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Multivariable LMI Approach for Robust Control of Tractor Trailer System with State Delay and Parametric Uncertainty

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ABSTRACT: *The design of controllers for systems which are subject to uncertain time delays is a challenging problem for control engineers. This is due to the potential for closed-loop instability contributed by the uncertainty in the processphase.This paper gives a robust method to control a Wheeled Mobile Robot trailer system with time delay. This articulated vehicle being a complicated non holonomic system, the number of control inputs is less compared to the number of generalized coordinates. A robust H-infinity output feedback controller is synthesized using LMI algorithms for the nominal plant considering the model uncertainties in unstructured form and also the output disturbances. Analysis of the system without the output disturbance, with a first order output disturbance and with a second order output disturbance have been carried out. The simulation results indicate that the WMR trailer system is stable with the proposed LMI* H_∞ *controller.Also this paper describes a linearized model of WMR trailer system which is highly nonlinear in the presence of output disturbances, uncertainties and time delays.*

KEYWORDS: *LMI,robust control, time delay, tractor trailer, uncertainty.*

I. INTRODUCTION

The Tractor-Trailer system, especially the articulated form that uses several trailers, forms a unique class of control systems because of the variety of control issues existing in the motion of this type of articulated vehicles. In such systems, the complexity in tracing a pre-defined path for the forward and backward movementincreases as the number of trailers increases. These vehicles are assumed to be moving on plane surfaces without any guide ways or rail lines. Thus the controlled motions have to be achieved by the operation of the wheels both for traction as well as steering. Articulated vehicles made of wheeled mobile robots have advantages in terms of their suitability for experimentation. These types of vehicle have nonlinearity in the control loops and there are many constraints. The two wheeled mobile robot can be connected as tractor trailer system so that the effect of steering control for trailer units as a subsidiary control can be studied. It is seen that when the vehicle is having freedom to move without connecting power or data wires linked to it from the terrestrial control station, then we have two constraints: one requirement is to conserve battery and extend the life, and secondly, to communicate from the control station with the WMR drive circuit on board the WMR. This data link could be wireless and packet switched scheme can be employed to make use of the technology of sensor networks. This networking of sensors and controllers may cause time delays in the system model. Hence such a scenario is investigated here.

Control of these articulated systems is ahighly nonlinear problem. Nonlinear control strategies for a WMR based on feedback linearization were proposed in [1-2] where the emphasis was for solving the trajectory tracking and the posture stabilization, by employing the kinematics of the system. Tracking control schemes, where adaptive tracking controllers for kinematic model and adaptive back stepping controllers for dynamic models were designed for nonholonomic mobile robots with unknown parameters [3]. Adaptive control using back stepping designed for tracking reference trajectory and point stabilization were proposed in [4-5] where the asymptotic convergence of tracking error to zero is said to be achieved. A robust tracking control design for wheeled vehicle systems with trailer, based on adaptive fuzzy elimination technique was proposed in [6]. Here, anH-infinity minimax control equipped with adaptive fuzzy elimination scheme is used to achieve a robust tracking performance, despite the parameter uncertainties and external disturbance.

The static control method used requires tuning with constant control law updating when subjected to disturbances. In [7] a robust H-infinity output feedback controller is designed using mu synthesis for a tractor trailer system subjected to model uncertainties in unstructured form and output disturbances. An LMI based H_∞ design is proposed in [8] for both the steering control and the fault detection. But in all the above cases the effect of time delay was not taken in to account. In the conventional mathematical description of a physical process, one generally assumes that the behavior of the considered process depends on the current process. However, there exist situations where such an assumption is not satisfied and the use of a 'current state' model in analysis would cause a poor performance or even destabilization. These systems are called time delay systems.The performance and stability of a control system can be directly affected by a time-delay located either in its input, output, or both. In the case of a mobile robot, an input time-delaymay become critical in different situations, such as when vision is used as the localization technique and a high frame per second rate is demanded, or when centralized control of multiple agents is desired, or even if very accurate regulation or tracking performance is required [9]. The time delay affects the system due to the fact that the controller and the robot are linked via a delay inducing communication channel, by which the performance and stability of the system are possibly compromised. The time delay can be imposed in transmission of the data between the system components. It is known that the existence of time delay in a system is the main source of instability and poor performance. In previous researches, scattering theory and passivity based control are used to guarantee the stability in case time delay exists [10-11].This paper gives a robust method to control a wheeled mobile robot trailer system with time delay. A robust H-infinity output feedback controller is synthesized using LMI algorithms for the system when subjected to model uncertainties in unstructured form and output disturbances. Analysis of the system without considering the output disturbance and also with a first order and second order output disturbance have been performed for different values of time delays. The simulation results indicate that the WMR trailer system is stable with the proposed LMI H_∞ controller. Also this paper describes a linearized model of WMR trailer system which is highly nonlinear in the presence of output disturbances, uncertainties and time delays.

The remaining part of the paper is organized as follows: In Section 2, system description and problem formulation are discussed. The mathematical modeling of the WMR trailer system with time delay is discussed in Section 3.Section 4deals with the control Design. Section 5deals with theH_∞ control scheme for WMR trailer system.The simulation results are presented in Section 6.Finally, Section 7 concludes the paper followed by the references used.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a wheeled mobile robot trailer system with time delay described by the state equation

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_d x(t-\tau) + B_1 w(t) + B_2 u(t) \\ z(t) &= Cx(t) \end{aligned} \tag{1}$$

where$\boldsymbol{x(t)} \in \boldsymbol{R^n}$is the state, $\boldsymbol{u(t)} \in \boldsymbol{R^m}$ is the control input,$\boldsymbol{w(t)} \in \boldsymbol{R^l}$ is the external disturbance, which belongs to $\boldsymbol{L_2}[\mathbf{0},\infty]$ and $\boldsymbol{z(t)} \in \boldsymbol{R^p}$ is the controlled system output.$\boldsymbol{A_1}$,$\boldsymbol{A_d}$,$\boldsymbol{B_1}$,$\boldsymbol{B_2}$and $\boldsymbol{C}$are known constant matrices with appropriate dimensions, $\boldsymbol{\tau > 0}$ is the time delay. One important assumption for completing the description of dynamic system of (1) is that all of the system states are measurable.

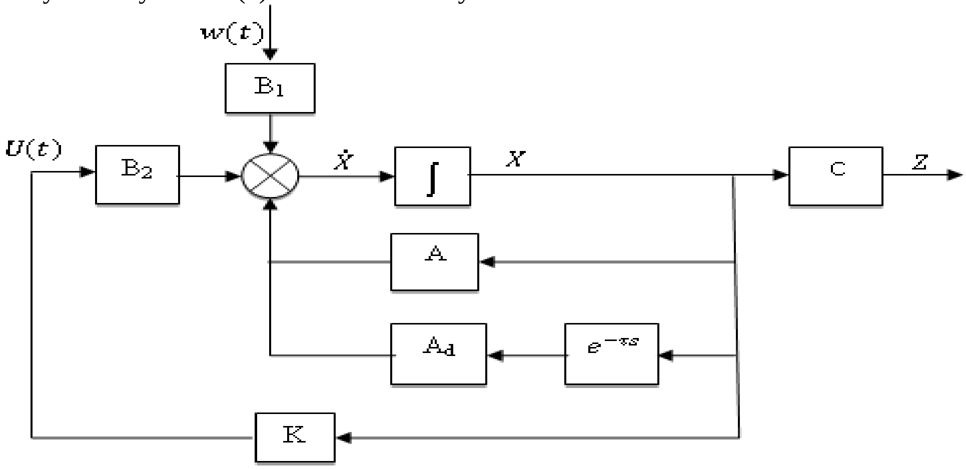


Figure:1. Control scheme for a WMR Trailer time delay system represented by Eqn (1)

The output feedback structure for the proposed Hinfinity control scheme is shown in Fig.2. The system consists of a linear time invariant augmented system *P* which belongs to the class P of uncertain system.*P* comprises the nominal model and weighting functions corresponding to model uncertainties and disturbances. The inputs w and u represents the exogenous and control inputs whereas zandy represent the controlled and measured outputs.

The relation between output and input can be represented as

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{2}$$

Considering the invariance of the plant P and the separability of the variables for the linear setting, the system has been partitioned into the state space for as given in Eq.(3)

$$\begin{gathered} \dot{x} = A_1 x + B_1 w + B_2 u + A_d \, x(t - \tau) \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{gathered} \tag{3}$$

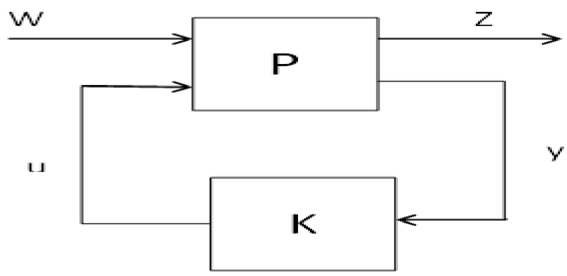


Figure:2. H-infinity feedback control

Assume that (A_1,B_1) is stabilizable; (A,C_1) is detectabl;(A,B_2) is controllable;and (A,C_2) is observable. With a feedback $u=K(s)\,y$, the closed loop transfer function from disturbance w to controlled output z is as in (4)

$$F_l(P,K) = P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21} \tag{4}$$

With the above cited assumptions, we can obtain a proper real rational controller for the closed loop system so that the system remains internally stable. The control problem is: with a feedback $u=K(s)\,y$, find an admissible internally stabilizing control K which would be attenuating disturbances such that the norm of the stable closed loop system from the disturbances to the controlled outputs is less than γ which is equal to 1 for optimal and slightly greater than 1 for suboptimal control [7]. The control objective is stated in the mathematical form in Eq.(5)

$$\|F_l(P,K)\|_\infty = sup_{\omega \epsilon R} /F_l(P,K)(j\omega)/ < \gamma (5)$$

III. MATHEMATICAL MODEL OF THE WHEELED MOBILE ROBOT TRAILER SYSTEM WITH TIME DELAY

The schematic diagram shown in Fig.3 represents the kinematics of the WMR trailer system having time delay. As shown in the figure, there is the front mobile differential drive robot made of rigid body and non-deforming wheels which acts as the tractor. It is assumed that the vehicle moves on a plane without slipping. The robotic tractor has a platform with two driving wheels mounted on the same axis with independent actuators and one free castor wheel for balancing purposes. The mobile robot is steered by changing the relative angular velocities of the driving wheels. The trailer is neither powered nor controlled in its motion.

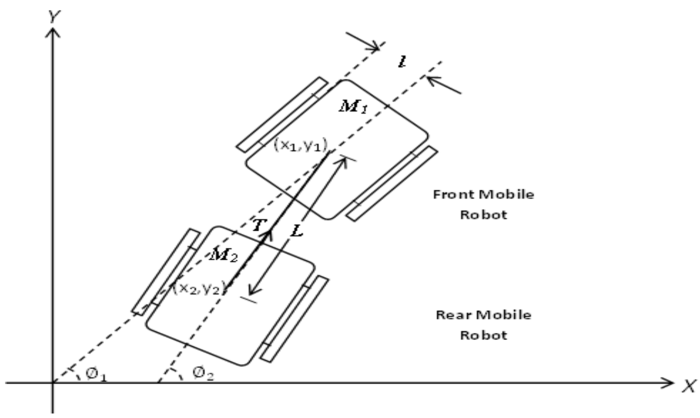


Figure:3. Wheeled mobile robot with trailer

Let the wheeled vehicle with two independent driving wheels be rigid, moving on the plane as shown in Fig. 3, where X-Y indicates the absolute (fixed) coordinates on the plane. The following assumptions are made for dealing with the wheeled vehicle.

Assumptions

1) The running wheeled vehicle satisfies both the pure rolling and non-slipping conditions.
2) The link bar of the wheeled vehicle crosses the centers of gravity of the front and rear wheeled vehicles, i.e.,$d_L = 0$

Under the above assumptions, the model of such wheeled vehicle can be composed of the following equations [12-13].

1) The dynamic equation of the front mobile robot

$$I_v \ddot{\emptyset}_1 - T d_L = D_r l - D_l l \tag{6}$$
$$M_1 \dot{v}_1 - T\cos(\emptyset_2 - \emptyset_1) = D_r + D_l$$

2) The dynamic equation of the driving systems for right and left wheels of the front mobile robot

$$I_w \ddot{\theta}_r + c\dot{\theta}_r = ku_r - rD_r \tag{7}$$
$$I_w \ddot{\theta}_l + c\dot{\theta}_l = ku_l - rD_l \tag{8}$$

3) Pure rolling constraints

$$r\dot{\theta}_r = v_1 + l\dot{\emptyset}_1 \tag{9}$$
$$r\dot{\theta}_l = v_1 - l\dot{\emptyset}_1 \tag{10}$$

4) Nonslipping constraints

$$\dot{x}_1 \sin\emptyset_1 - \dot{y}_1 \cos\emptyset_1 = 0 \tag{11}$$
$$\dot{x}_2 \sin\emptyset_2 - \dot{y}_2 \cos\emptyset_2 = 0 \tag{12}$$

Or

$$\dot{x}_1 = v_1 \cos\emptyset_1 \tag{13}$$
$$\dot{y}_1 = v_1 \sin\emptyset_1 \tag{14}$$
$$\dot{x}_2 = v_2 \cos\emptyset_2 \tag{15}$$
$$\dot{y}_2 = v_2 \sin\emptyset_2 \tag{16}$$

5) Relationships between front and rear wheeled vehicles

$$T = M_2 \dot{v}_2 \tag{17}$$
$$v_2 = v_1 \cos(\emptyset_2 - \emptyset_1) \tag{18}$$
$$\dot{\emptyset}_2 = -\frac{1}{L} v_1 \sin(\emptyset_2 - \emptyset_1) \tag{19}$$

Substituting Eqn(9) &Eqn (10) and its derivatives in Eqn(7)&Eqn(8) we get

$$\frac{I_w}{r}(\dot{v}_1 + l\ddot{\emptyset}_1) + \frac{c}{r}(v_1 + l\dot{\emptyset}_1) = ku_r - rD_r \tag{20}$$
$$\frac{I_w}{r}(\dot{v}_1 + l\ddot{\emptyset}_1) + \frac{c}{r}(v_1 - l\dot{\emptyset}_1) = ku_l - rD_l \tag{21}$$

From Eqn(20) &Eqn(21) we have

$$D_r = \frac{k}{r}u_r - \frac{I_w}{r^2}(\dot{v}_1 + l\ddot{\emptyset}_1) - \frac{c}{r^2}(v_1 + l\dot{\emptyset}_1) \tag{22}$$
$$D_l = \frac{k}{r}u_l - \frac{I_w}{r^2}(\dot{v}_1 - l\ddot{\emptyset}_1) - \frac{c}{r^2}(v_1 - l\dot{\emptyset}_1) \tag{23}$$

Because of the second assumption$d_L = 0$ Eqn(6) becomes

$$I_v \ddot{\emptyset}_1 = D_r l - D_l l \tag{24}$$

Substituting Eqn(22)&Eqn(23) in Eqn(24) we will get

$$(I_v + \frac{2l^2}{r^2} I_w)\ddot{\emptyset}_1 = -\frac{2l^2 c\dot{\emptyset}_1}{r^2} + \frac{lk}{r}(u_r - u_l) \tag{25}$$

Again from Eqn(6) we have

$$M_1 \dot{v}_1 = T\cos(\emptyset_2 - \emptyset_1) + D_r + D_l \tag{26}$$

With

$$T = M_2 \cos(\emptyset_2 - \emptyset_1)\dot{v}_1 + \frac{1}{L} M_2 v_1 \sin^2(\emptyset_2 - \emptyset_1) v_1 + M_2 v_1 \sin(\emptyset_2 - \emptyset_1)\dot{\emptyset}_1 \tag{27}$$

SubstitutingEqns (22),(23) & (27) in Eqn(26) ,we have

$$\left[M_1 + \frac{2I_w}{r^2} - M_2 cos^2(\emptyset_2 - \emptyset_1)\right]\dot{v}_1 = \frac{k}{r}(u_r + u_l) - \frac{2c}{r^2}v_1 + \frac{1}{L}M_2 \sin^2(\emptyset_2 - \emptyset_1)\cos(\emptyset_2 - \emptyset_1)v_1 + M_2 v_1 \sin(\emptyset_2 - \emptyset_1)\cos(\emptyset_2 - \emptyset_1)\dot{\emptyset}_1 \quad (28)$$

From Eqns(13),(15),(19),(25)&(28), the state equations of the above system can be written as

$$\dot{x}_1 = x_5 \cos x_3$$
$$\dot{x}_2 = x_5 \sin x_3$$
$$\dot{x}_3 = x_6$$
$$\dot{x}_4 = -\frac{x_5}{L}\sin(x_4 - x_3)$$
$$\dot{x}_5 = \frac{k}{rP}((u_r + u_l) - \frac{2cx_5}{r^2 P} + \frac{M_2}{LP}\sin^2(x_4 - x_3)\cos(x_4 - x_3)x_5 + \frac{M_2}{P}\sin(x_4 - x_3)\cos(x_4 - x_3)x_5 x_6$$
$$\dot{x}_6 = \frac{lk}{r\vartheta}(u_r - u_l) - \frac{2l^2 c}{r^2\vartheta}x_6$$

Where $P = M_1 + \frac{2I_w}{r^2} - M_2 cos^2(\emptyset_2 - \emptyset_1)and\vartheta = (I_v + \frac{2l^2}{r^2}I_w)$.

The nominal parameters of this wheeled vehicle system [14] are shown in Table 1.

Table 1: The nominal parameters of the WMR Trailer system

Parameter	**Nominal value**
$I_v(kgm^2)$	10
$I_w(kgm^2)$	0.005
$M_1(kg)$	200
$M_2(kg)$	100
$l(m)$	0.3
c(kgm^2/s)	0.05
$r(m)$	0.1
$L(m)$	1.2
K	5

Time delay in WMR system appears in two situations1) In local controller, position signal from robot to sensor and the execute signal from actor to robot cause time delay; 2)In remote network controller, the signal transmission cause time delay. Also the transmission of control error signals from the control center to the robot can cause the delay. So in order to accommodate the time delay of the system, a matrix A_d is also introduced in the system.

Substituting the nominal values of the parameters in the nominal model linearized at a constant velocity of 1m/s yields G_{nom} for the system.Thus the state space representation of the nominal system with time delay can be represented as

$$\dot{x}_n = A_n x_n + B_n u + A_d x(t - \tau)$$
$$y = C_n x$$

Where$A_n = \begin{bmatrix} 0 & 0 & -0.3536 & 0 & 0.7071 & 0 \\ 0 & 0 & 0.3536 & 0 & 0.7071 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.3915 & -0.3915 & 0.285 & 0 \\ 0 & 0 & 0.16 & -0.16 & -0.0075 & -0.1426 \\ 0 & 0 & 0 & 0 & 0 & -0.0892 \end{bmatrix}$ $B_n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.4437 & 0.4437 \\ 1.4866 & -1.4866 \end{bmatrix}$

$C_n = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ $A_d = \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 & 0 & 0 \\ 0.1 & 0 & 0.2 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

IV. CONTROL DESIGN

In order to synthesize a robust H-infinity output feedback controller using LMI algorithms for the nominal plant when subjected to model uncertainties in unstructured form and output disturbances, the corresponding linear matrix inequalities have to be formulated first. The LMI solvers in MATLAB® are used for solving the inequalities.

Theorem: The system (1) is asymptotically stable and the norm of system transfer function from the disturbance w to the output y, $\|T_{wy}\| <= \gamma, \gamma > 0$, for $\tau > 0$ if there exist real symmetric matrices $P > 0, Q > 0$ satisfying the Arithmetic Riccati Equations

$$PA + A^T P + Q + PA_d Q^{-1} A_d{}^{-1} P + C^T C + \gamma^{-2} PB_1 B_1{}^T P < 0$$

The above inequality can be expressed as an LMI.

A Lyapunov function candidate for the system represented by Eqn(1) is given by

$$V(x,t) = x(t)^T Px(t) + \int_{t-\tau}^{t} x(\sigma)^T Qx(\sigma)d\sigma \qquad (29)$$

where $P, Q \in R^{nxn}$ are positive definite symmetric matrices [15]. If $P > 0, Q > 0$ satisfies $\dot{V}(x,t) < 0$ for every x satisfying Eqn(1), then the system (1) is stable, i.e., $x(t) \to 0$ as $t \to \infty$

Let $u(t) = Kx(t)$, then the resulting closed loop system is

$$\dot{x} = (A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t) \qquad (30)$$

The time derivative of $V(x,t)$ along the trajectory of the system represented by Eqn(30) is given by $L(x,t) = \dot{V}(x,t)$

$$= x(t)^T P[(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)] + [(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]^T Px(t) + x(t)^T Qx(t) - x(t-\tau)^T Qx(t-\tau)$$

Introduce the following performance measure

$$J = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt$$

This can be rewritten as $J \le [\int_0^\infty (z^T(t)z(t) - \gamma^2 w^T(t)w(t) + L(x,t)]dt$ (31)

Substituting $z(t)$ and $L(x,t)$ in the Eqn(31), it can be written as

$$J \le \int_0^\infty \{[cx(t)]^T [cx(t)] - \gamma^2 w^T(t)w(t) + x(t)^T P[(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)] + [(A_1 + B_2 K)x(t) + A_d x(t-\tau) + B_1 w(t)]^T Px(t) + x(t)^T Qx(t) - x(t-\tau)^T Qx(t-\tau)\} dt$$

From this performance measure inequality we get the linear matrix inequality for the system represented by Eqn (1) as

$$\begin{bmatrix} (A_1 X + B_2 Y)^T + A_1 X + B_2 Y + Q & B_1 & (C_1 X)^T & A_d X \\ B_1^T & -\gamma I & 0 & 0 \\ C_1 X & 0 & -\gamma I & 0 \\ X A_d^T & 0 & 0 & -Q \end{bmatrix} < 0 \qquad (32)$$

X>0

Where $X (= X^T)$, Q and Y are the matrices and γ is the H_∞ performance attenuation bound. A_1, A_d, B_1, B_2 and C_1 are known constant matrices with appropriate dimension. Using LMI tool box in MATLAB® we can get suitable matrices X and Y by the MATLAB code X=dec2mat(lmis,xfeas,x) and Y=dec2mat(lmis,xfeas,y).Then a state feedback robust H_∞ controller $u = YX^{-1}x(t)$ can be obtained to guarantee the stability of the system [16].

V. H-INFINITY CONTROL SCHEME

Because of the uncertainty in the system model, the model parameters may not be exact and are subjected to variations. These uncertainties due to the factors like modeling errors, parameter uncertainties and nonlinearities of the system should also be taken in to account while designing a robust controller,so that we get satisfactory control over a wider range of the operating variables. Based on the nominal model and the set of perturbed models in the neighborhood of the nominal model, a formulation of uncertainty is obtained and is represented as the uncertainty weight matrix. The perturbed model is represented in multiplicative input uncertainty form with the mathematical equation,

$$G_p = G_{nom}(I + W_m \Delta_m), \|\Delta_m\|_\alpha \le 1$$

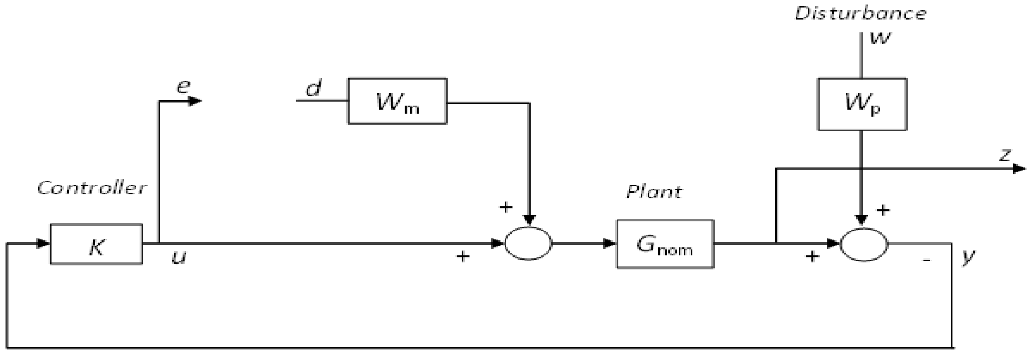


Figure:4. Augmented structure of Wheeled mobile robot with trailer

A suitable value of W_m can be selected in such a way that its magnitude covers all model perturbations. This model uncertainty weighting W_m can be represented as $W_m = C_m(sI - A_m)^{-1}B_m$.Again consider an output disturbance weighting W_p acting on the system which can be represented as $W_p = C_p(sI - A_P)^{-1}B_p$.

The augmented structure of the perturbed plant with Δ_m open is shown in Fig.4. Then the augmented state space representation of the perturbed system is in the form

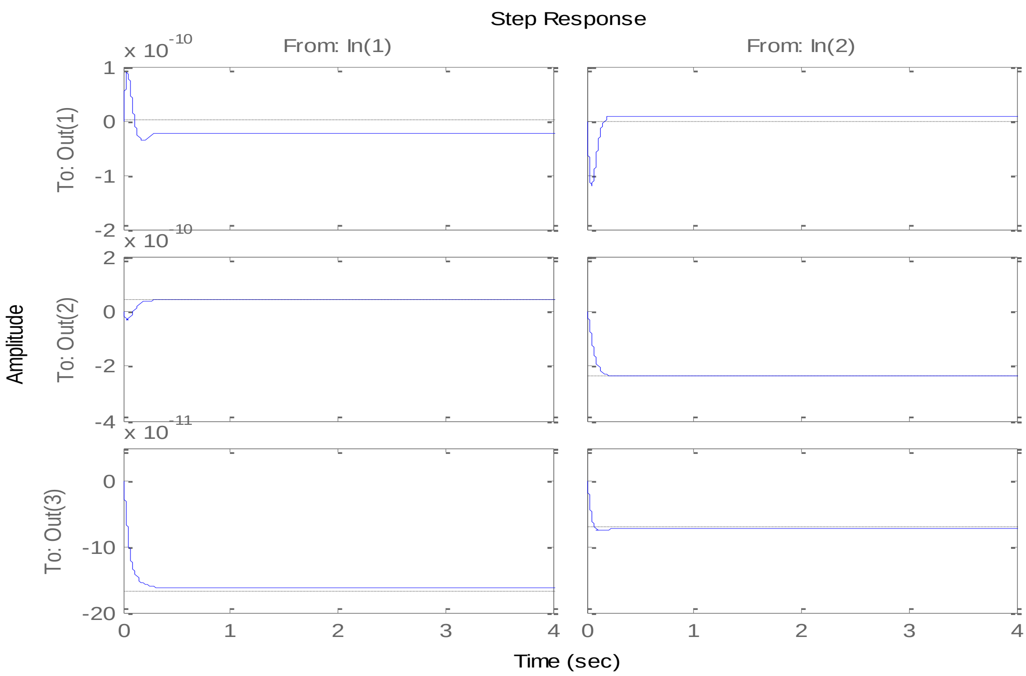
$$\dot{x} = Ax + B_1 w + B_2 u + A_d x(t - \tau)$$
$$y = C_2 x$$
$$z = C_1 x$$

Where $A = \begin{bmatrix} A_n & B_n C_m & 0 \\ 0 & A_m & 0 \\ 0 & 0 & A_P \end{bmatrix} B_1 = \begin{bmatrix} B_n D_m & 0 \\ B_m & 0 \\ 0 & B_p \end{bmatrix}$

$$B_2 = \begin{bmatrix} B_n \\ 0 \\ 0 \end{bmatrix} \qquad C_1 = \begin{bmatrix} 0 & 0 & 0 \\ c_z & 0 & 0 \end{bmatrix} \qquad C_2 = \begin{bmatrix} -C_n & 0 & -C_p \end{bmatrix}$$

VI. SIMULATION RESULTS AND DISCUSSION

In order to establish the effect of time delay in the WMR trailer system, three cases are considered. In the first case there is no output disturbance to the system. In this case the output disturbance weighting W_p becomes zero, and then only the model uncertainty weighting W_m is acting on the system. In the second case a first order output disturbance weighting W_p is acting on the system along with the model uncertainty weighting.



In the third case the output disturbance weighting W_p acting on the system is of second order. For all the above three cases an H-infinity controller is synthesized in MATLAB®with the help of LMI solvers. The closedloop responses of the system for different values of timedelay are plotted. Fig.5 & Fig.6 shows the closed loop responses of the given WMR trailer system having the time delay 0.1 sec& 0.5 sec respectively. From this

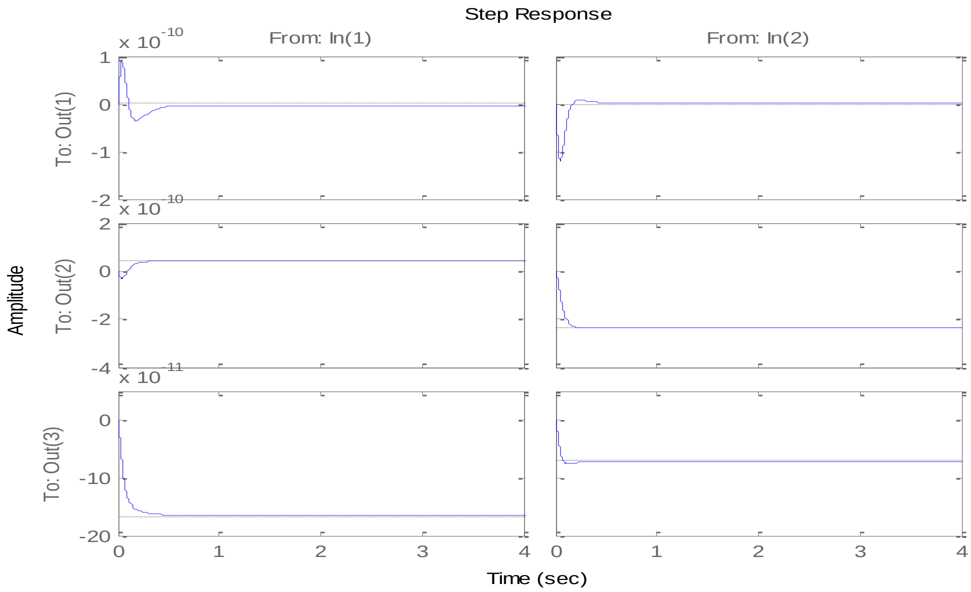


figure it can be seen that the given WMR system is stabilized by the proposed LMI H_∞control.

Figure:5. Closed loop response of the given WMR trailer system with time delay 0.1 sec.

Figure:6. Closed loop response of the given WMR trailer system with time delay 0.5 sec.

Table 2 gives the Performance specification of the above WMR trailer system having different values of time delay .The optimum value of disturbance attenuation bound is obtained as 0.00025.

Table 2 Performance specification of the WMR Trailer system having different values of time delay

Time delay (sec)	Settling time(sec)	Steady state error
0	0.715	0
0.1	0.73	0
0.5	0.527	0
1	0.828	0
2	0.828	0
3	0.828	0
4	0.828	0
5	0.828	0

From the Table 2 it can be seen that, as the value of time delay increases from 0 sec to 0.1 sec, the settling time also increases from 0.715 sec to 0.73 sec. When the time delay becomes 0.5 sec, the settling time decreases to 0.527 sec. As the delay reaches 1sec, the settling time increases to 0.828 sec and then remains constant for different values of delays.

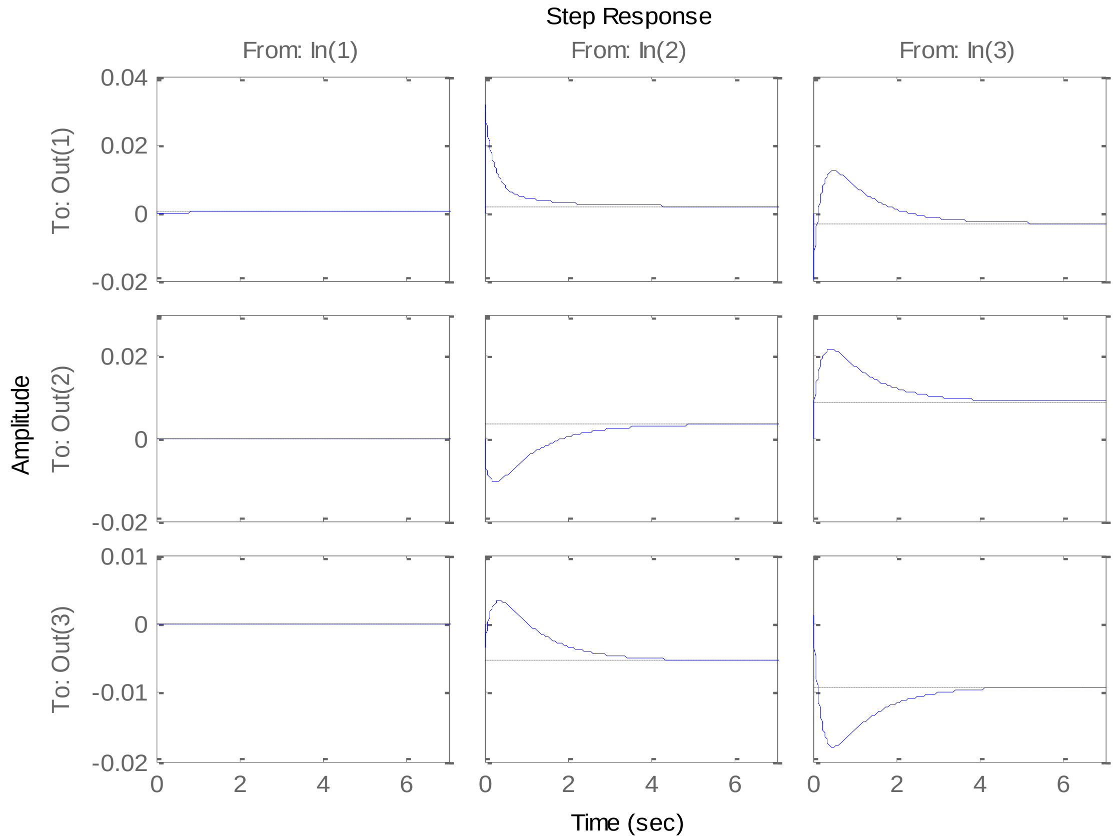


Figure:7. Closed loop response of the given WMR trailer system with $W_p = \frac{.0019s+.6}{s+1}$ having no delay

Fig.7 shows the closed loop responses of the given WMR trailer system with $W_p = \frac{.0019s+.6}{s+1}$ having no delay. As in the previous case here also it can be seen that the given WMR trailer system is stabilized by the proposed LMI H_∞ controller. Table 3 gives the Performance specification of the above WMR trailer systemwith $W_p = \frac{.0019s+.6}{s+1}$ having different values of time delay. The optimum value of disturbance attenuation bound is obtained as 0.2027.

Table: 3 Performance specification of the WMR Trailer system with $W_p = \frac{.0019s+.6}{s+1}$ having different values of time delay.

Time delay(sec)	Settling time(sec)	Steady state error
0	4.59	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
0.1	4.59	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
0.5	4.71	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
1	4.99	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
2	5.85	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
3	6.23	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
4	6.29	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919
5	6.3	0.00178&-0.0033 0.00337&0.00885 -0.00537&-.00919

From the Table 3 it can be seenthat,the settling time increases as the time delay increases.But the steady state error remains constant for every values of delays

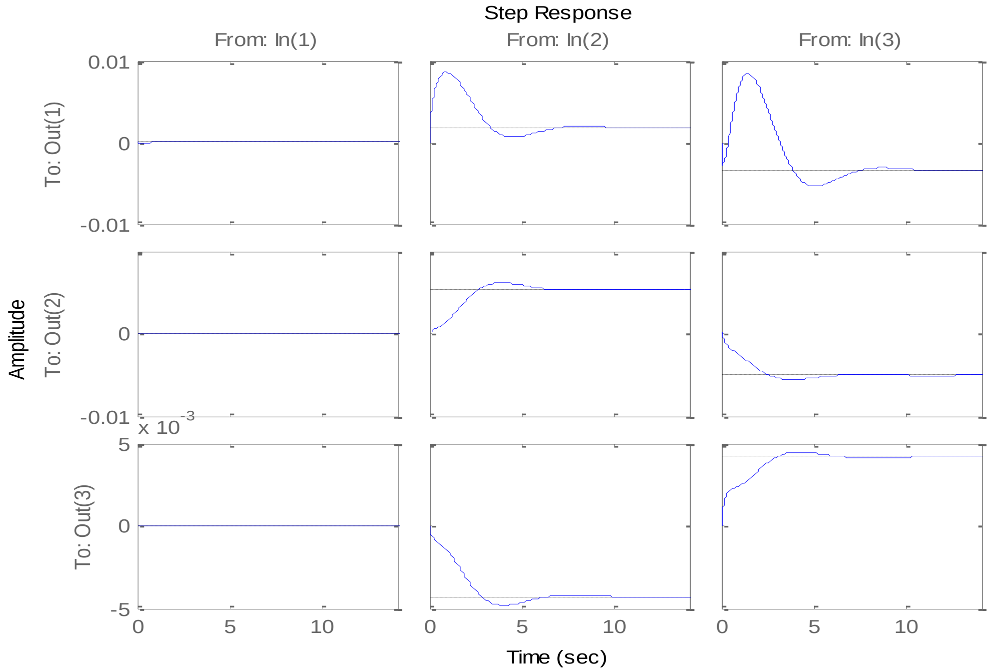


Figure:8. Closed loop response of the given WMR trailer system with $W_p = \frac{.0019s+.6}{s^2+s+1}$ having the delay 0.1 sec.

Fig.8 shows the closed loop responses of the given WMR trailer system with$W_p = \frac{.0019s+.6}{s^2+s+1}$having the delay 0.1 sec. As in the previous case here also it can be seen that the given WMR trailer system is stabilized by the proposed LMI H_∞controller. Table 4 gives the Performance specification of the above WMR trailer systemwith $W_p = \frac{.0019s+.6}{s^2+s+1}$having different values of time delay . The optimum disturbance attenuation bound obtained is 0.1594.

Table 4 Performance specification of the WMR Trailer system with $W_p = \frac{.0019s+.6}{s^2+s+1}$having different values of time delay

Time delay(sec)	Settling time(sec)	Steady state error
0	9.52	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
0.1	9.53	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
0.5	9.61	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
1	9.75	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
2	9.9	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
3	10.9	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
4	11.9	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418
5	13.1	0.00179&-0.00343 0.00542&-0.00505 -0.00429&0.00418

As in the previous case,here also it can be seen that as the time delay increases the settling time also increases, but the steady state error remains constant.It can be seen that even though the robust control of time delay systems are more complicated than that of the system without time delay, the proposed LMI H_∞controller is successful in controlling the WMR trailer system with both delay and uncertain disturbances.

VII. CONCLUSIONS

Robust control of WMR trailer system in the presence of disturbance as well as delay using LMI techniques was discussed here.Also this paper gives a linearized model of the WMR trailer system in the presence of disturbances, uncertainties and delays.A robust H-infinity output feedback controller is synthesized using LMI algorithms for the nominal plant when subjected to model uncertainties in unstructured form and output disturbances.The LMI solvers inMATLAB®were madeuse of for solving the inequalities.Analysis of the system without the output disturbance, with a first order output disturbance and with a second order output disturbance have been performed for different values of delays. The simulation results indicate that the WMR trailer system is stable with the proposed LMI H_∞ controller.

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