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A Brief Note on Quasi Static Thermal Stresses In A Thin Rectangular Plate With Internal Heat Generation

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Abstract: - The present paper deals with the determination of quasi static thermal stresses in a thin rectangular plate with internal heat generation. A thin rectangular plate is considered having zero initial temperature and subjected to arbitrary heat supply at $x = 0$ and $x = a$ where as thin rectangular plate is insulated at $y = 0$ and $y = b$. Here we modify Kulkarni (2007). The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature, displacement and stresses have been computed numerically and illustrated graphically.

Keywords: - *Quasi static thermal stresses, thermoelastic problem, internal heat generation, thin rectangular plate.*

I. INTRODUCTION

Gogulwar and Deshmukh (2004) determined the thermal stresses in rectangular plate due to partially distributed heat supply. Nasser M. *et al*. (2004) solved two dimensional problem of thick plate with heat sources in generalized thermoelasticity. Khandait and Deshmukh (2010) studied thermoelastic problem in a rectangular plate with heat generation.

Recently Patil *et al*. (2013) determined the thermal stresses in a rectangular slab with internal heat source, now here a thin rectangular plate is considered having zero initial temperature and and subjected to arbitrary heat supply at $x = 0$ and $x = a$ where as the plate is insulated at $y = 0$ and $y = b$. Here we modify Kulkarni (2007). To obtain the temperature distribution, cosine integral transform and Laplace transform are applied. The results are obtained in series form in terms of Bessel's functions and the temperature change, displacement and stresses have been computed numerically and illustrated graphically. A mathematical model has been constructed of a thin rectangular plate with the help of numerical illustration by considering steel (0.5% carbon) rectangular plate. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant and gas power plant.

II. FORMULATION OF THE PROBLEM

A thin rectangular plate occupying the space D: $0 \le x \le a,\ 0 \le y \le b, a \ne b$ is considered. A thin rectangular plate is considered having zero initial temperature and subjected to arbitrary heat supply at $x = 0$ and $x = a$ where as the plate is insulated at $y = 0$ and $y = b$. Here the plate is assumed sufficiently thin and considered free from traction. Since the plate is in a plane stress state without bending. Airy stress function method is applicable to the analytical development of the thermoelastic field. The equation is given by the relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U = -a_t\, E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T \tag{1}$$

where a_t, E and U are linear coefficient of the thermal expansion, Young's modulus elasticity of the material of the plate and Airys stress functions respectively.

The displacement components u_x and u_y in the X and Y direction are represented in the integral form and the stress components in terms of U are given by

$$u_x = \int \left\{ \frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - v\frac{\partial^2 U}{\partial x^2}\right) + a_t T \right\} dx \tag{2}$$

$$u_y = \int \left\{ \frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - v\frac{\partial^2 U}{\partial y^2}\right) + a_t T \right\} dy \tag{3}$$

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2} \tag{4}$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2} \tag{5}$$

and

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{6}$$

$$\sigma_{xy} = 0 \text{ at } y = b. \tag{7}$$

where v is the Poisson's ratio of the material of the rectangular plate.

The temperature of the thin rectangular plate at time t satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{8}$$

$$T(x,y,t) = 0 \; at \; t = 0 \quad 0 \le x \le a, \; 0 \le y \le b \tag{9}$$

$$T(x,y,t) = f_1(y,t) \; at \; x = 0, \quad 0 \le y \le b \tag{10}$$

$$T(x,y,t) = f_2(y,t) \; at \; x = a, \quad 0 \le y \le b \tag{11}$$

$$\frac{\partial T}{\partial y} = 0 \; at \; y = 0, \quad 0 \le x \le a \tag{12}$$

$$\frac{\partial T}{\partial y} = 0 \;\; at \; y = b, \quad 0 \le x \le a \tag{13}$$

$$q(x,y,t) = \delta(y - y_0)\, sin[\, \beta_m(x + a)](1 - e^{-t}), \; 0 < y_0 < b \tag{14}$$

where α is the thermal diffusivity of the material of the plate, k is the thermal conductivity of the material of the plate, q is the internal heat generation and $\delta(r)$ is well known dirac delta function of argument r.

Eq. (1) to Eq. (14) constitute mathematical formulation of the problem.

III. SOLUTION

To obtain the expression for temperature $T(x,y,t)$, we introduce the cosine integral transform and its inverse transform are

$$\bar{T}(x,\beta_m,t) = \int_0^b K_0(\beta_m, y)\; T(x,y,t)dy \tag{15}$$

$$T(x,y,t) = \sum_{m=1}^{\infty} K_0(\beta_m, y)\, \bar{T}(x,\beta_m,t) \tag{16}$$

where the kernel

$$K_0(\beta_m, y) = \sqrt{\frac{2}{b}}\, cos(\beta_m y) \tag{17}$$

where β_m is the m[th] root of the transcendental equation sin $(\beta_m b) = 0, \beta_m = \frac{m\pi}{b}, m = 1,2, ...$

On applying the cosine integral transform defined in the Eq. (15), its inverse transform defined in Eq. (16), applying Laplace transform and its inverse by residue method successively to the Eq. (1), one obtains the expression for temperature as

$$\begin{aligned} T(\mathrm{x,y,t}) = \sum_{\mathrm{m}=1}^{\infty}\sum_{\mathrm{n}=1}^{\infty} \sqrt{\frac{\mathbf{2}}{\mathbf{b}}}\, cos\beta_m y \; \Bigg\{ &\left(\frac{-2\mathrm{n}\pi\alpha}{\mathrm{a}^2(-1)^n}\right) \sin\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) \\ &+ \left(\frac{-2\mathrm{n}\pi\alpha}{\mathrm{a}^2(-1)^n}\right) \sin\left(\frac{n\pi}{a}x\right) Q_2(t) + \frac{\alpha}{\mathrm{k}}\sqrt{\frac{2}{b}}\, cos\beta_m y_0 \, \sin[\beta_m(\mathrm{x+a})]\, Q_3(t) \Bigg\} \end{aligned} \tag{18}$$

where

$$\begin{aligned} Q_1(t) = \int_0^t e^{-\alpha\left[\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right](t-u)} \Bigg\{ & \sqrt{\frac{2}{b}}\, cos\beta_m y_0 \; \frac{\alpha \sin\beta_m a}{\mathrm{k}} \\ & \times \left(\frac{1}{2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-u}}{1-2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-2\alpha\beta_\mathrm{m}{}^2 u}}{2\alpha\beta_\mathrm{m}{}^2(2\alpha\beta_\mathrm{m}{}^2-1)}\right) - F_1(\beta_m, \mathrm{u})\, \mathrm{du} \Bigg\} \end{aligned}$$

$$\begin{aligned} Q_2(t) = \int_0^t e^{-\alpha\left[\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right](t-u)} \Bigg\{ & F_2(\beta_m, \mathrm{u}) - \sqrt{\frac{2}{b}}\, cos\beta_m y_0 \; \frac{\alpha \sin\beta_m a}{\mathrm{k}} \\ & \times \left(\frac{1}{2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-u}}{1-2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-2\alpha\beta_\mathrm{m}{}^2 u}}{2\alpha\beta_\mathrm{m}{}^2(2\alpha\beta_\mathrm{m}{}^2-1)}\right) \mathrm{du} \Bigg\} \end{aligned}$$

and

$$Q_3(t) = \left(\frac{1}{2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-t}}{1-2\alpha\beta_\mathrm{m}{}^2} + \frac{e^{-2\alpha\beta_\mathrm{m}{}^2 t}}{2\alpha\beta_\mathrm{m}{}^2(2\alpha\beta_\mathrm{m}{}^2-1)}\right)$$

Airy stress function U

Using Eq.(18) in Eq.(1), one obtains the expression for Airy's stress function U as

$$U = a_t\ E \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\frac{2}{b}} \frac{\cos \beta_m y}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)} \left\{ \left(\frac{-2n\pi\alpha}{a^2(-1)^n}\right) \sin\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \left(\frac{-2n\pi\alpha}{a^2(-1)^n}\right) \sin\left(\frac{n\pi}{a}x\right) Q_2(t)\right.$$

$$\left. + \frac{\alpha}{2\beta_m{}^2 k} \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \sin[\beta_m(x+a)]\, Q_3(t) \left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)\right\} \tag{19}$$

Displacement and Stresses

Now using Eqs. (18) and (19) in Eqs. (2) to (6) one obtains the expressions for displacement and stresses as

$$u_x = a_t \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\frac{2}{b}} \cos\beta_m y \left\{ \left(\frac{2\alpha\pi^2 n^2}{a^3(-1)^n}\right) \frac{(1+v)}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)}\right.$$
$$\times \left[\cos\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \cos\left(\frac{n\pi}{a}x\right) Q_2(t)\right]$$
$$\left. - \frac{\alpha}{2k} \frac{(1+v)}{\beta_m} \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \cos[\beta_m(x+a)]\, Q_3(t)\right\} \tag{20}$$

$$u_y = a_t \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\frac{2}{b}} \frac{\sin\beta_m y}{\beta_m} \left\{ \left(\frac{-2n\pi\alpha}{a^2(-1)^n}\right) \frac{\beta_m{}^2(1+v)}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)}\right.$$
$$\times \left[\sin\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \sin\left(\frac{n\pi}{a}x\right) Q_2(t)\right]$$
$$\left. + \frac{\alpha}{2k}(1+v) \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \sin[\beta_m(x+a)]\, Q_3(t)\right\} \tag{21}$$

$$\sigma_{xx} = a_t E \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\frac{2}{b}} \frac{(-\beta_m{}^2)\cos\beta_m y}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)} \left\{ \left(\frac{-2n\pi\alpha}{a^2(-1)^n}\right)\right.$$
$$\times \left[\sin\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \sin\left(\frac{n\pi}{a}x\right) Q_2(t)\right]$$
$$\left. + \frac{\alpha}{2k\beta_m{}^2}\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right) \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \sin[\beta_m(x+a)]\, Q_3(t)\right\} \tag{22}$$

$$\sigma_{yy} = a_t E \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sqrt{\frac{2}{b}} \frac{\left(-\frac{\pi^2 n^2}{a^2}\right)\cos\beta_m y}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)} \left\{ \left(\frac{-2n\pi\alpha}{a^2(-1)^n}\right)\right.$$
$$\times \left[\sin\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \sin\left(\frac{n\pi}{a}x\right) Q_2(t)\right]$$
$$\left. + \frac{\alpha\, a^2}{2k\pi^2 n^2}\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right) \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \sin[\beta_m(x+a)]\, Q_3(t)\right\} \tag{23}$$

$$\sigma_{xy} = a_t E \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\beta_m \sin\beta_m y}{\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right)} \left\{ \left(\frac{-2n^2\pi^2\alpha}{a^3(-1)^n}\right)\right.$$
$$\times \left[\cos\left[\frac{n\pi}{a}(x-a)\right] Q_1(t) + \cos\left(\frac{n\pi}{a}x\right) Q_2(t)\right]$$
$$\left. + \frac{\alpha}{2k\beta_m{}^2}\left(\beta_m{}^2 + \frac{\pi^2 n^2}{a^2}\right) \sqrt{\frac{2}{b}} \cos(\beta_m y_0) \cos[\beta_m(x+a)]\, Q_3(t)\right\} \tag{24}$$

IV. SPECIAL CASE AND NUMERICAL CALCULATIONS

Setting

$$f_1(y,t) = f_2(y,t) = \delta(y-y_1)\delta(t-t_0),\ \ 0 \le y_1 \le b,\ \ 0 < t_0 < \infty$$

$$F_1(\beta_m,t) = F_2(\beta_m,t) = \sqrt{\frac{2}{b}} \cos(\beta_m y_1)\ \ \delta(t-t_0)$$

$a = 1m,\ b = 2m,\ t_0 = 0, 2, 4, 6, 8$ sec and $y_0 = y_1 = 1m.$

4.1 Material Properties

The numerical calculation has been carried out for steel (0.5% carbon) rectangular plate with the material properties defined as

Thermal diffusivity $\alpha = 14.74\times 10^{-6}\ \mathrm{m^2 s^{-1}}$,

Specific heat $c_\rho = 465$ J/kg,

Thermal conductivity k = 53.6 W/m K,

Poisson ratio $\vartheta = 0.35$,

Young's modulus $E = 130\ G$ pa,
Lame constant $\mu = 26.67$,
Coefficient of linear thermal expansion $a_t = 13 \times 10^{-6}\ {}^{1}/_{K}$

4.2 Roots of Transcendental Equation

The $\beta_1 = 3.1414,\ \beta_2 = 6.2828,\ \beta_3 = 9.4242,\ \beta_4 = 12.5656,\ \beta_5 = 15.707, \beta_6 = 18.8484$ are the roots of transcendental equation $\sin(\beta_m b) = 0$. The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

V. DISCUSSION

In this paper a thin rectangular plate is considered which is free from fraction and determined the expressions for temperature, displacement and stresses due to arbitrary heat supply on the edges $x = 0$ and $x = a$ of plate whereas the plate is insulated at $y = 0$ and $y = b$. A mathematical model is constructed by considering steel (0.5% carbon) rectangular plate with the material properties specified above.

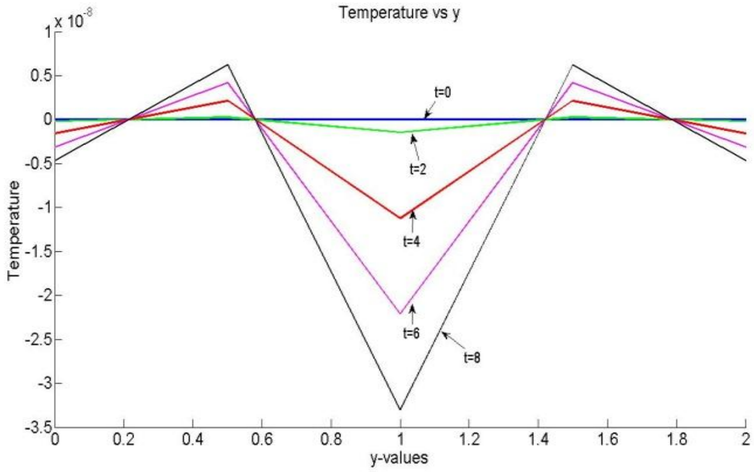

Fig. 1 Temperature T in Y- direction.

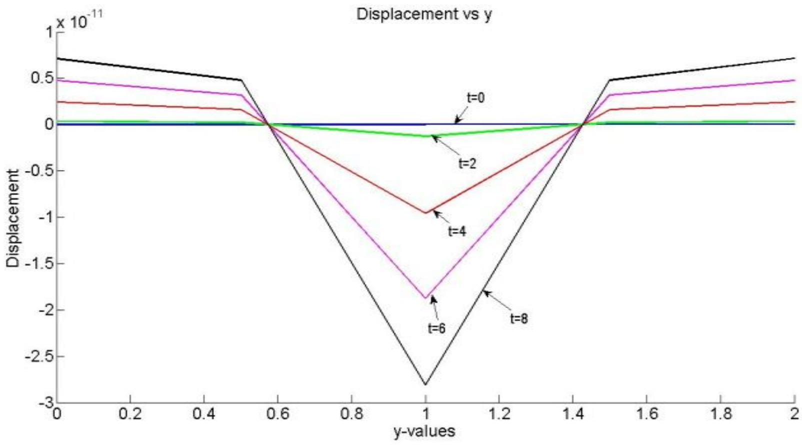

Fig. 2 The displacement u_x in Y- direction.

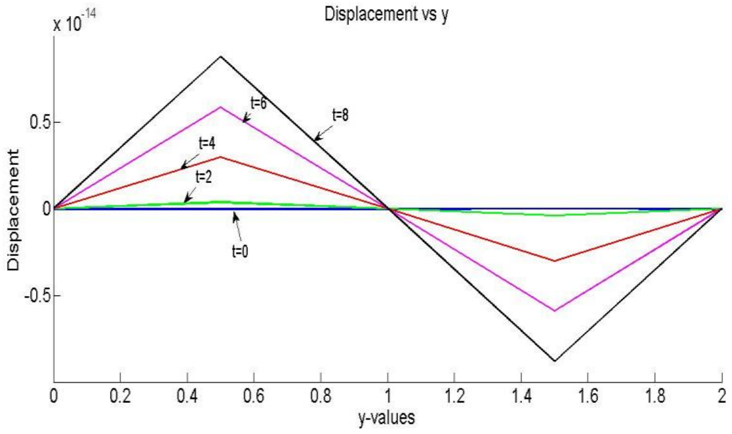

Fig. 3 The displacement u_y in Y- direction.

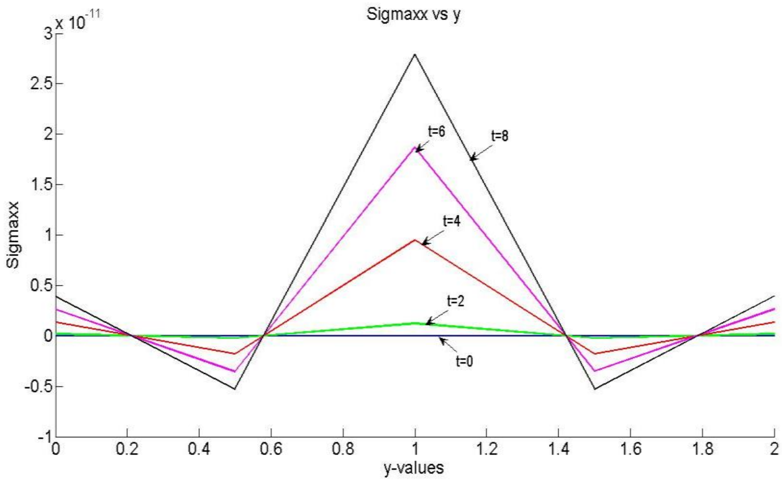

Fig. 4 Thermal stresses σ_{xx} in Y-direction

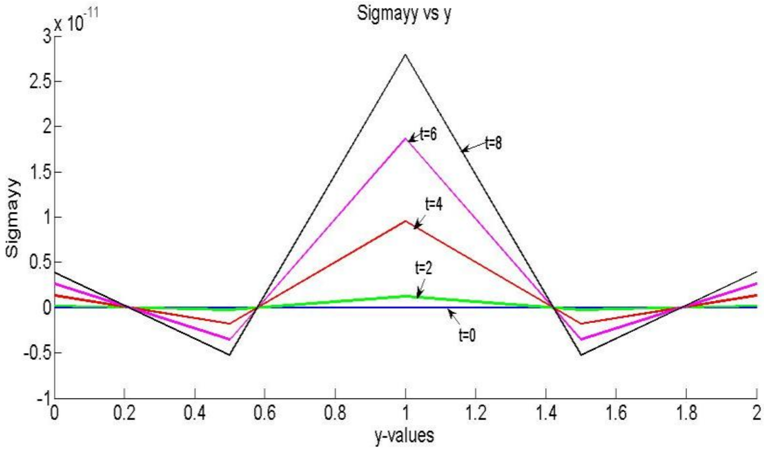

Fig. 5 Thermal stresses σ_{yy} in Y-direction.

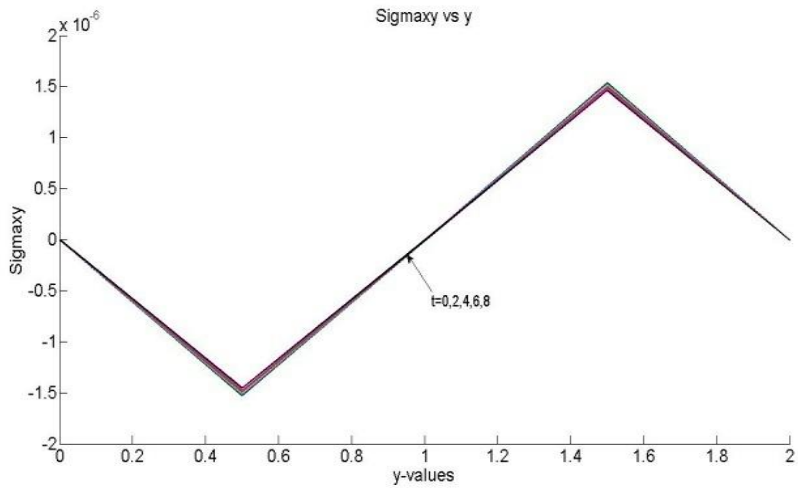

Fig. 6 Thermal stresses σ_{xy} in Y-direction.

From figure 1, it is observed that temperature T decreases as the time increases. The overall behavior of temperature is decreasing and it is symmetric about y= 1 in a thin rectangular plate with internal heat generation along Y-direction.
From figure 2, it is observed that displacement u_x decreases as the time increases. The overall behavior of temperature is decreasing and it is symmetric about y= 1 in a thin rectangular plate with internal heat generation along Y-direction.
From figure 3, it is observed that the displacement u_y is increasing for $0 \leq y \leq 0.5$, $1.5 \leq y \leq 2$ and decreasing for $0.5 \leq y \leq 1.5$. The overall behavior of displacement u_y is decreasing along Y-direction and it is antisymmetric about y= 1 in a thin rectangular plate with internal heat generation along Y-direction.
From figure 4 and 5, it is observed that thermal stresses σ_{xx}, σ_{yy} increases as the time increases. Maximum value of stresses σ_{xx} , σ_{yy} occure near heat source and it is tensile in nature in a thin rectangular plate with internal heat generation along Y-direction.
From figure 6, it is observed that the thermal stresses σ_{xy} is decreasing for $0 \leq x \leq 0.5, 1.5 \leq x \leq 2$ and increasing for $0.5 \leq x \leq 1.5$. The overall behavior of stresses σ_{xy} is increasing and it is antisymmetric about y= 1 in rectangular plate with internal heat generation along Y-direction.

VI. CONCLUSION

We can conclude that temperature T, displacement u_x and u_y are decreasing with time in a thin rectangular plate with internal heat generation along Y-direction. The thermal stresses σ_{xx}, σ_{yy} are tensile in nature in a thin rectangular plate with internal heat generation along Y-direction. The thermal stresses σ_{xy} is increasing and it is antisymmetric about $y = 1$ in a thin rectangular plate with internal heat generation along Y-direction. The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin rectangular plate, base of furnace of boiler of a thermal power plant and gas power plant.

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