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Stochastic Analysis of Concrete Strength In An Ongoing Construction

Onwuka D.O.[1] and Sule, S.[2]

[1]*Department of Civil Engineering Federal University of Technology, Owerri, Imo State.*
[2]*Department of Civil and Environmental Engineering, University of Port Harcourt, P.M.B 5323 Port Harcourt, Rivers State - Nigeria.*

Abstract: - Structural safety evaluation is a task of paramount importance at every stage of a building process. In this paper, the result of stochastic analysis of concrete strength in an ongoing construction is discussed. Convolution theory was employed in the reliability estimation. The parameters used in the stochastic analysis were obtained from the schmidt hammer test carried out on the Laboratory Block at College of Continuing Education, University of Port Harcourt, Rivers State, Nigeria. The strength parameters were assumed to be random and stochastic. The obtained geometric index was found to be 2.97 which is less than the target safety index of 4.5 for slabs, 4.9 for beams in bending or flexure, 3.6 for beams in shear and 3.9 for columns under dead and live load combination. Also, the failure probability corresponding to the estimated geometric index (1.49E-3) when compared with the tolerable risk levels (10^{-3}) for structures in society showed that the structure is not safe and can lead to a very serious accident which may result in loss of lives and damage of properties on collapse.

Keywords: *- structural safety, building process, stochastic analysis, convolution theory, reliability estimation*

I. INTRODUCTION

Building failures in Nigeria has led to loss of lives and damage of properties. As a result reliability appraisal of structures becomes a necessity at every stage of a building process as a guide against structural failure and eventual collapse of structures [1-3]. Structural deterioration is a common reason for structural appraisal [4]. Condition assessment of a building is a necessity at every stage of a building construction rather than sitting down and watch the building collapse [5]-[6]. According to Afolayan [5-6], once the nature of the risk has been recognized the next step is the determination and implementation of measures to reduce the risk or reduce the effect of the loss or both at an economical cost. Eventually, the need for loss financing will be reduced in most instances and losses will be avoided or reduce to the bearest minimum.

Application of safety factors in the conventional design cannot guarantee structural safety as the applied loads are probabilistic in nature.

The best way to assess the safety of an existing or deteriorating structure is by probability of failure [8]. In structural design, structural loading and intensities cannot be predicted with certainty and probabilistic concept has become an important tool for any realistic, quantitative and rational analysis and any conceivable condition is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the structural significance of a specified condition can be assessed. Since it is not possible to achieve absolute reliability in the uncertain world, a probabilistic approach to the evaluation of structural safety becomes a sensible solution [9]. According to Afolayan [10], it has been the directional effort of the engagement of probabilistic thinking to systematically assess the effect of uncertainty on structural performance. The probabilistic concept may not provide answers to all issues of unknown but has played a very important role in the integrity appraisal of many engineering structures.

This paper highlights the use of probabilistic concept to assess the structural integrity of an ongoing construction. The probabilistic model is simple and straightforward and can be manually achieved.

II. FORMULATION OF STOCHASTIC MODEL

Let X and Y be the applied stress random variable and allowable stress random with statistical properties described by first and second moment, (μ_x, σ_x) and (μ_y, σ_y) respectively.

The limit state function is given by:

$$Z = X - Y \tag{1}$$

According to equation (1),
Violation of limit state occurs when:

$$z > 0, \tag{2}$$

Again, using equation (1), the probability of failure is given by:

$$P_f = \int_0^{\infty} g(z)dz. \tag{3}$$

The capacity demand are assumed to statistically independent.

$$f(x)\, f(y)\, dx\, dy.$$

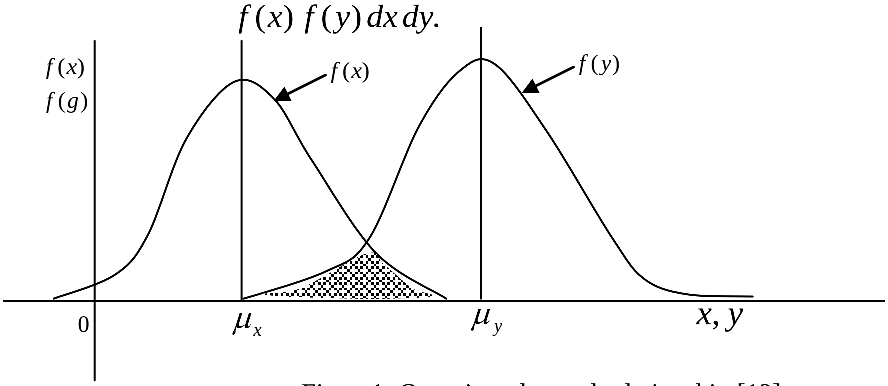

Figure1: Capacity –demand relationship [13]

Using equation (1), the joint density function of capacity and demand is transformed as:

$$f(x)\, f(x-z)\, dx.dz. \tag{4}$$

Using equation (1) and applying convolution theorem, the probability density function of Z given by:

$$g(z) = \int_a^b f(x)\, f(x-z)\, dx\, dz \tag{5}$$

where a and b represent the structural stress limits.

From Figure 1, X and Y are assumed to be normally distributed. Therefore, the probability density functions are given by equations (6) and (7) respectively [13].

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x} \right)^2 \right] \quad (-\infty, \ \infty), \tag{6}$$

$$f(y) = \frac{1}{\sigma_y \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \quad (-\infty, \ \infty), \tag{7}$$

Substituting for $f(x)$ and $f(x-z)$ using equation (6) and (7) gives:

$$g(z) = \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_x\sigma_z} \exp\left[-\frac{(x-\mu_x)^2}{2\sigma_x^{\ 2}} - \frac{(x-z-\mu_y)^2}{2\sigma_y^{\ 2}} \right] dx \tag{8}$$

Let the expression in the bracket be denoted by λ.
Therefore,

$$\lambda = -\frac{(x-\mu_x)^2}{2\sigma_x^{\ 2}} - \frac{(x-z-\mu_y)^2}{2\sigma_y^{\ 2}} \tag{9}$$

Multiplication of top and bottom of equation (9) by $\sigma_x^2 + \sigma_y^2$ gives:

$$\lambda = \frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x 2\sigma_y{}^2}\left(\frac{-2\sigma_y{}^2(x-\mu_x)^2 - 2\sigma_x{}^2(x-\cancel{z}-\mu_y)^2}{\sigma_x{}^2+\sigma_y{}^2}\right) \tag{10}$$

Simplification of bracketed terms in equation (10) gives equation (11).

$$\lambda = \frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x{}^2\, 2\sigma_y{}^2}\left[\frac{x^2 - 2x(\mu_x\sigma_z{}^2 + z\sigma_x{}^2 + \mu_y\sigma_x{}^2)}{\sigma_x{}^2+\sigma_y{}^2} + \mu_x{}^2\sigma_y{}^2 + \frac{(z^2+\mu_y{}^2+2z\mu_z).\sigma_x{}^2}{\sigma_x{}^2+\sigma_y{}^2}\right]. \tag{11}$$

Multiplying the top and bottom of the last term of equation (11) by $\sigma_x{}^2 + \sigma_y{}^2$ gives:

$$\lambda = -\frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x{}^2\, 2\sigma_y{}^2}\left\{\frac{x^2 - 2x(\mu_x\sigma_y{}^2 + (z+\mu_y)\sigma_x{}^2]}{\sigma_x{}^2+\sigma_y{}^2} + \frac{\mu_x{}^2\sigma_x{}^2\sigma_y{}^2 + (z^2+2z\mu_y+\mu_y{}^2)\sigma_x{}^4 + \mu_x{}^2\sigma_y{}^4 + (z^2+2z\mu_y+\mu_x{}^2)\sigma_x{}^2\sigma_y{}^2}{(\sigma_x{}^2+\sigma_y{}^2)^2}\right\} \tag{12}$$

According to Haugen [13], separation of the two middle terms of the last fraction of equation (12) from the other two terms followed by addition and subtraction of expression

$$\frac{2\mu_x(z+\mu_y)\sigma_x{}^2\sigma_y{}^2}{(\sigma_x{}^2+\sigma_y{}^2)^2}$$

transforms equation (12) to:

$$\lambda = \frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x{}^2\sigma_y{}^2}\left[x^2 - 2x\frac{\mu_x\sigma_y{}^2 + (z+\mu_y)\sigma_x{}^2}{\sigma_x{}^2+\sigma_y{}^2} + \frac{\mu_x{}^2\sigma_y{}^4 + 2\mu_x(z+\mu_y)\sigma_x{}^2\sigma_y{}^2 + (z+\mu_y)^2\sigma_x{}^4}{(\sigma_x{}^2+\sigma_y{}^2)} + \frac{\mu_x{}^2 - 2\mu_x(z+\mu_y) + (z+\mu_y)^2}{(\sigma_x{}^2+\sigma_y{}^2)^2}\sigma_x{}^2\sigma_y{}^2\right] \tag{13}$$

Also, multiplying the last term of equation (13) by $-\dfrac{\sigma_x{}^2+\sigma_y{}^2}{2\sigma_x{}^2\sigma_y{}^2}$ transforms equation (13) to:

$$\lambda = -\frac{\sigma_x{}^2 + \sigma_y{}^2}{2\sigma_x{}^2\sigma_y{}^2}\left(x - \frac{\mu_x\sigma_y{}^2 + z + \mu_y)\sigma_x{}^2}{\sigma_x{}^2+\sigma_y{}^2}\right)^2 - \frac{(z+\mu_y-\mu_x)^2}{2(\sigma_x{}^2+\sigma_y{}^2)} \tag{14}$$

Using equation (14), equation (8) now becomes:

$$g(z) = \frac{1}{\sqrt{2\pi(\sigma_x{}^2+\sigma_y{}^2)}}\exp\left\{-\frac{\sigma_x{}^2+\sigma_y{}^2}{2\sigma_x{}^2 y^2}\left[x - \frac{\mu_x\sigma_y{}^2 + (z+\mu_y)\sigma_x{}^2}{\sigma_x{}^2+\sigma_y{}^2}\right] - \frac{(z+\mu_y-\mu_x)^2}{2(\sigma_x{}^2+\sigma_y{}^2)}\right\}dx, \tag{15}$$

Let

$$\alpha = \int_{-\infty}^{\infty} \frac{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}}{\sqrt{2\pi\sigma_x\sigma_z}} \exp\left\{ \frac{-\sigma_x^{\ 2} + \sigma_y^{\ 2}}{2\sigma_x^{\ 2} 2\sigma_z^{\ 2}} \left[\frac{x - \mu_x\sigma_y^{\ 2} + (z + \mu_y)^2}{\sigma_x^{\ 2} + \sigma_y^{\ 2}} \right]^2 \right\} dx$$

(16)

Equation (15) now becomes:

$$g(z) = \frac{1}{\sqrt{2\pi(\sigma_x^{\ 2} + \sigma_y^{\ 2})}} \exp\left[-\frac{(z + \mu_y - \mu_x)}{2(\sigma_x^{\ 2} + \sigma_y^{\ 2})} \right] \alpha, \qquad (17)$$

From equation (14), let

$$t = \frac{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}}{\sqrt{\sigma_x\sigma_y}} \left[x - \frac{\mu_x\sigma_y^{\ 2} + (z + \mu_y)\sigma_x^{\ 2}}{\sigma_x^{\ 2} + \sigma_y^{\ 2}} \right], \qquad (18)$$

Differentiating t with respect to x in equation (18) yields:

$$dt = \frac{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}}{\sigma_x\sigma_y} dx, \qquad (19)$$

Substituting for t *and* dx in equation (16), we have:

$$\alpha = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-(t^2/2)} dt \qquad (20)$$

From statistics and probability,

$$\int_{-\infty}^{\infty} pdfdx = 1 \qquad (21)$$

Therefore, equation (15) now transforms to:

$$g(z) = \frac{1}{\sqrt{2\pi(\sigma_x^{\ 2} + \sigma_y^{\ 2})}} \exp\left[-\frac{1}{2} \frac{(z + \mu_y - \mu_x)^2}{(\sigma_x^{\ 2} + \sigma_y^{\ 2})} \right], \qquad (22)$$

From Figure 1, Z is a normally distributed random variable. The mean and standard deviation are therefore:

$$\mu_z = \mu_x - \mu_y\,, \qquad (23)$$

$$\sigma_z = \sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}} \qquad (24)$$

The probability that the structure fulfils the intended purpose is structural reliability defined by:

$$\text{Reliability} = \int_0^{\infty} g(z)\,dz. \qquad (25)$$

Again, let

$$t = \frac{z + \mu_y - \mu_x}{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}}; \qquad (26)$$

Differentiation of equation (26) with respect to x yields:

$$dt = \frac{d_z}{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}} \qquad (27)$$

Using equation (27), equation (25) now transforms to:

$$\text{Reliability} = \int_0^{\infty} g(z)dz = \int_0^{\infty} \frac{\mu_y - \mu_x}{\sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}}} \frac{1}{2\pi} e^{-(t/2)}\, dt \qquad (28)$$

Using equation (28), the transformation which relates μ_x, μ_y and standard normalized variable z is given by:

$$z = \frac{\mu_y - \mu_x}{\sqrt{\sigma_x^2 + \sigma_y^2}} \tag{29}$$

Applied stress = 0

Therefore,

$$y = \mu_y = \sigma_y = 0 \tag{30}$$

Using equation (29) now reduces to:

$$z = \frac{|\mu_x|}{\sigma_x} \tag{31}$$

Let σ_{cu} *and* x represent the concrete cube strength and strength of concrete in an ongoing construction respectively.

According to BS8110 [11], the mean design strength is given by:

$$\mu_x = 0.67\sigma_{cu} \tag{32}$$

To cater for error in the formulated reliability mode, errors due to test procedures and errors due to in-batch variabilities of concrete strength reinforcement strength and dimensional variability, the resultant coefficient of variation of concrete strength is given as:

$$COV_{Resultant} = \left(COV_y^2 + COV_{testing}^2 + COV_{in-batch}^2\right)^{1/2} \tag{33}$$

Where:

$\mu_x, \sigma_x =$ mean value and standard deviation of structural capacity respectively.

COV_y is a function of the mix design

According to Ranganathan [2], $COV_{testing} = COV_{in-batch} = 0.10$.

Structural failure occurs when $X < \sigma_{all}$. Therefore, the probability of failure (P_{fi}) for a particular structural member is given as:

$$P_{fi} = P(X_i < \sigma_{all}) \tag{34}$$

Where:

P, σ_{all} represents probability operator and allowable concrete stress in axial compression respectively.

According to BS8110 [11]

$$\sigma_{all} = 0.33\sigma_{cu} \tag{35}$$

Assuming X to be normally distributed, the probability of failure is the structure is given by:

$$P_f = \phi\left(\frac{\sigma_{all} - \mu_x}{\sigma_x}\right) \tag{36}$$

Using equations (32) and (35), equation (36) can be written as:

$$P_f = \phi\left(\frac{0.33\sigma_u - 0.67\sigma_{cu}}{COV_{Resultant}(0.67\sigma_{cu})}\right), \left(\sigma_x = COV_{Resultant} \times 0.67\sigma_{cu}\right) \tag{37}$$

According to Ranganathan [2], the probability of structural failure can be approximated as:

$$P_f \approx \phi(-\beta) \tag{38}$$

Where:

$\phi(.)$ is the standard Gaussian cumulated function and

$$\beta = \min \| u \| = \left(\sum_{i=1}^{n} X_i^2\right)^{1/2}$$

$\|u\|$= minimum distance between the origin and the failure surface in the normalized coordinate
u_{ii} represents an appropriate probabilistic transformation.

III. RESULTS AND DISCUSSION

Table 1: Results of Schmidt hammer test on concrete [1].

S/No	Location	Rebound Hammer readings	Average Rebound	Concrete Strength from Rebound Test (y)
1	Middle panel	23,23	23	18
2	Edge panel	23,23	23	18
3	Beam 2	20,20	20	14
4	Slab 2	24,24	24	20
5	Slab 1	18, 19	19	8
6	Beam 1	12,12	12	5
7	Staircase	23.3, 19	21.2	15
8	Middle column	35,27	31	29
9	Corner column	27,27	27	2.5
10	Column footing	12.5,6	9	4
				$\mu_y = \sum_{i=1}^{10} \frac{y_i}{10} = 15\,N/mm^2$

Table 2: Stochastic model [2].

Variable	Mix	Specified strength	Mean (μ_y) (N/mm^2)	Std deviation $\sigma_y\,(N/mm^2)$	COV (σ_y) (%)	Probability distribution	Quality control
Cube strength	Grade 15	15	17.56	2.69	15.33	Normal	Design mix

From Table 2, $\mu_{\sigma cu} = 17.56\,N/mm^2, \sigma_{\sigma cu} = 2.69\,N/mm^2$ and $COV_y = 0.1533.$
Using equation (33),

$$COV_{Resultant} = (0.1533^2 + 0.10^2)^{1/2} = 0.18$$

From equation (32),

$$\mu_x = 0.67\,\sigma_{cu} = 0.67 \times 17.56 = 11.76\,N/mm^2$$

$$\sigma_x = COV_{Resultant}\,(0.67\,\sigma_{cu}) = 0.67 \times 0.18 \times 17.56 = 2.24\,N/mm^2$$

From equation (35),

$$\sigma_{all} = 0.34\,\sigma_{cu} = 0.34 \times 15 = 5.10\,N/mm^2$$

From equation (37), the probability of failure of concrete is structure is:

$$P_f = \varphi\left(\frac{5.10 - 11.76}{2.24}\right) = \varphi(-2.97) = 1.49\,x10^{-3}$$

IV. DISCUSSION OF RESULTS AND CONCLUSION

The results of stochastic appraisal of an ongoing construction using convolution theory has been presented. From Table 1, it can seen that the average strength of concrete in the as constructed structure is about 15N/mm^2. The as-constructed safety appraisal gave a geometric index value of 2.97 which is below the target value of 4.9 for beams in bending or flexure, 3.6 for beams in shear, 4.5 for slabs, and 3.9 for columns subjected to both dead and live load combination.

In conclusion, the structure cannot perform satisfactorily in service and can cause serious accident and serious damage to properties on collapse. The structure is therefore, recommended for careful demolition to give rise to a new structure and more stringent supervision should be carried out Also, reliability prediction using convolution theory gave the same result (geometric index = 2.97) as those of the previous models showing the effectiveness of the convolution theory in the reliability prediction of an ongoing construction.

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