soundalgekar [10], Agrawal *et al.* [11], Jha and Singh [12], Jha and Prasad [13] have paid attention to the study of MHD free convection and mass transfer flows. Abdusattar [14] and Soundalgekar *et al.* [15] also analyzed about MHD free convection through an infinite vertical plate. Acharya *et al.* [16] have presented an analysis to study MHD effects on free convection and mass transfer flow through a porous medium with constant auction and constant heat flux considering Eckert number as a small perturbation parameter. This is the extension of the work of Bejan and Khair [17] under the influence of magnetic field.

In our present work, we study about the effects of thermal radiation and chemical reaction on mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate through porous medium in the presence of magnetic hall current. The dimensionless governing equations are reduced to a set of ordinary differential equation. Then we solve these equations with the help of transformed boundary conditions. We have used MATHMATICA to draw graph and to find the numerical results of the equation.

II. MATHEMATICAL FORMULATION

Consider the two-dimensional flow of an electrically conducting, viscous, incompressible, radiating, fluid of density ρ through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface. Fig.1 shows the physical model where the \bar{x} -axis is taken along the vertical and \bar{y} axis is horizontal perpendicular to the plate. A uniform magnetic field B_0 is applied normally to the flow region.



Figure 1 The Physical Co-ordinate System

 $\frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{1}$

$$\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = v\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + g\beta\left(\overline{T} - \overline{T}_{\infty}\right) + g\overline{\beta}\left(\overline{C} - \overline{C}_{\infty}\right) - \frac{\sigma B_0^2}{\rho\left(1 + m^2\right)}\overline{u} - \frac{v}{\overline{k}}\overline{u}$$
(2)

$$\overline{v}\frac{\partial\overline{T}}{\partial\overline{y}} = \frac{\kappa}{\rho C}\frac{\partial^2 \overline{T}}{\partial\overline{y}^2} + \frac{\nu}{C}\left(\frac{\partial\overline{u}}{\partial\overline{y}}\right)^2 - \frac{1}{\rho C}\frac{\partial q_r}{\partial\overline{y}}$$
(3)

$$\overline{v}\frac{\partial \overline{C}}{\partial \overline{y}} = D\frac{\partial^2 \overline{C}}{\partial \overline{y}^2} - Kc\left(\overline{C} - \overline{C}_{\infty}\right)$$
(4)

The corresponding boundary conditions are

$$\overline{y} = 0 \quad : \quad \overline{u} = 0, \quad \overline{T} = \overline{T}_{w}, \quad \overline{C} = \overline{C}_{w}$$

$$\overline{y} \to \infty \quad : \quad \overline{u} \to 0, \quad \overline{T} \to \overline{T}_{\infty}, \quad \overline{C} \to \overline{C}_{\infty}$$

$$(5)$$

Here \overline{u} and \overline{v} are the velocity components along the \overline{x} and \overline{y} - directions respectively where \overline{T} is the fluid temperature; \overline{T}_w is the temperature of the fluid at the plate, T_∞ is the fluid temperature far away from the plate, gis the acceleration due to gravity, β is the coefficient of thermal expansion, κ is the thermal conductivity, ρ is the density of the fluid, C_p is the specific heat at constant pressure, σ is the electrical conductivity, D is the molecular diffusivity, v_0 is the uniform velocity, \overline{C} is the concentration of species, \overline{C}_w is the mean concentration, \overline{C}_∞ is the concentration of species for uniform flow, B_0 is the uniform applied magnetic field, ρ is the density, vis the kinematic viscosity, β is the coefficient of volume expansion and $\overline{\beta}$ is the coefficient of volume expansion with concentration and the other symbols have their usual meaning.

To make dimensionless the governing equations from (1) to (4) under the boundary conditions (5) we now introduce the following dimensionless quantities.

$$u = \frac{1}{v_0}\overline{u}, \quad y = \frac{v_0}{v}\overline{y}, \quad \theta = \frac{\overline{T} - \overline{T_{\infty}}}{\overline{T_W} - \overline{T_{\infty}}}, \quad F = \frac{4I_1v^2}{\kappa v_0^2}, \quad Gr = \frac{g\beta v (T_w - T_{\infty})}{v_0^3}, \quad Gm = \frac{g\beta v (C_W - C_{\infty})}{v_0^3} \right\}$$

$$Pr = \frac{\mu C_p}{\kappa}, \quad Sc = \frac{v}{D}, \quad Kc = \frac{v}{v_0^2}\overline{Kc}, \quad Ec = \frac{v_0^2}{C_p (\overline{T_w} - \overline{T_{\infty}})}, \quad M = \frac{\sigma B_0^2 v}{\rho v_0^2}, \quad k = \frac{v_0^2}{v_0^2}\overline{k}, \quad \phi = \frac{\overline{C} - \overline{C_{\infty}}}{\overline{C_w} - \overline{C_{\infty}}} \right\}$$
(6)

Equation (1) implies $\overline{v} = -v_0$ (say)

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(7)

In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self absorption, but it does absorb radiation emitted by the boundaries. Mahaptra *et al.* [18] showed that in the optically thick limit for a non gray near equilibrium as

$$\frac{\partial q_r}{\partial \overline{y}} = 4 I_1 \left(\overline{T} - \overline{T_{\infty}} \right)$$
(8)

Thus the non-dimensional form of the governing equations (1), (2) and (3) are respectively as follows: $u'' + u' - Nu = -Gm\phi - Gr\theta$

where
$$N = \frac{M}{1+m^2} + \frac{1}{k}$$

 $\theta'' + Pr\theta' - F\theta = -PrEcu'^2$
(10)
 $\phi'' + Sc\phi' - KcSc\phi = 0$
(11)

where dashes denote differentiation with respect to y.

The corresponding boundary conditions (5) in non-dimensional forms are

$$y = 0 \quad : u = 0, \quad \theta = 1, \quad \phi = 1$$

$$y \to \infty \quad : u \to 0, \quad \theta \to 0, \quad \phi \to 0$$

$$(12)$$

III. METHOD OF SOLUTION

To solve the equations (9), (10) and (11) with boundary conditions (12), we use the following simple perturbation. The governing equations (9), (10) and (11) are expanded in power of Eckert number Ec(<<1). So we consider Ec as a perturbation quantity. Also we consider a second order correction of it.

$$u(y) = u_0(y) + Ecu_1(y) + O(Ec^2)$$

$$\theta(y) = \theta_0(y) + Ec\theta_1(y) + O(Ec^2)$$

$$\phi(y) = \phi_0(y) + Ec\phi_1(y) + O(Ec^2)$$

$$(13)$$

Substituting (13) in equations (9), (10) and (11) we get

$$u_{0}^{\,\prime\prime} + u_{0}^{\,\prime} - N u_{0} = -G \, m \, \phi_{0} - G \, r \, \theta_{0} \tag{14}$$

$$u_1'' + u_1' - Nu_1 = -Gm\phi_1 - Gr\theta_1$$
(15)

$$\theta_0^{\prime\prime} + Pr\theta_0^{\prime} - F\theta_0 = 0 \tag{16}$$

$$\theta_1'' + Pr\theta_1' - F\theta_1 = -Pru_0'^2 \tag{17}$$

$$\phi_0'' + Sc\phi_0' - ScKc\phi_0 = 0 \tag{18}$$

 $\phi_1^{\ \prime \prime} \,+\, S\,c\,\phi_1^{\ \prime} \,-\, S\,c\,K\,c\,\phi_1 \,=\, 0$

subject to the boundary conditions

$$\begin{array}{l} y = 0 & : u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0, \quad \phi_0 = 1, \quad \phi_1 = 0 \\ y \to \infty & : u_0 \to 0, \quad u_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0, \quad \phi_0 \to 0, \quad \phi_1 \to 0 \end{array} \right\}$$
(20)

Solving equations (14) to (19) with boundary condition (20), the following results are obtained

$$u = k_{3}e^{-k_{1}y} + (k_{4} + Eck_{29})e^{-k_{2}y} + (k_{5} + Eck_{30})e^{-k_{6}y}$$

$$+ Ec\left(k_{23}e^{-2k_{1}y} + k_{24}e^{-2k_{2}y} + k_{25}e^{-2k_{6}y} + k_{26}e^{-k_{11}y} + k_{27}e^{-k_{13}y} + k_{28}e^{-k_{15}y}\right)$$
(21)

$$\theta = \left(1 + Eck_{22}\right)e^{-k_2y} + Ec\left(k_{16}e^{-2k_1y} + k_{17}e^{-2k_2y} + k_{18}e^{-2k_6y} + k_{19}e^{-k_{11}y} + k_{20}e^{-k_{13}y} + k_{21}e^{-k_{15}y}\right)$$
(22)

$$\phi = e^{-k_1 y}$$

Now we want to calculate the skin friction, the rate of heat transfer (Nusselt number) and the rate of mass transfer (Sherwood number). For this purpose we differentiate u, θ and ϕ with respect to y and then for y=0 we can write, where y is the dimensionless coordinate axis normal to the plate.

$$\begin{aligned} \tau &= -k_1k_3 - k_2k_4 - k_5k_6 - Ec\left(2k_1k_{23} + 2k_2k_{24} + 2k_6k_{25} + k_{11}k_{26} + k_{13}k_{27} + k_{15}k_{28} + k_2k_{29} + k_6k_{30}\right)\\ Nu &= k_2 + Ec\left(2k_1k_{16} + 2k_2k_{17} + 2k_6k_{18} + k_{11}k_{19} + k_{13}k_{20} + k_{15}k_{21} + k_2k_{22}\right)\\ Sh &= k_1\end{aligned}$$

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(19)

(23)

(9)

IV. RESULTS AND DISCUSSION

In order to get the physical insight into situation of the problem of our study the effects of Prandtl number (Pr), Schmidt number (Sc), magnetic parameter (M), Grashof number (Gr), modified Grashof number (Gm), Eckert number (Ec), Hall parameter (m), chemical reaction parameter (Kc), permeability parameter (k), radiation parameter (F) on velocity field, temperature field, concentration field, skin-friction, the rate of heat transfer in terms of Nusselt number (Nu) and the rate of mass transfer in terms of Sherwood number (Sh) are studied taking different numerical values. To observe the effects of these parameters, the values of Schmidt number (Sc) are chosen for hydrogen (Sc=0.22), water-vapor (Sc=0.60), ammonia (Sc=0.78) at 25°C and one atmosphere pressure. The values of Prandtl number (Pr) are chosen for sodium (Pr=0.01), air (Pr=0.71) and water (Pr=7.0). We also choose the Grashof number for heat transfer is Gr=5.0, 6.0, 10.0 and modified Grashof number for mass transfer is Gm=2.0, 3.0, 4.0. The values of magnetic parameter are given M=1.0, 3.0, 5.0 arbitrarily.

The velocity profiles u for different values of the above parameters are illustrated in Fig.2 to Fig.11, the temperature profiles θ for different values of the parameters are described in Fig.12 to Fig.14 and the concentration profiles ϕ for different values of the above parameters are expressed in Fig.15 and Fig.16. Also the numerical values of Skin-Friction (τ), the rate of heat transfer (*Nu*) and the rate of Mass Transfer (*Sh*) are shown in the Table 1 to Table 3.

In the Fig.2 it is observed that the velocity decreases with the increase of magnetic parameter (M). Physically this is true as the magnetic force retards he flow, velocity decreases. Fig.3 shows the velocity distributions for different values of Hall parameter (m). After analysing the figure it is noticed that the velocity increases with the increase of Hall parameter (m). It is described in the Fig.4 that the velocity distributions for different values of chemical reaction parameter (Kc). In this figure we observe the velocity decreases with the increase of permeability parameter (k). Fig.6 indicates that the velocity decreases with the increase of radiation parameter (F). From Fig.7 the velocity increases with the increase of Grashof number (Gr). At y=0 the velocity profiles are zero. In the Fig.8 it is observed that the velocity increases with the increase of Schmidt number (Sc). Also Fig.10 marks that the velocity decreases with the increase of Prandtl number (Pr). Physically it is true because the increase in the Prandtl number due to increasing the viscosity of the fluid which makes the fluid thick and hence decrease the velocity of fluid. The velocity distributions for different values of Eckert number (Ec).

From Fig.12 it is clear that the temperature decreases with the increase of radiation parameter (*F*). At y=0 the temperature profiles attain the maximum value 1.0 and then decrease smoothly and attain to zero with the increase of *y*. It is observed in the Fig.13 that the temperature distributions for different values of Prandtl number (*Pr*). In this figure it is noticed that the temperature decreases with the increase of Prandtl number (*Pr*). The temperature profiles attain the maximum value 1.0 at y=0 and then gradually attain nearly to zero for large values of *y*. Fig.14 shows the temperature distributions for different values of Eckert number (*Ec*). After analysing the figure it is noticed that the temperature increases with the increase of Eckert number (*Ec*).

Fig.15 depicts that the concentration decreases with the increase of reaction parameter (*Kc*). In this figure maximum value of concentration profiles for y=0 is 1.0 the concentration profiles decrease smoothly and attain to zero for large value of y. In the Fig.16 it marks that the concentration decreases with the increase of Schmidt number (*Sc*). We get the maximum value of concentration profiles for y=0 the concentration profiles gradually attain zero with the increase of y.





Figure 2 Velocity profiles where Gr=6.0, Gm=3.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.



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Figure 4 Velocity profiles where M=3.0, Gr=6.0, Gm=3.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.



Figure 6 Velocity profiles where M=3.0, Gr=6.0, Gm=3.0, Kc=0.04, Sc=0.22, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.



Figure 8 Velocity profiles where M=3.0, Gr=6.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.



Figure 10 Velocity profiles where M=3.0, Gm=3.0, Gr=6.0, F=0.5, Kc=0.04, Sc=0.22, m=2, k=0.5 and Ec=0.01 against y.



Figure 5 Velocity profiles where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2 and *Ec*=0.01 against *y*.



Figure 7 Velocity profiles where M=3.0, Gm=3.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.



Figure 9 Velocity profiles where M=3.0, Gm = 3.0, Gr=6.0, F=0.5, Kc=0.04, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.



Figure 11 Velocity profiles where M=3.0, Gm=3.0, Gr=6.0, F=0.5, Kc=0.04, Sc=0.22, m=2, k=0.5 and Pr=0.71 against y.

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Figure 12 Temperature profiles where *M*=3.0, *Gr*=6.0, *Gm* =3.0, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.



Figure 14 Temperature profiles where M=3.0, Gm=3.0, Gr=6.0, F=0.5, Kc=0.04, Sc=0.22, m=2, k=0.5 and Pr=0.71 against y.



Figure 16 Concentration profiles where M=3.0, Gr=6.0, Gm=3.0, F=0.5, Kc=0.04, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.

Table 1 Numerical values of Skin-Friction (τ)

SL No	М		F	Kc	k	τ
1	1.0	2.0	0.5	0.04	0.5	4.99799
2	3.0	2.0	0.5	0.04	0.5	4.65836
3	5.0	2.0	0.5	0.04	0.5	4.38312
4	3.0	1.0	0.5	0.04	0.5	4.10228
5	3.0	2.0	0.5	0.04	0.5	4.65836
6	3.0	3.0	0.5	0.04	0.5	4.90566
7	3.0	2.0	0.5	0.04	0.5	4.65836
8	3.0	2.0	5.0	0.04	0.5	3.65714
9	3.0	2.0	10.0	0.04	0.5	3.35018
10	3.0	2.0	0.5	-0.04	0.5	4.79238
11	3.0	2.0	0.5	0.00	0.5	4.70956
12	3.0	2.0	0.5	0.04	0.5	4.65836
13	3.0	2.0	0.5	0.04	0.1	2.50745
14	3.0	2.0	0.5	0.04	0.5	4.65836
15	3.0	2.0	0.5	0.04	1.0	5.70216

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Figure 13 Temperature profiles where M=3.0, Gm=3.0, Gr=6.0, F=0.5, Kc=0.04, Sc=0.22, m=2, k=0.5 and Ec=0.01 against y.



Figure 15 Concentration profiles where M=3.0, Gr=6.0, Gm=3.0, F=0.5, Sc=0.22, Pr=0.71, m=2, k=0.5 and Ec=0.01 against y.

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Sl. No.	М	т	F	Кс	k	Nu
1	1.0	2.0	0.5	0.04	0.5	1.11868
2	3.0	2.0	0.5	0.04	0.5	1.12324
3	5.0	2.0	0.5	0.04	0.5	1.12659
4	3.0	1.0	0.5	0.04	0.5	1.12970
5	3.0	2.0	0.5	0.04	0.5	1.12324
6	3.0	3.0	0.5	0.04	0.5	1.11996
7	3.0	2.0	0.5	0.04	0.5	1.12324
8	3.0	2.0	5.0	0.04	0.5	2.60881
9	3.0	2.0	10.0	0.04	0.5	3.52950
10	3.0	2.0	0.5	-0.04	0.5	1.12154
11	3.0	2.0	0.5	0.00	0.5	1.12260
12	3.0	2.0	0.5	0.04	0.5	1.12324
13	3.0	2.0	0.5	0.04	0.1	1.14178
14	3.0	2.0	0.5	0.04	0.5	1.12324
15	3.0	2.0	0.5	0.04	1.0	1 10770

Table 2 Numerical values of the rate of Heat Transfer (Nu)

Table 3	Numerical	values of	the rate of	Mass	Transfer ((Sh)

Sl. No.	Sc	Kc	Sh
1	0.22	0.04	0.254568
2	0.60	0.04	0.637639
3	0.78	0.04	0.818135
4	0.22	-0.04	0.167446
5	0.22	0.0	0.220000
6	0.22	0.04	0.254568

V. CONCLUSION

In the present research work, we have studied the effects of Hall current, chemical reaction and radiation on MHD free convection flow through a vertical plate embedded in porous medium. The results are given graphically to illustrate the variation of velocity, temperature and concentration with different parameters. Also the Nusselt number, Sherwood number and skin-friction are presented in tables. From the analysis of the study the following conclusions are made:

The velocity profiles increase with the increase of Hall parameter (m), permeability parameter (k), Grashof number (Gr) and modified Grashof number (Gm). On the other hand, it decrease with the increase of magnetic parameter (M), chemical reaction parameter (Kc), radiation parameter (F), Schmidt number (Sc) and Prandtl number (Pr). The temperature distributions decrease with the increase of radiation parameter (F) and Prandtl number (Pr). The concentration distributions decrease with the increase of chemical reaction parameter (Kc) and Schmidt number (Sc).

The skin friction (τ) increases with the increase of Hall parameter (m) and permeability parameter (k) whereas it decreases with the increase of magnetic parameter (M), radiation parameter (F) and chemical reaction parameter (Kc). The heat transfer rate expressed in terms of the Nusselt number(Nu) increases with the increase of magnetic parameter (Kc) and decreases with the increase of magnetic parameter (Kc) and decreases with the increase of Hall parameter (m) and permeability parameter (k). The mass transfer rate expressed in terms of Sherwood number (Sh) increases with the increase of Schmidt number (Sc) and chemical reaction parameter (Kc).

APPENDIX

$$\begin{aligned} k_{1} &= \frac{1}{2} \Big\{ Sc + \sqrt{Sc^{2} + 4ScKc} \Big\} \,, \qquad k_{2} &= \frac{1}{2} \Big\{ Pr + \sqrt{Pr^{2} + 4F} \Big\} \,, \qquad k_{3} &= -\frac{Gm}{k_{1}^{2} - k_{1} - N} \,, \qquad k_{4} &= -\frac{Gr}{k_{2}^{2} - k_{2} - N} \\ k_{5} &= -k_{3} - k_{4} \,, \quad k_{6} &= \frac{1}{2} \Big\{ 1 + \sqrt{1 + 4N} \Big\} \,, \quad k_{7} &= k_{1}^{2} k_{3}^{2} \,, \quad k_{8} &= k_{2}^{2} k_{4}^{2} \,, \quad k_{9} &= k_{5}^{2} k_{6}^{2} \,, \quad k_{10} &= 2k_{1}k_{2}k_{3}k_{4} \,, \quad k_{11} &= k_{1} + k_{2} \\ k_{12} &= 2k_{1}k_{3}k_{5}k_{6} \,, \qquad k_{13} &= k_{1} + k_{6} \,, \qquad k_{14} &= 2k_{2}k_{4}k_{5}k_{6} \,, \qquad k_{15} &= k_{2} + k_{6} \,, \qquad k_{16} &= \frac{-Prk_{7}}{4k_{1}^{2} - 2Prk_{1} - F} \\ k_{17} &= \frac{-Prk_{8}}{4k_{2}^{2} - 2Prk_{2} - F} \,, \qquad k_{18} &= \frac{-Prk_{9}}{4k_{6}^{2} - 2Prk_{6} - F} \,, \qquad k_{19} &= \frac{-Prk_{10}}{k_{11}^{2} - Prk_{11} - F} \,, \qquad k_{20} &= \frac{-Prk_{12}}{k_{13}^{2} - Prk_{13} - F} \\ k_{21} &= \frac{-Prk_{14}}{k_{15}^{2} - Prk_{15} - F} \,, \qquad k_{22} &= -k_{16} - k_{17} - k_{18} - k_{19} - k_{20} - k_{21} \,, \qquad k_{23} &= \frac{-Grk_{16}}{4k_{1}^{2} - 2k_{1} - N} \,, \qquad k_{24} &= \frac{-Grk_{17}}{4k_{2}^{2} - 2k_{2} - N} \\ \end{array}$$

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$$k_{25} = \frac{-Grk_{18}}{4k_6^2 - 2k_6 - N}, \quad k_{26} = \frac{-Grk_{19}}{k_{11}^2 - k_{11} - N}, \quad k_{27} = \frac{-Grk_{20}}{k_{13}^2 - k_{13} - N}, \quad k_{28} = \frac{-Grk_{21}}{k_{15}^2 - k_{15} - N}, \quad k_{29} = \frac{-Grk_{22}}{k_{2}^2 - k_{2} - N}, \quad k_{29} = \frac{-Grk_{21}}{k_{2}^2 - k_{2} - N}, \quad k_{29} = \frac{-Grk_{21}}{k_{2} - K}, \quad k_{29} = \frac{-Gr$$

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