

## Mathematical Modeling of Optimizing Power Stream Measurement Using Genetic Algorithm

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**Abstract:** - An electrical engineer always tries to generate transmit and distribute electrical energy at affordable cost while satisfying the constraints. So optimal power flow is the problem which is mathematically modeling this objective. OPF is the allocation of optimal load to each committed generators while satisfying the power flow and plant constraints. The objective is to minimize the fuel cost and reduce the total losses by maintaining the generation power in limits. In this proposed work two case studies are carried out on and IEEE-30 bus systems. The solution methodology is developed as a software tool in Matlab 7.0.1. In this project fuel cost is taken as an objective function & it is compared with the results of Matpower package. The GA tool box is utilized for these two case studies.

**Keywords:** - OPF (Optimal Power Flow), GA (Genetic Algorithm), NR (Newton Raphson), PSO (Practical Swarm Optimization).

### I. INTRODUCTION

The definition of optimal power flow and solution of optimal power flow by conventional methods given in [1] and [2] will be briefly explained. Effective optimal power flow is limited by (i) the high dimensionality of power systems and (ii) the incomplete domain dependent knowledge of power system engineers. GAOPF requires two load flow to be performed per entity, per iteration because all convenient variables are included in the fitness. In this project, a simple genetic algorithm applied to the problem of optimal power flow in large power distribution systems. OPF is a tool used for both the operation and planning of a power system. It can be intuitively explained in the following way. If we are to provide a given requirement, and if we have generation units committed (participating in the dispatch), OPF gives an answer as to how much power each unit has to produce (dispatch) as well as how to adjust transformer settings in order to supply demand most economically, while respecting all the constraints imposed on the system.

### II. PROBLEM FORMULATION

The standard OPF predicament can be written in the subsequent form,

Minimize  $F(x)$  (the objective function) subject to :

$$h_i(x) = 0, \quad i=1,2,\dots,n \quad (\text{parity constraints}) \quad \text{-----}(2.1)$$

$$g_i(x) \leq 0, \quad j=1,2,\dots,m \quad (\text{disparity constraints}) \quad \text{-----}(2.2)$$

where  $x$  is the vector of the control variables, that is those which can be varied by a control center operator (generated active and reactive powers, cohort bus voltage magnitudes, transformers taps etc.); The essence of the optimal power flow problem resides in reducing the objective function and concurrently satisfying the load flow equations (parity constraints) without violating the dissimilarity constraints.

### III. OBJECTIVE FUNCTION

The most commonly used objective in the OPF problem formulation is the minimization of the total cost of real power generation. The individual costs of each generating unit are assumed to be function, only, of

active power generation and are represented by quadratic curves of second order. The objective function for the entire power system can then be written as the sum of the quadratic cost model at each generator.

$$c(p) = ap^2 + bp + c \quad \text{-----(2.3)}$$

Where p is in MW (or per unit) output of the generator and a, b, c are constant coefficients.

**IV. CONSTRAINTS**

As we stated, the OPF is a constrained optimization problem. The set of constraints can be divided into parity constraints and disparity constraints. The parity constraint set typically consists of power balance (active and reactive) at each node of the network which results from Kirchoff’s current law. Another set of constraints are disparity constraints, which are usually limits resulting from network constituent boundaries. A frequent set of disparity constraints consists of:

- Generator power constraints (P and Q)
- Line power constraints (P)
- Voltage, tap ratios, and phase shifter angle constraints

Generators are rated by maximum apparent power which they can produce. The combination of P, Q produced by a generator must obey the apparent circle equation  $P^2 + Q^2 \leq S_{max}^2$ . The maximum active power ( $P_{max}$ ) produced by generator is limited by the turbine’s physical limits, while maximum reactive power ( $Q_{max}$ ) is often determined so that heating of the rotor is within a pre specified tolerance. Likewise, a minimum generation level is usually precise. Therefore for each and every generator in the network is subject to the following constraints:

$$P_{min} \leq P \leq P_{max} \quad \text{----- (2.4)}$$

$$Q_{min} \leq Q \leq Q_{max} \quad \text{----- (2.5)}$$

Besides generators, transformers provide an additional means of control of the flow of both active and reactive power.

There are two types of controllable transformers: tap changers and phase shifters, even though some transformers control both the magnitude and phase angle. Controllable transformers are those which provide a small adjustment of voltage magnitude, usually in the range  $\pm 10\%$  or which shift the phase angle of the line voltages. A type of transformer considered for small adjustments of voltage rather than for changing voltage levels is called a regulating transformer.

**V. TYPES OF PARITY CONSTRAINTS**

While minimizing the cost function, it’s necessary to make sure that the generation still supplies the load demands plus losses in transmission lines. Usually the power flow equations are used as parity constraints.

$$P_i(V, \theta) - (P_{gi} - P_{di}) = 0 \quad \text{-----(2.6)}$$

$$Q_i(V, \theta) - (Q_{gi} - Q_{di}) = 0 \quad \text{-----(2.7)}$$

Where active and reactive power injection at bus i are defined in the following equation:

$$P_i = \sum |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \quad \text{-----(2.8)}$$

$$Q_i = \sum |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \quad \text{-----(2.9)}$$

Where i=bus no. & k=1,2,3.....n

**VI. TYPES OF DISPARITY CONSTRAINTS**

The disparity constraints of the OPF replicate the limits on physical devices in the power scheme as well as the limits created to ensure system protection. The most natural types of disparity constraints are advanced bus voltage limits at generations and load buses, lower bus voltage confines at load buses, var. confines at production buses, greatest active power limits corresponding to lower limits at some generators, maximum line loading limits and limits on tap setting of TCULs and phase shifter. The disparity constraints on the dilemma variables measured include:

- i) Upper and lower bounds on the active generations at generator buses  $P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max}, i = 1, ng.$
- ii) Upper and lower bounds on the reactive power generations at generator buses and reactive power injection at buses with VAR compensation  $Q_{gi}^{min} \leq Q_{gi} \leq Q_{gi}^{max}, i = 1, npv$
- iii) Upper and lower bounds on the voltage magnitude at the all buses  $V_i^{min} \leq V_i \leq V_i^{max} \quad i = 1, nbus.$
- iv) Upper and lower bounds on the bus voltage phase angles:  $\theta_i^{min} \leq \theta_i \leq \theta_i^{max} \quad i=1$  to n bus.

It can be seen that the comprehensive objective function  $F$  is a non-linear, the number of the parity and disparity constraints boost with the size of the power allotment systems. Applications of a predictable optimization technique such as the gradient-based algorithms to a large power allocation system with a very non-linear objective functions and enormous quantity of constraints are not good enough to solve this problem. Because it depend on the subsistence of the first and the second derivatives of the objective function and on the well computing of these derivative in huge investigate space.

## VII. EXPERIMENTAL INVESTIGATIONS

For experimental investigation the knowledge of genetic algorithm tool box is necessary. The Genetic Algorithm toolbox is a collection of functions that extend the capabilities of the Optimization Toolbox and the MATLAB numeric computing environment. The Genetic Algorithm toolbox includes routines for solving optimization problems using Genetic algorithm. This algorithm enables you to solve a variety of optimization problems that lie outside the scope of the standard Optimization Toolbox.

All the toolbox functions are MATLAB M-files, made up of MATLAB statements that implement specialized optimization algorithms. The capabilities of the Genetic Algorithm toolbox can be extended by writing own M-files, or by using the toolbox in combination with other toolboxes, or with MATLAB or Simulink.

### Genetic algorithm in optimal power flow

The genetic algorithms are part of the evolutionary algorithms family, which are computational models, inspired in the Nature. Genetic algorithms are powerful stochastic search algorithms based on the mechanism of natural selection and natural genetics.

GAs works with a population of binary string, searching many peaks in parallel. By employing genetic operators, they exchange information between the peaks, hence reducing the possibility of ending at a local optimum.

GAs are more flexible than most search methods because they require only information concerning the quality of the solution produced by each parameter set (objective function values) and not lake many optimization methods which require derivative information, or worse yet, complete knowledge of the problem structure and parameters.

### GA Applied to optimal power flow

A simple Genetic Algorithm is an iterative procedure, which maintains a constant size population  $P$  of candidate solutions. During each iteration step (generation) three genetic operators (reproduction, crossover, and mutation) are performing to generate new populations (offspring), and the chromosomes of the new populations are evaluated via the value of the fitness which is related to cost function. Based on these genetic operators and the evaluations, the better new populations of candidate solution are formed.

With the above description, a simple genetic algorithm is given as follow [6]:

1. Generate randomly a population of binary string
2. Calculate the fitness for each string in the population
3. Create offspring strings through reproduction, crossover and mutation operation.
4. Evaluate the new strings and calculate the fitness for each string (chromosome).
5. If the search goal is achieved, or an allowable generation is attained, return the best chromosome as the solution; otherwise go to step 3.

## VIII. CROSSOVER

Crossover is the primary genetic operator, which promotes the exploration of new regions in the search space. For a pair of parents selected from the population the recombination operation divides two strings of bits into segments by setting a crossover point at random, i.e. Single Point Crossover.

The segments of bits from the parents behind the crossover point are exchanged with each other to generate their offspring. The mixture is performed by choosing a point of the strings randomly, and switching their segments to the left of this point. The new strings belong to the next generation of possible solutions. The strings to be crossed are selected according to their scores using the roulette wheel [6]. Thus, the strings with larger scores have more chances to be mixed with other strings because all the copies in the roulette have the same probability to be selected.

**IX. MUTATION**

Mutation is a secondary operator and prevents the premature stopping of the algorithm in a local solution. The mutation operator is defined by a random bit value change in a chosen string with a low probability of such change. The mutation adds a random search character to the genetic algorithm, and it is necessary to avoid that, after some generations, all possible solutions were very similar ones.

All strings and bits have the same probability of mutation. For example, in the string 110011101101, if the mutation affects to time bit number six, the string obtained is 110011001101.

**X. REPRODUCTION**

Reproduction is based on the principle of survival of the better fitness. It is an operator that obtains a fixed number of copies of solutions according to their fitness value. If the score increases, then the number of copies increases too. A score value is of associated to a given solution according to its distance of the optimal solution (closer distances to the optimal solution mean higher scores).

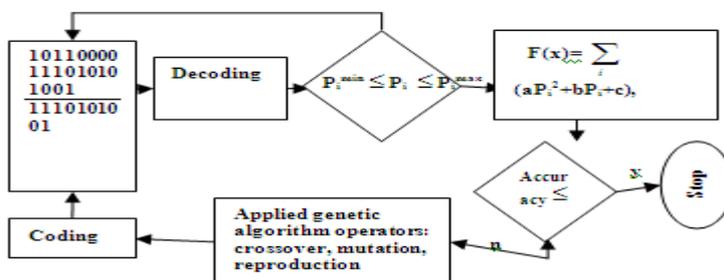
**XI. COST FUNCTION**

The cost function is defined as:

$$F(x) = \sum_i (aP_i^2 + bP_i + c) \quad P_i^{\min} \leq P_i \leq P_i^{\max} \quad \text{----- (3.1)}$$

Our objective is to search the generation powers in their admissible limits to achieve the optimization problem of OPF.

Using the above components, a standard GA procedure for solving the optimal power flow problem is summarized in the diagram of the Fig 1.



**Fig 1. Simple flow chart of the GA OPF**

The use of penalty functions in many OPF solutions techniques to handle generation bus reactive power limits can lead to convergence problem due to the distortion of the solution surface. In this method no penalty functions are required. Because only the active power of generators are used in the fitness. And the reactive levels are scheduled in the load flow process. Because his essence of this idea is that the constraints are partitioned in two types of constraints, active constraints are checked using the GA procedure and the reactive constraints are updating using an efficient Newton-Raphson Load flow procedure.

**XII. LOAD FLOW CALCULATION**

After the search goal is achieved, or an allowable generation is attained by the genetic algorithm. It's required to performing a load flow solution in order to make fine adjustments on the optimum values obtained from the GAOPF procedure. This will provide updated voltages, angles and transformer taps and points out generators having exceeded reactive limits.

- Employing the simple GA to solve the optimal power flow problem.

**i) IEEE-30 BUS SYSTEM:**

**CHROMOSOME CODING AND DECODING:**

GAs works with a population of binary string, not the parameters themselves. With the binary coding method, the active generation power set of 30 bus system ( $P_1, P_2, P_5, P_8, P_{11}, P_{13}$ ) would be coded as binary string of 0's and 1's with length  $B_1, B_2, B_5, B_8, B_{11}$  and  $B_{13}$  (may be different), respectively. Each parameter  $P_i$  have upper bound  $U_i$  and lower bound  $L_i$ . The choice of  $B_1, B_2, B_5, B_8, B_{11}$  and  $B_{13}$  for the parameters is concerned with the resolution specified by the designer in the search space. In the binary coding method, the bit length  $B_i$  and the corresponding resolution

$R_i$  is related by

$$R_i = (U_i - L_i) / (2^{Bi} - 1)$$

where  $R_i$  = resolution

As result, the  $P_i$  set can be transformed into a binary string (chromosome) with certain length and then the search space is explored. Note that each chromosome presents one possible solution to the problem. Power generation limits & generator cost parameters of IEEE-30 bus system are shown in Table 1.

**Table 1: Power generation limits & generator cost parameters of IEEE-30 bus system in p.u. (Sb=100mva)**

buss	Pmin	Pmax	Vmin	Vmax	a	b	c
1	0.50	2.0	0.95	1.10	200	200	0
2	0.20	0.80	0.95	1.10	175	175	0
5	0.15	0.50	0.95	1.10	625	100	0
8	0.10	0.35	0.95	1.10	83.4	325	0
11	0.10	0.30	0.95	1.10	250	300	0
13	0.10	0.40	0.95	1.10	250	300	0

a in (\$/MW<sup>2</sup>hr), b in (\$/MWhr) and c in (\$/hr)

Depending on the resolution the parameter set:

( $P_1, P_2, P_5, P_8, P_{11}, P_{13}$ ) can be coded according to the following Table 2.A.

**Table 2 (A): Coding of pi parameter set**

$P_1$	code	$P_2$	code	$P_5$	code
0.5	0000	0.25	0000	0.15	0000
0.6	0001	0.30	0001	0.175	0001
0.7	0010	0.35	0010	0.20	0010
0.8	0011	0.40	0011	0.225	0011
0.9	0100	0.45	0100	0.25	0100
1.0	0101	0.50	0101	0.275	0101
1.1	0110	0.55	0110	0.30	0110
1.2	0111	0.60	0111	0.325	0111
1.3	1000	0.65	1000	0.35	1000
1.4	1001	0.70	1001	0.375	1001
1.5	1010	0.75	1010	0.40	1010
1.6	1011	0.80	1011	0.425	1011
1.7	1100	0.85	1100	0.45	1100
1.8	1101	0.90	1101	0.475	1101
1.9	1110	0.95	1110	0.50	1110
2.0	1111	1.00	1111	0.525	1111

**Table 2 (B): Coding of pi parameter set**

$P_8$	code	$P_{11}$	code	$P_{13}$	code
0.10	0000	0.10	0000	0.10	0000
0.12	0001	0.12	0001	0.12	0001
0.14	0010	0.14	0010	0.14	0010
0.16	0011	0.16	0011	0.16	0011
0.18	0100	0.18	0100	0.18	0100
0.20	0101	0.20	0101	0.20	0101
0.22	0110	0.22	0110	0.22	0110
0.24	0111	0.24	0111	0.24	0111
0.26	1000	0.26	1000	0.26	1000
0.28	1001	0.28	1001	0.28	1001
0.30	1010	0.30	1010	0.30	1010
0.32	1011	0.32	1011	0.32	1011
0.34	1100	0.34	1100	0.34	1100
0.36	1101	0.36	1101	0.36	1101
0.38	1110	0.38	1110	0.38	1110
0.40	1111	0.40	1111	0.40	1111

If the candidate parameters set is (1.9, 0.80, 0.50,0.38,0.32,0.30), then the chromosome is a binary string 1110|1011|1110|1110|1011|1010. The decoding procedure is the reverse procedure.

The first step of any genetic algorithm is to generate the initial population. A binary string of length L is associated to each member (individual) of the population. The string is usually known as a chromosome and represents a solution of the problem. A sampling of this initial population creates an intermediate population. Thus, some operators (reproduction, crossover and mutation) are applied to this new intermediate population in order to obtain a new one.

Process, that starts from the present population and leads to the new population, is named as generation. When executing a genetic algorithm for IEEE -30 bus system, the results after first generation are shown in Table 3.

For this IEEE-30 bus system to apply GA we require initial population. This initial population can be obtained by using NR method. By NR method,

Results after 3<sup>rd</sup> iteration are P<sub>1</sub>=130 MW, P<sub>2</sub>=60.2MW, P<sub>5</sub>=27.5MW, P<sub>8</sub>=34MW, P<sub>11</sub>=18MW, P<sub>13</sub>=16MW. By substituting these values in cost function  $F(x) = \sum_i (aP_i^2 + bP_i + c)$ , we get total generation cost=51,320 rs/hr.

Results after 4<sup>th</sup> iteration are P<sub>1</sub>=139.9 MW, P<sub>2</sub>=57.56MW, P<sub>5</sub>=24.5MW, P<sub>8</sub>=35MW, P<sub>11</sub>=17.9MW, P<sub>13</sub>=16.9MW. By substituting these values in cost function  $F(x) = \sum_i (aP_i^2 + bP_i + c)$ , we get total generation cost=50,520 rs/hr.

Table 3: First generation of GA process for 30 bus system

	Chromo- some	initial population	cost(rs/hr)
3 <sup>rd</sup> iteration	1	1000 0111 0101 1100 0100 0011	51,320
4 <sup>th</sup> iteration	2	1001 0110 0100 1100 0100 0011	50,520
After single pt Crossover	3	1000 0110 0100 1100 0100 0011	48,040
	4	1001 0111 0101 1100 0100 0011	43,600
After Mutation	5	1001 0011 0101 1100 0100 0011	39,200

After 100 generations we get chromosome as 0010|1110|1010|1100|0110|0101. In decoded form P<sub>1</sub>=71.56MW, P<sub>2</sub>=97.63MW, P<sub>5</sub>=41.54MW, P<sub>8</sub>=34.8MW, P<sub>11</sub>=22.06MW, P<sub>13</sub>=20.02MW. For this case total generation cost=31,960 rs/hr & it is the optimal solution by using GA.

The corresponding IEEE -30 bus system is shown Fig-2

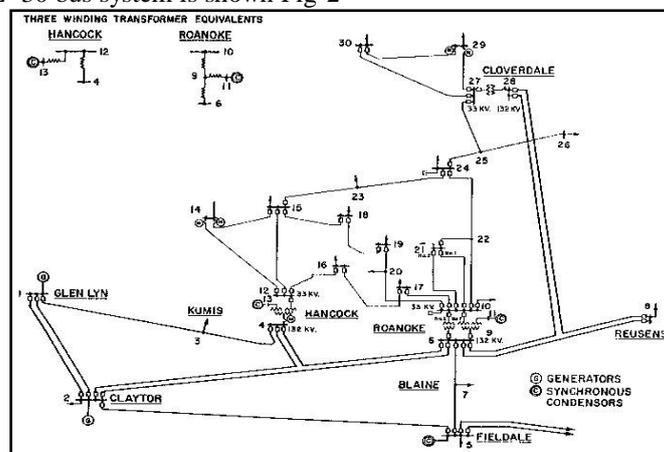


Fig 2: IEEE- 30 BUS SYSTEM

### XIII. RESULTS AND ANALYSIS

Results of OPF using GA and Matpower for IEEE-30 buses will be given and they are compared.

#### Results

**Table 4: Total cost and Losses for IEEE-30 bus system using matpower:**

PARAMETER	VALUE
i) TOTAL COST	32,000 (rs/hr)
ii) TOTAL LOSSES	4.34 (MW)

Description: For IEEE-30 bus system using Matpower we get,  
 Total generation cost =32,000 (rs/hr)  
 Total transmission losses =4.34(MW)

**Table 4.1 Generated power for IEEE-30 bus system using matpower:**

VARIABLE	VALUE (MW)
P <sub>1</sub>	69.93
P <sub>2</sub>	96.56
P <sub>5</sub>	41.71
P <sub>8</sub>	36.45
P <sub>11</sub>	22.31
P <sub>13</sub>	20.76

Description: Total active power generated for IEEE-30 bus system using  
 Matpower =P<sub>1</sub>+P<sub>2</sub>+P<sub>5</sub>+P<sub>8</sub>+P<sub>11</sub>+P<sub>13</sub>=287.74(MW)  
 And total load demanded =283.4 (MW)

**Table 4.2 total cost and losses for IEEE-30 bus system using GA:**

PARAMETER	VALUE
i) TOTAL COST	31,960 (rs/hr)
ii) TOTAL LOSSES	4.23 (MW)

Description: For IEEE-30 bus system using GA we get,  
 Total generation cost =31,960(rs/hr)  
 Total transmission losses =4.23(MW)

**Table 4.3 Generated power for IEEE 30 bus system with GA:**

VARIABLE	VALUE (MW)
P <sub>1</sub>	71.56
P <sub>2</sub>	97.63
P <sub>5</sub>	41.54
P <sub>8</sub>	34.8
P <sub>11</sub>	22.16
P <sub>13</sub>	20.02

Description: Total active power generated for IEEE-30 bus system using  
 GA = P<sub>1</sub>+P<sub>2</sub>+P<sub>5</sub>+P<sub>8</sub>+P<sub>11</sub>+P<sub>13</sub>=287.63(MW)  
 And total load demanded =283.4 (MW)

#### XIV. COMPARISON OF RESULTS OF GA WITH MATPOWER

**Table 4.4. Total cost and Losses for IEEE-30 bus system:**

PARAMETER	WITH GA	WITH MATPOWER
i) TOTAL COST	31,960 (rs/hr)	32,000 (rs/hr)
ii) TOTAL LOSSES	4.23 (MW)	4.34 (MW)

Description: To meet the load demand of 283.4 MW for IEEE-30 bus system, total generated power cost using  
 Matpower=32,000(rs/hr) & using GA =31,960(rs/hr).  
 Since demand is constant for both the methods, losses are less for GA compared to Matpower.

Table 4.5 Generated power for IEEE-30 bus system

VARIABLE	WITH MATPOWER	WITH GA
$P_1$ (MW)	69.93	71.56
$P_2$ (MW)	96.56	97.63
$P_5$ (MW)	41.71	41.54
$P_8$ (MW)	36.45	34.8
$P_{11}$ (MW)	22.31	22.06
$P_{13}$ (MW)	20.76	20.02

Description :Here  $P_1, P_2, P_5, P_8, P_{11}$  and  $P_{13}$  are the generated powers at buses 1,2,5,8,11 & 13 resp. To meet the demand of 283.4MW for IEEE-30 bus system ,total power generated using Matpower =287.74(MW) & using GA=287.63(MW)

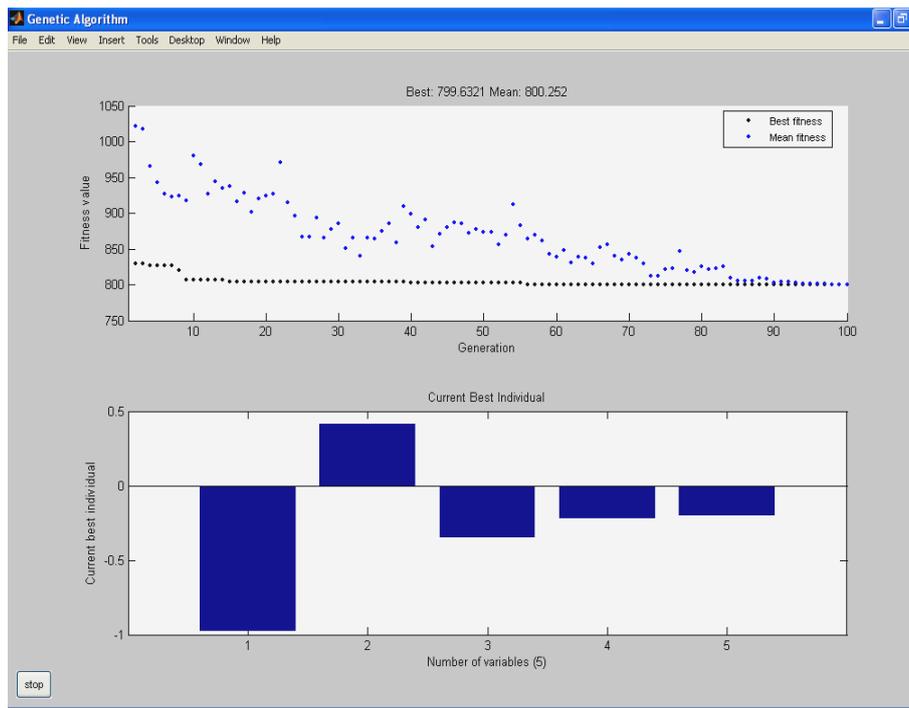


Fig 3: Total cost curve of a IEEE-30 bus system

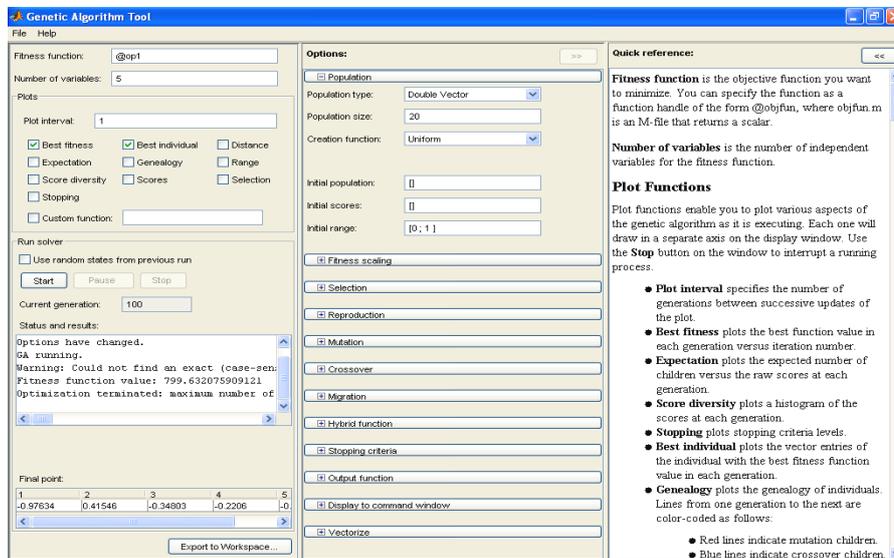


Fig 4: GA tool for IEEE -30 bus system

## XV. CONCLUSION

Application of Genetic approach to Optimal Power Flow has been explored and tested. A simulation results show that a simple genetic algorithm can give a best result using only simple genetic operations such as proportionate reproduction, simple mutation, and one-point crossover in binary codes. It's recommended to indicate that in large-scale system the numbers of constraints are very large consequently the GA accomplished in a large CPU time.

To save an important CPU time, the constraints are to be decomposing in active constraints and reactive ones. The active constraints are the parameters whose enter directly in the cost function and the reactive constraints are infecting the cost function indirectly. With this approach, only the active constraints are taken to calculate the optimal solution set. And the reactive constraints are taking in an efficient load flow by recalculate active power of the slack bus. The developed system was then tested and validated on the IEEE-30 bus systems. Solutions obtained with the developed Genetic Algorithm Optimal Power Flow program has shown to be almost as fast as the solutions given by Matpower package.

## XVI. FUTURE WORK

In this project OPF is solved by using GA method for IEEE-30 bus systems. OPF problem using GA in combination with the Particle Swarm Optimization technique can give better results compared to GA method alone.

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