

## Experimental Validation of Hottel's Transmittance Model for Estimating Beam Radiation In Makurdi Location

<sup>1</sup>Edeoja, Alex Okibe and <sup>1</sup>Eloka-Eboka, C. Andrew

<sup>1</sup>Mechanical Engineering Department, University of Agriculture, Makurdi, Nigeria

**Abstract:** - Hottel's transmittance model for beam radiation was experimentally validated for Makurdi, Nigeria. Hourly, daily, weekly and monthly average beam radiation were determined from measured total/global radiation using a daystar sun-meter over a period July and November when the sun was at the northern hemisphere. These were correlated and compared with the average predicted by Hottel's model using analysis of variation (ANOVA) at 5% and 1% levels of significance, mean bias difference (MBD) and Root mean square difference (RMSD). The results indicated high significant differences at all levels tested with a variation of 2.9029 of the MBD and 3.1237 of RMSD and correlation coefficients of -0.8202 with measured normal radiation and -0.6397 with measured total radiation. This indicates a slim suitability of Hottel's model in Makurdi location. Climatic factors such as humidity, seasonal variation and weather may have caused the variation because they were not directly taken into consideration in the development of the model. The model could however be used by applying appropriate correction factors that can be obtained and employed which may make up for the limitation(s) for Makurdi and other locations. A model for Makurdi location like Hottel's is now of interest.

**Keywords:** - Hottel's model, beam radiation, Analysis of variation, Mean bias difference, Root mean square difference, Climatic factors.

### I. INTRODUCTION

Solar energy is one of the most significant sources of energy on the globe. One of the most outstanding factors in predicting, studying and designing solar energy systems in each site is to have the exact information and statistics regarding the degree of solar radiation on that site, to the extent that it is necessary to estimate the amount of solar energy in each area before measurement and scheduling for its use [1]. The radiation emitted per unit area of the sun is approximately  $6.33 \times 10^7$  W/hectare. If the energy coming from just 10 hectares of the surface of the sun could be harvested, it would be enough to supply sufficient energy to the world. This is however not possible since the amount of radiation leaving a unit area of the sun surface is not the same amount reaching equal unit area on the earth surface. The reason for this reduction is due to some predictable factors like the earth displacement from the sun, the earth's atmosphere and around the earth. As a result of these reductive factors, engineers and designers have found it important to know the estimation of radiation reaching the earth's surface based on the location for its usefulness in proper analysis of solar collection systems. The hourly and also instantaneous distribution of solar radiation is needed in various applications in the field of solar energy. The most reliable predictions of solar system performance are based on pyranometer data taken over a period of year at the place of interest. In many instances, however, available solar radiation data that have been collected are presented as a sum of integrated daily values [2–4]. In addition, the intensity of atmospheric radiant energy reaching the earth's surface could also be affected by the extent of cloud cover and dust particles as aerosol over a location. The radiation reaching the earth is reduced by 30% on a clear and by 90% on a hazy (cloudy) day. Therefore, it is appropriate to say that solar designed systems perform better on a clear day since there are more radiations. So, the need to ascertain the estimation of radiation (direct or beam, diffuse and total radiation) to a surface (vertical, horizontal and tilted) on a clear day in a particular location (s) using either analytical approach (models) or experimental measurements becomes very paramount and essential. These analytical methods whether simple or complex, were developed by different authors at different locations for

estimating solar radiations.

The successful design and effective utilization of solar energy systems and devices for application in various facets of human endeavors, such as power and water supply for industrial, agricultural and domestic uses, largely depend on the availability of information on solar radiation characteristic of the location in which the system and devices should be situated. [5]. Accurately computing solar irradiance on external facades is a prerequisite for reliably predicting thermal behavior and cooling loads of buildings, and other solar energy systems. Validation of radiation models and algorithms implemented in building energy simulation codes or other areas then becomes an essential endeavor for evaluating solar gain models [6-11].

Hourly radiation data is largely inexistent in Nigeria and in most places in the sub-Sahara regions of Africa, hence designers of solar energy systems usually resort to empirical models to evaluate hourly inputs of diffuse and beam irradiance for designs and optimization of different solar systems or more recently satellite information. There are several models which are available in literature used in predicting hourly solar irradiance. [12] presented an empirical model for determining the monthly average daily global solar radiation on a horizontal surface for Makurdi, Nigeria (Latitude 7°7'N and Longitude 8°6'E) was developed using the Angstrom-Page equation. The solar radiation ( $W/m^2$ ), hours of bright sunshine and cloudiness were measured hourly from 0600 H to 1800 H daily for 18 months. The constants 'a' and 'b' of the Angstrom linear type equation were determined by plotting the clearness index ( $H/H_0$ ) against the possible sunshine hours ( $n/N$ ) to obtain the line of best fit. The constant 'a' was obtained from the intercept of the line on the y-axis while the constant 'b' was obtained from the slope of the line. The developed model for determining the global horizontal solar radiation at the location was

$$H = H_0 \left[ 0.17 + 0.68 \left( \frac{n}{N} \right) \right] \quad (1)$$

with a coefficient of correlation of 0.78. The mean bias error and root mean square error that were used to test the performance of the constants were 0.17% and 1.22% respectively. The measured solar radiation was compared with the solar radiation predicted by the model and no significant difference was found between them using F-LSD at  $P \geq 0.05$ . [8] proposed a mono-variable model of monthly mean daily diffuse solar radiation on horizontal surfaces for some cities between 2°N and 5°N of Cameroon (Bertoua, Yokadouma, Yaoundé, Kribi, Kumba). The estimation was based on a correlation between clearance index and diffuse to global solar radiation ratio and was computed using monthly mean daily data set for global solar radiation on horizontal surfaces. The predictive efficiency of the proposed model was compared with the observed values and those believed to be universally applicable. The results suggest that the existing methods could be replaced by the developed model for a diffuse solar radiation data generation scheme. [13] investigated the effectiveness of an innovative procedure to calculate the global real sky irradiance of a mountain urban region, the city of Trento (Italy). The proposed methodology improves the predictive Bird's real-sky model by introducing in it both atmospheric parameters, specifically defined for the analyzed site, and a local cloud cover factor, based on experimental data, to calculate the global real sky irradiance. A multiple linear regression was applied to explain the relationship among Angstrom coefficients and geographical and meteorological data sets which were monthly mean clear sky or extraterrestrial radiation, the ratio of sunshine hours to day length, ambient and soil temperatures, relative humidity, sine of declination angle [14]. Variables in these equations were used to estimate the global solar radiation. Values calculated from models were compared with the meteorological values. Twenty four models having the best determination coefficient were compared with measured meteorological values by using statistical tests. In this study, first of all it was seen that the clear sky radiation can be used to estimate the global solar radiation in Bishkek. Finally, the using of the geographical and meteorological variables commonly has given the good results in estimating global solar radiation in Bishkek, Kyrgyzstan.

However, the complexity of some of them makes it difficult to put to appropriate use in Nigeria for radiation prediction. This is because the models require in many cases, meteorological inputs that may not be readily accessible due to the non-availability of measuring instrumentation. Angstrom turbidity coefficient,  $\beta$ , or Linke's turbidity factor, for example, cannot be easily measured in Nigeria. Other parameters not readily available include ground and sky albedos, ozone amount, predictable water vapor, illuminance, pyrheliometric instruments at narrow wavelengths and spectral irradiance. All these are contributory factors why complex models cannot be easily employed for estimation of solar radiation and therefore are not put to use. [9, 15-17]. There are some other very simple models which may not require any need of meteorological inputs to determine clear sky irradiance. However, as earlier established the utilization of simple or complex models require some level of validation in order to have an idea of the degree of relevance or applicability.

This present work considers an experimental validation of a simple model, Hottel's transmittance model for beam radiation on a clear day, for use in estimating solar irradiance in Makurdi generally and particularly for University of Agriculture, Makurdi as an experimental case point for data collection and

measurement.

## 1.2 Theoretical Background

Several radiation models have been postulated and are been utilized to estimate radiation values for various locations. For convenience, these models may be classified into those for inclined surfaces and the ones for horizontal surfaces. Many of these models are assumed to be applicable in locations with at least similar environmental conditions.

Hottel presented a simple model for the estimation of the transmittance of beam radiation in clear sky conditions [18]. The required inputs are the altitude of the location (in km above sea level), day number in the year and the zenith angle of the location. According to this model, the atmospheric transmittance for beam radiation  $\tau_b$  is given by (2):

$$\tau_b = \frac{I_b}{I_o} \quad (2)$$

where,  $I_b$  is the hourly normal beam radiation, and  $I_o$  is the hourly beam radiation outside the Earth's atmosphere (extraterrestrial radiation) given by (3):

$$I_o = I_{SC} \left[ 1 + 0.033 \left( \frac{360N}{365.25} \right) \right] \quad (3)$$

where  $I_{SC}$  is the solar constant approximately equal to  $1367 \text{W/m}^2$  and  $1 \leq N \leq 365$ .

The clear sky beam normal radiation is given by (4):

$$I_{cnb} = I_o \tau_b \quad (4)$$

The clear sky horizontal beam radiation is given as in (5):

$$I_b = I_o \tau_b \cos \theta_z \quad (5)$$

For a horizontal surface, the angle of incidence  $\theta$  is obtained from (6):

$$\cos \theta = \cos \theta_z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \omega \quad (6)$$

The declination angle  $\delta$  which accounts for the seasonal changes of the sun's path through the sky throughout the months of the year is given in degrees by (7):

$$\delta = 23.45 \sin^{-1} \left[ 360 \left( \frac{284 + N}{365} \right) \right] \quad (7)$$

The hour angle  $\omega$  can be determined from the expression given below in (8):

$$\omega = 15(ST - 12) \quad (8)$$

ST is the solar time in hours and  $\varphi$  is the latitude of the location. Hottel defined the atmospheric transmittance as in (9):

$$\tau_b = a_o + a_1 e^{(-k/\cos \theta_z)} \quad (9)$$

The constants  $a_o$ ,  $a_1$  and  $k$  can be determined using the correction factors shown below in (10) for all climates:

$$r_o = \frac{a_o}{\hat{a}_o}, \quad r_1 = \frac{a_1}{\hat{a}_1}, \quad \text{and} \quad r_k = \frac{k}{\hat{k}} \quad (10)$$

For tropical locations,  $r_o = 0.95$ ,  $r_1 = 0.98$  and  $r_k = 1.02$ . For altitudes less than 2.5km the constants  $\hat{a}_o$ ,  $\hat{a}_1$  and  $\hat{k}$  are given by (11 – 13):

$$\hat{a}_o = 0.4237 - 0.0082(6 - A)^2 \quad (11)$$

$$\hat{a}_1 = 0.5055 + 0.0595(6.5 - A)^2 \quad (12)$$

$$\hat{k} = 0.2711 + 0.01858(2.5 - A)^2 \quad (13)$$

For an urban haze atmosphere, the constants are given by (14 – 16):

$$\hat{a}_o = 0.2538 - 0.0063(6 - A)^2 \quad (14)$$

$$\hat{a}_1 = 0.7678 + 0.01858(6.5 - A)^2 \quad (15)$$

$$\hat{k} = 0.249 + 0.081(2.5 - A)^2 \quad (16)$$

where  $A$  is the altitude of the location above sea level in kilometers. Thus, the transmittance of the standard atmosphere for beam radiation can be determined for any zenith angle and altitude below 2.5km.

## II. MATERIALS AND METHODS

### 2.1 Generation of Data

Hourly, daily, weekly and monthly data were generated by experimental measurements at the Engineering Complex of the Federal University of Agriculture, Makurdi, Benue State of Nigeria. Measurements were carried out daily from 0800 – 1700 hours between July 2nd to November, 17<sup>th</sup> 2007 excluding Sundays and cloudy days. Hourly, daily, weekly and monthly data were also generated using Hottel's clear day model.

**2.1.1 Experimental measurements of total/global radiation**

Experimental measurements of the total radiation were obtained by the use of sun-meter during the 10 hours periods of experimentation (0800 -1700 hours) for insolation, bright cloud and cloudy parameters and converted using the prescribed equations. Hourly conversion from total radiation to direct normal radiation was calculated using equation (2).

**2.1.2 Analytical method for beam radiation estimation using Hottel’s model**

Analytical data generation employed (9) with  $a_0$ ,  $a_1$  and  $k$  being determined using correction factor as described by (10 – 14). The declination angle,  $\delta$ , which accounted for the seasonal changes of the sun’s path through the sky throughout the months of the year, was used and it is given by (7). For the horizontal surface, the incidence angle,  $\theta$ , was calculated by (6) and the hour angle,  $\omega$ , was determined using (8); where ST is the solar time in hours and it is in reference to a particular location, in this case, Makurdi.

**2.2 Analysis of Data**

The data generated from the experimental and Hottel’s model was statistically correlated, analyzed and compared using analysis of variance at different levels of significance; also with the mean bias difference (MBD) and the root square difference (RMSD) using the relationships (16 – 18):

$$MBD = \frac{\sum(I_p - I_m)}{\sum I_m} \tag{16}$$

$$RMSD = \frac{\sqrt{\frac{\sum(I_p - I_m)^2}{N}}}{\frac{\sum I_m}{N}} \tag{17}$$

$$\gamma = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n \sum x^2 - (\sum x)^2] \cdot [n \sum y^2 - (\sum y)^2]}} \tag{18}$$

where N is the number of data,  $I_p$  and  $I_m$  are the predicted and measured hourly, weekly or monthly clear sky irradiance respectively;  $\gamma$  is the correlation coefficient for the relationship.

**III. RESULTS**

The mean weekly radiation predicted by Hottel’s model, the total measured radiation and the beam radiation computed from the measured total radiation are shown in Table 1 representing the daily and monthly results for the period of the study. Tables 2 and 3 show the results of analysis of variation at 5 and 1% level of significance respectively.

Table 1: Average Weekly Hottel’s Predicted Radiation and Measured Radiation for Makurdi Solar Location

Hottel's Direct Radiation (Weekly) (W/m <sup>2</sup> )	Makurdi Measured Weekly Normal Radiation (W/m <sup>2</sup> )	Makurdi Measured Weekly Total Radiation (W/m <sup>2</sup> )
544.45	78.88	307.32
547.45	168.32	397.84
551.7	105.84	341.14
556.81	122.33	323.25
562.03	35.39	246.01
566.81	23.56	244.68
570.51	57.43	342.63
572.41	23.89	232.35
572.01	20.97	259.53
569.17	Cloudy	Cloudy
563.64	104.19	322.8
555.6	112.83	341.17
545.19	183.92	382.48
533.42	161.42	362.35
521.57	341.6	432.03
507.89	462.4	475.59
495.08	194.47	372.19

Table 2: Analysis of Variance (ANOVA) of the Hottel’s Model with Measured Values at 5% Significant Level

<i>SUMMARY</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
Hottel's Direct Radiation	17	9335.74	549.1612	524.3668
Measured normal radiation	17	2197.44	129.2612	14661.56
Measured Total Radiation	17	5383.36	316.6682	11095.22

  

ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Columns	1504446	2	752222.8	111.235	3.91E-15	3.294537
Error	216398.8	32	6762.464			
Total	1924944	50				

Ho:  $F \leq F_{crit}$   $\alpha = 0.05$   
 Ha:  $F > F_{crit}$

Table 3: Analysis of Variance (ANOVA) of the Hottel’s Model with Measured Values at 1% Significant Level

<i>SUMMARY</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
Hottel's Direct Radiation	17	9335.74	549.1612	524.3668
Measured normal radiation	17	2197.44	129.2612	14661.56
Measured Total Radiation	17	5383.36	316.6682	11095.22

  

ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Columns	1504446	2	752222.8	111.235	3.91E-15	5.336343
Error	216398.8	32	6762.464			
Total	1924944	50				

Ho:  $F \leq F_{crit}$   $\alpha = 0.01$   
 Ha:  $F > F_{crit}$

Table 4: Pearson Correlation Analysis of Hottel’s Model with measured Data

	<b>Hottel's Direct Radiation</b>	
<b>Hottel's Direct Radiation</b>	Pearson Correlation Coefficient	1
<b>Measured normal radiation</b>	Pearson Correlation Coefficient	-0.820170315
	R Standard Error	0.021821377
	t	-5.552172763
	Significance Level	5.54E-05
	Ho (5%)	rejected
<b>Measured Total Radiation</b>	Pearson Correlation Coefficient	-0.639715163
	R Standard Error	0.039384301
	t	-3.223480713
	Significance Level	0.005683868
	Ho (5%)	rejected

Source: Correlation Analysis using Biostat 2008 Professional

#### IV. DISCUSSION

The statistical tests carried out indicated that there was great variation between the radiation values predicted by Hottel's model and those computed from the measured values as indicated by the closeness of the MBD and RMSD values of 2.9029 and 3.1237 respectively. This variation could be due to climatic changes which are obviously not under control nor predictable. More significantly, the simplified Hottel's transmittance model, though generalized for tropical altitudes below 2.5km, may have considered some other factors that are not applicable to Makurdi location with an altitude of about 111m. Moreover, some natural occurrences which could not have been accounted for when the model was developed since 1976 are being reported daily all over the world. This includes reoccurring climate changes and actions of mitigations.

A closer look at Table 1 however, indicates some daily/weekly measured values which were higher than the values predicted by the model with glaring significant differences at 5% and 1% respectively. The significant difference at 1% is of 99% confidence level. The correlation coefficients are negative, quite distant from 'unity' of correlation. Though tangible conclusions cannot now be made based on these, they however indicate some magnitude of validity that remains to be verified. It should be noted however, that the general statistical trend in this case invalidates the model in Makurdi.

Obviously, a significant error margin may result from the use of models that are developed for and in a particular region when used in other regions. A possibility of reducing this error exists by obtaining some correction factors to apply to the results. The process of obtaining such factors is a subject for further study. The use of radiation models will definitely continue for quite a while due to the absence of basic equipment for direct measurement in many locations in the developing world where incidentally most of the solar radiation is obtainable and especially in Nigeria. There is therefore a need to develop a Makurdi-based model which may be in the mold of Hottel's simplified model taking into consideration the local factors required. As is the usual criteria for validity, this will take several years. It is however, a worthwhile considering the handicap of designers of solar systems requiring the use of beam radiation due to lack of basic equipment for measurement.

#### V. CONCLUSION AND RECOMMENDATION

Hottel's simplified transmittance model for topical altitudes below 2.5km may not be very suitable for use in Makurdi location with the knowledge that significant error margin exists. However, the accuracy will improve significantly if some correction factors are determined and applied to the predicted values of radiation.

A Makurdi-based model patterned after Hottel's model needs to however be developed that will incorporate the peculiar factors affecting beam radiation in the location. Further study is being conducted in order to obtain correction factors that could be applied to reduce the error margin and marginal variation associated with the use of Hottel's model in Makurdi.

#### REFERENCES

- [1] F. Raeiszadeh & M. S. Behbahanizadeh, The Application of Empirical Models to Compute the Solar Radiation Energy in Shahrekord, *J. Basic. Appl. Sci. Res.*, 2(11), 2012, 10832-10842.
- [2] K. Bakirci, Correlations for Estimation of Solar Radiation on Horizontal Surfaces, *J. Energy Eng.*, 134(4), 2008, 130-134.
- [3] M. E. Vieira da Silva, A. Sandro de Araújo & M. R. Q. Medeiros, Adjustment of the Clear Sky Coefficients for the Transmission of Solar Radiation under the Ambient Conditions in Fortaleza, *World Climate & Energy Event, January 6-11, 2002, RIO 02*, 85 - 88.
- [4] B. Y. H. Liu & R. C. Jordan, (1960). The Inter-relationship and Characteristic Distribution of direct, Diffuse and Total Solar Radiation, *Solar Energy*, 4(3), 1-19.
- [5] J.L. Bosch, F.J. Batlles, L.F. Zarzalejo & G. López, Solar resources estimation combining digital terrain models and satellite images techniques, *Renewable Energy*, 35, 2010, 2853 - 2861.
- [6] P.G. Loutzenhiser, H. Manz, C. Felsmann, P.A. Strachan, T. Frank, G.M. Maxwell, Empirical validation of models to compute solar irradiance on inclined surfaces for building energy simulation, *Solar Energy*, 81, 2007, 254-267.
- [7] C. A. Gueymard, Direct solar transmittance and irradiance predictions with broadband models. Part I: detailed theoretical performance assessment, *Solar Energy*, 74, 2003, 355-379.
- [8] T. Lealea & R. Tchinda, Estimation of Diffuse Solar Radiation in the South of Cameroon, *Journal of Energy Technologies and Policy*, 3(6), 2013, ISSN 2224-3232 (Paper) ISSN 2225-0573 (Online)
- [9] M. J. Reno, C. W. Hansen, J. S. Stein, Global Horizontal Irradiance Clear Sky Models: Implementation and Analysis, Sandia Report, Sand2012-2389, Sandia National Laboratories, Albuquerque, New Mexico
- [10] S. A. Al-Sanea, M. F. Zedan & S. A. Al-Ajlan, Adjustment factors for the ASHRAE clear-sky model based on solar-radiation measurements in Riyadh, *Applied Energy*, 79(2), 2004, 215-237.
- [11] A. Louche, G. Simonnot, M. Iqbal & M. Mermier, (1982). Experimental Verification of some Clear Sky

- Insolation Models, *Solar Energy*, 41, 273-279.
- [12] J. K. Yohanna, I. N. Itodo & V. I. Umogbai, A model for determining the global solar radiation for Makurdi, Nigeria, *Renewable Energy*, 36, 2011, 1989 – 1992.
- [13] M. Grigante, F. Mottes, D. Zardi, M. de Franceschi, Experimental solar radiation measurements and their effectiveness in setting up a real-sky irradiance model, *Renewable Energy*, 36, 2011, 1 – 8.
- [14] İ. T. Toğrul, Estimation of Solar Radiation from Angstroms Coefficients by using Geographical and Meteorological Data in Bishkek, Kyrgyzstan, *J. of Thermal Science and Technology*, 29(2), 2009, 99-108.
- [15] M. M. Khan & M. J. Ahmad, Estimation of global solar radiation using clear sky radiation in Yemen, *Journal of Engineering Science and Technology Review*, 5(2), 2012, 12-19.
- [16] A. Ianetz, V. Lyubansky, I. Setter, B. Kriheli, E. G. Evseev, & A. I. Kudish, Inter-comparison of different models for estimating clear sky solar global radiation for the Negev region of Israel, *Energy Conversion and Management*, 48(1), 2007, 259–268
- [17] A. B. Aliyu & A. S. Sambo, (1991). Development of a Model for Total Component of Solar Radiation in Sokoto, *Nigerian Journal of Renewable Energy*, 2, pp. 10 -17.
- [18] H. C. Hottel, A Simple Model for Estimating the Transmittance of Direct Solar Radiation Through Clear Atmospheres, *Solar Energy*, 18(2), 1976, 129 – 134.