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Free Convective Heat Transfer From A Vertical Surface For The Case Of Linearly Varying Thermal Potential

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Abstract: - The present theoretical investigation deals with the problem of free convective heat transfer from a vertical plate having linear temperature gradient along its surface to the surrounding thermally stratified fluid. Integral method of analysis is adopted to investigate the effect of four parameters viz., the gradients of temperature in the fluid and the wall, Grashof number and Prandtl number on heat transfer coefficients. It is observed from the numerical results that an increase in the surface temperature gradient would result in higher heat transfer coefficients than those observed in isothermal wall case.

I. INTRODUCTION

In the atmosphere around us we continually encounter transport processes in fluids wherein the motion is driven by the buoyancy effect due to temperature differences. The problem of natural convection from a vertical surface is solved [1] for the case of either constant temperature of the surface or constant heat flux at the surface. Also, the problem of natural convection heat transfer from bodies submerged is stratified fluids arises in many important applications. A few studies on the effect of stratification are available [2,3,4]. However, in certain practical instances, the temperature of the wall may vary arbitrarily, thus not conforming to either of the two cases mentioned above, namely the case of constant wall temperature and the case of constant wall heat flux. In view of this fact, for the sake of completeness in the literature, presently a different situation is considered in which a linear variation in temperature along the vertical surface is imposed together with linear thermal stratification in the bulk fluid. The problem for this case solved theoritically to study the effect of heat transfer rate of various parameters such as Grashof number and Prandtl number of the fluid and thermal gradients of the surface and the fluid.

II. FORMULATION AND ANALYSIS

In the physical model under consideration, as shown in Fig.1 Heat transfer by free convection takes place from a vertical surface into a thermally stratified fluid. The heating conditions on the surface are maintained such that there is a linear variation in the temperature along the surface. Further, the thermal stratification in the fluid is assumed to be linear. Thus, the temperature variations along the surface and in the fluid medium are characterised by the following equations.

$$y = 0, \frac{dT_W}{dx} = m_1 = \text{constant} \quad .. \quad (1)$$

$$y = \delta, \frac{dT_\infty}{dx} = m_2 = \text{constant} \quad .. \quad (2)$$

The rate of heat transfer from the vertical surface to the fluid medium under the conditions stated above is governed by the conservation equations in integral form as shown below

$$\frac{d}{dx}\int_0^\delta u^2 dy = -\nu \frac{\partial u}{\partial y}|y=0 + g\,\beta \int_0^\delta (T - T_\infty) dy \quad ..(3)$$

$$\frac{d}{dx}\int_0^\delta (T - T_\infty) dy + m_2 \int_0^\delta u\, dy = -a \frac{\partial T}{\partial y}|y=0 \quad ..(4)$$

The following velocity and temperature profiles are chosen in the boundary layer, i.e. for $0 \le y \le \delta$

$$\frac{u}{u_0} = y^+(1-y^+)^2 \qquad ..(5)$$

$$\frac{T-T_\infty}{T_W-T_\infty} = (1-y^+)^2 \qquad ..(6)$$

where $y^+ = y/\delta$

Eqs. (5) and (6) obey the boundary conditions.

y = 0, u = 0 .. (7)

y = 0, $T = T_W$ where $T_W = T_\infty(x)$ .. (8)

y = $\delta, u = 0$ *and* $\frac{\partial u}{\partial y} = 0$ .. (9)

y = $\delta, \frac{\partial T}{\partial y} = 0$ *and* $T = T_\infty$

where $T_\infty = T_\infty(x)$ .. (10)

Eqs. (3) and (4) are evaluated with the aid of equations (5) and (6) to yield the system of constitutive differential equations in dimensionless form as given below:

$$\frac{d\delta^+}{dx^+} = \frac{15(7\,\mathrm{Pr}+8)}{\mathrm{Pr}\,Gr^{1/2}\delta^+U_0^+} - \frac{35\delta^+}{U_0^{+2}}$$

$$\frac{\delta^+}{\theta}(2\,M_1 + 3\,M_2) \qquad ..(11)$$

$$\frac{dU_0^+}{dx^+} = \frac{35\theta}{U_0^+} - \frac{15(7\,Pr+4)}{Pr\;Gr^+\delta^{+2}}$$

$$+\frac{U_0^+}{2\theta}(2M_1 + 3M_2) \qquad ..(12)$$

Eqs. (11) and (12) are numerically solved by Fourth-order Runge-Kutta method with the initial condition that at x = 0, $\delta^+ = 0$ *and* $U_0^+ = 0$. The heat transfer coefficient is defined by the equations

$$-\lambda\,\frac{\partial T}{\partial Y}\,|y=0 \quad = \alpha\,(T_W - T_\infty) \quad ..(13)$$

Results are presented to study the efforts of various system parameters on local as well as average heat transfer coefficients.

III. DISCUSSION OF RESULTS

In the present theoretical investigation effects of parameters M_1, M_2, Grashof number and Prandtl number on heat transfer rates are studied, where M_1 *and* M_2 signify the gradient in surface temperature and the degree of thermal stratification in the fluid respectively. In Fig.2 effect of M_2 on local Nusselt number is shown where M_1 is maintained constant at zero.

Fig.2 indicates that an increase in the degree of thermal stratification in the fluid is associated with an increase in the local Nusselt number, thus reaffirming the previous analyses [5,6]. It is found that a gradient in the surface temperature namely M_1 would cause a greater effect on local heat transfer coefficients, as evident from Fig.3. A positive gradient feature favours the local heat transfer rates, while a negative gradient cause a decrease in local heat transfer coefficient. However there is a limitation together on the values of M_1 *and* M_2 *such that at* $x = 1$, $(1 + M_1 - M_2)$ should always be greater than zero. A general observation of Fig.4, Fig.5, reveals the fact that the effect of M_1 on the average Nusselt number is more pronounced than that of M_2. In conclusion a combined effect of a gradient in the surface temperature and a thermal stratification in the fluid on the heat transfer rates is studied. The effect of these parameters is to greatly enhance the heat transfer rates in relation to the isothermal case of wall and ambient medium.

IV. NOMENCLATURE

$\bar{h}$ average heat transfer coefficient

m_1 gradient in the temperature of the surface, ${dT_W}/{dx}$

m_2 degree of thermal stratification in the fluid ${dT_x}/{dx}$

M_1 ${1m_1}/{(T_{W_0} - T_{\infty 0})}$

M_2 ${1m_2}/{(T_{W_0} - T_{\infty 0})}$

$\bar{N}u$ average nusselt number, ${\bar{h}1}/{K}$

U_0 convection velocity
U_0^+ $u_0 / [g\, 1\, \beta\, (T_{W_0} - T_{\infty_0})]$
x^+ $x / 1$
y^+ y / δ
δ thickness of boundary layer
δ^+ $\delta / 1$
θ $(T_W - T_\infty) / (T_{W_0} - T_{\infty_0})$

Subscripts
o $at\ x = o$
w wall
∞ $bulk\ fluid$

V. ACKNOWLEDGEMENT

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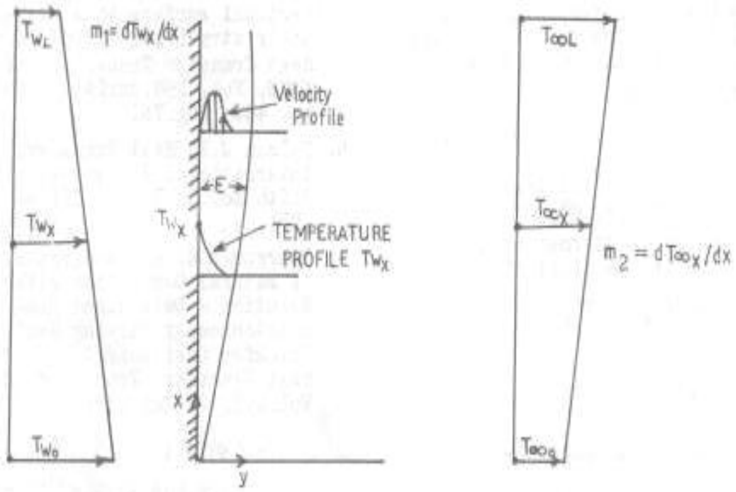

Fig.1 PHYSICAL MODEL

Figures

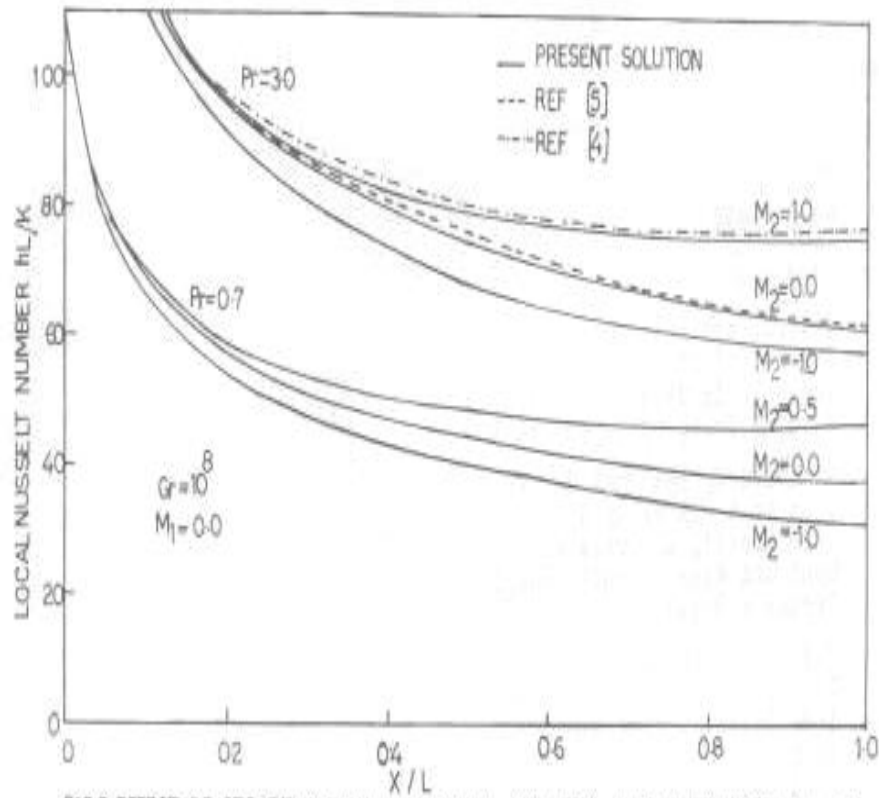

FIG.2 EFFECT OF STRATIFICATION IN FLUID M_2 ON LOCAL NUSSELT NUMBER, M_1=0.0

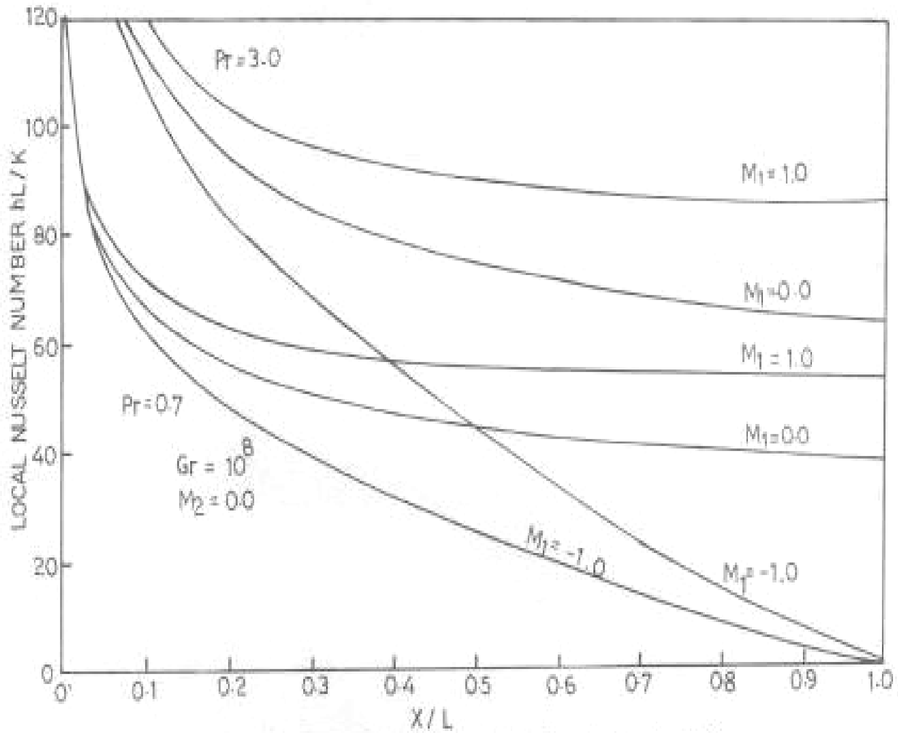

FIG.3 EFFECT OF PARAMETER M_1 ON LOCAL NUSSELT NUMBER, M_2=0.0

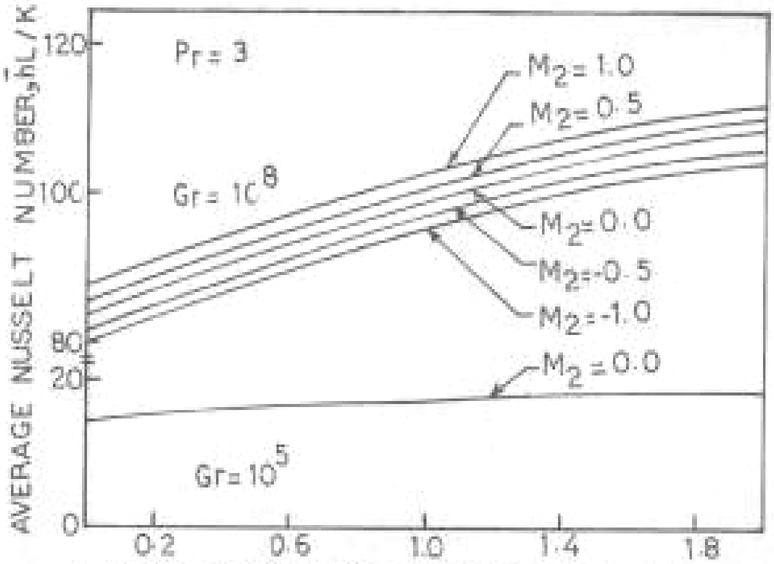

FIG.4 EFFECT OF M_1 ON $\bar{N}u$ WITH M_2 AS PARAMETER

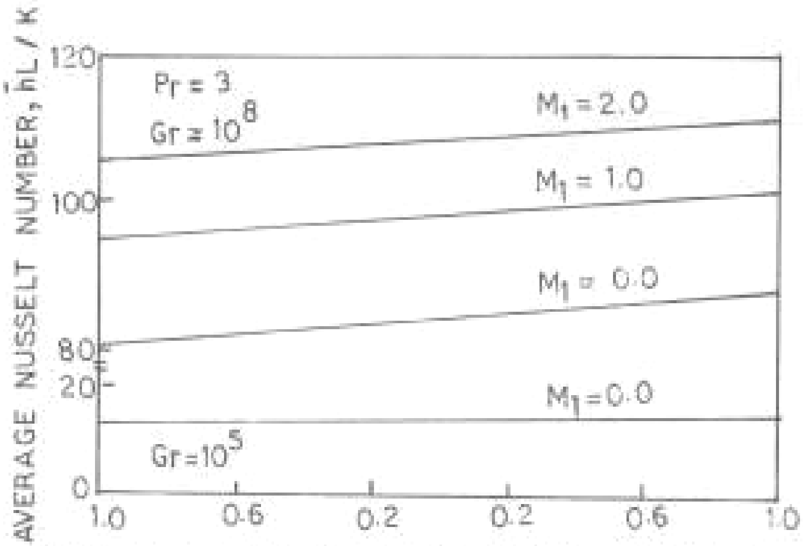

FIG.5 EFFECT OF M_2 ON $\bar{N}u$ WITH M_1 AS PARAMETER.