

## Developments in structural analysis of composites for subsea shells

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**Abstract:** - Use of subsea shells have increased tremendously in the last years of twentieth century. Isotropic shells are slowly being replaced by laminated fiber reinforced composite shells, the material properties of which can be changed as desired. These materials have high strength to weight ratio which is the basic quality required for any subsea shell material. The scheme of analysis for laminated composite thin shells is different from that of thin isotropic shells and that itself is a challenge to the structural engineer. Subsea shells are subjected to external hydrostatic pressure which causes compressive stress resultants in the shell membrane leading to buckling failure of these shells. Finite element method has simplified the tedious analyses and henceforth various finite element packages are commercially available for the analysis of composite subsea shells. This paper analyses laminated composite thin shells under external pressure using two different finite element softwares viz., ANSYS and ABAQUS. Acceptable results for a reasonable mesh size are not easy in the case of finite elements available in these packages. The necessity for the development of a doubly curved, shear deformable, thin composite shell finite element which can be specifically used for the analysis of laminated composite subsea shells has been justified in this paper

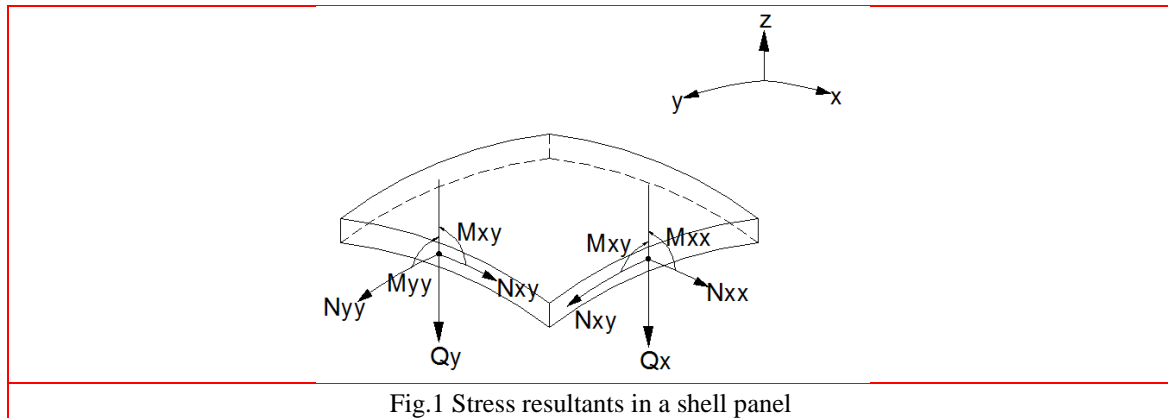
**Keywords:** - *subsea shells, laminated fiber reinforced composites, linear elastic analysis, linear buckling analysis, shear deformable element*

### I. INTRODUCTION

The structural components of underwater structures basically consist of shell forms to resist the external hydrostatic pressure. The main advantage of using shell forms is that it does not have any sharp edge, which suddenly obstructs the flow of water. Also the lack of sharp edges reduces the stress concentrations in the structure. The research on subsea structures have been initiated from the middle of last century which resulted into a variety of underwater structures being used today. The main subsea structure developed during 1950's was the submarines, which were used in for the naval warfare. Then came the era of underwater vehicles that were smaller than the submarines and these were then used for oceanographic investigations. Autonomous underwater vehicles which could complete the prescribed task according to the instructions given from onshore structures reduced the risk of man's life in the ocean. A further development of underwater vehicles is the autonomous underwater glider, which does not need power to propel itself forward. Underwater gliders permit the oceanographic investigations without disturbing the aquatic habitat and the organisms. Subsea shells are also used for underwater storage structures, underwater pipelines, structures used for tapping of tidal energy, underwater habitats etc. Shells of constant strength or echinodomes, a doubly curved shell, are used for the underwater storage structures. Despite the external components of tidal energy tapping structure, the turbine blades are doubly curved shell forms that are subjected to dynamic wave loading. Underwater habitats come in various forms with combinations of singly and doubly curved shell.

The basic structural component of underwater vehicles, whether it is submarine or autonomous underwater vehicle, is a cylindrical body with a hemi ellipsoidal or spherical enclosure. Subsea gliders have wings that are either singly curved or doubly curved shell panels. Thin shells that are subjected to external hydrostatic pressure are prone to buckling failure due to the compressive stress resultants developed in the shell membrane. During initiation of buckling failure the stresses in the shell membrane does not reach the ultimate

values and hence material failure does not take place; the structure fails by form failure. Various stress resultants acting on the shell membrane are shown in Fig.1.



The isotropic thin shells of revolution are not subjected to any shear stresses either inplane or transverse. This is not the case for anisotropic thin shells of revolution. Both the inplane and the transverse shear stresses will be present in such shells. These values are zero at the inner and outer surface, while vary across the thickness of the shell.

Table 1 Properties of candidate materials used for underwater vehicles (Wang et.al, 2009)

Material	Density (kg/dm <sup>3</sup> )	Yield strength (MPa)	Tensile modulus (GPa)	Specific strength (kNm/kg)
High strength Steel (HY80)	7.86	550	207	70
Aluminium alloy (7075-6)	2.9	503	70	173
Titanium alloy (6-4 STOA)	4.5	830	120	184
GFRP (Epoxy/S-Iass)	2.1	1200	65	571
CFRP (Epoxy/HS)	1.7	1200	210	706
MMC (6061 Al/SiC)	2.7	3000	140	1111
Acrylic	1.2	103	3.1	86
PVC	1.4	48	35	34

Steel, titanium and aluminium alloys have been the most commonly used structural material for subsea applications till about last quarter of twentieth century. The thickness warranted by subsea shells made of these materials has been so much that they would merely sink down to the ocean floor due to selfweight. This fact contributed to the application of composites in subsea, whose properties could be changed to match the designer's necessity. Laminated composites consist of layers of laminae bonded to each other by the bonding matrix material. A variety of fibers and matrices are used in the production of laminates, which are combinations of laminae. The most commonly used composite for marine vehicles is Glass-Fiber Reinforced Plastic (GFRP). GFRP is cheap with respect to other composites and has a very high strength to weight ratio. Carbon Fiber Reinforced Composites (CFRP) are about 3 times expensive than GFRP, but have a much higher tensile modulus than GFRP (Wang, 2009). Table 1 shows the properties of candidate materials used for underwater vehicles, from which it is evident that Metal Matrix Composite (MMC) is superior to other materials in terms of strength, but it is expensive than other materials (about 15 times more expensive than GFRP) (Wang et.al, 2009).

In isotropic thin shells, the transverse shear deformations are insignificant compared to other deformations and are often neglected. But when the material becomes anisotropic or orthotropic, such as laminated composites with various layers across the thickness, the transverse shear deformations of each layer depends on transverse shear stiffnesses. The transverse stiffness of laminated composites is much lower than the longitudinal stiffness due to the absence of fibers along transverse direction. Because of this, higher value of transverse shear deformation is obtained even for smaller forces acting on the structure and these cannot be neglected. So these should be taken into account while assuming the displacement field for the structure. Subsequently, the results of linear elastic and buckling analyses are affected by the inclusion of transverse shear deformations.

## II. FINITE ELEMENTS FOR SUBSEA SHELL APPLICATIONS

The subsea shells falls into the category of cylindrical, spherical, hemi ellipsoidal, shell panels, etc., or a combination of some of the above shapes. Two types of finite elements have been developed for the analysis of these shell forms, viz., axisymmetric and general shell elements which can either be singly curved or doubly curved. Among these the doubly curved elements can be used for analysing shell panels as well.

These shell elements can be triangular or quadrilateral elements and in general the shell geometry can effectively be modeled using doubly curved triangular elements. Higher order triangular elements can represent the shell geometry efficiently. Analysis of laminated composite thin shells under external pressure using two different finite element softwares viz., ANSYS and ABAQUS has been reported in this paper.

## III. FORMULATION OF FINITE ELEMENT FOR COMPOSITE SUBSEA SHELLS

Finite elements that are developed for the analysis of composite subsea shells should mainly possess the capability of conducting linear elastic analysis and buckling analysis besides incorporating the effect of hydrostatic follower force. The linear elastic analysis can be conducted based on linear strain-displacement relations derived for general shell element while the buckling analysis is based on nonlinear strain-displacement relations.

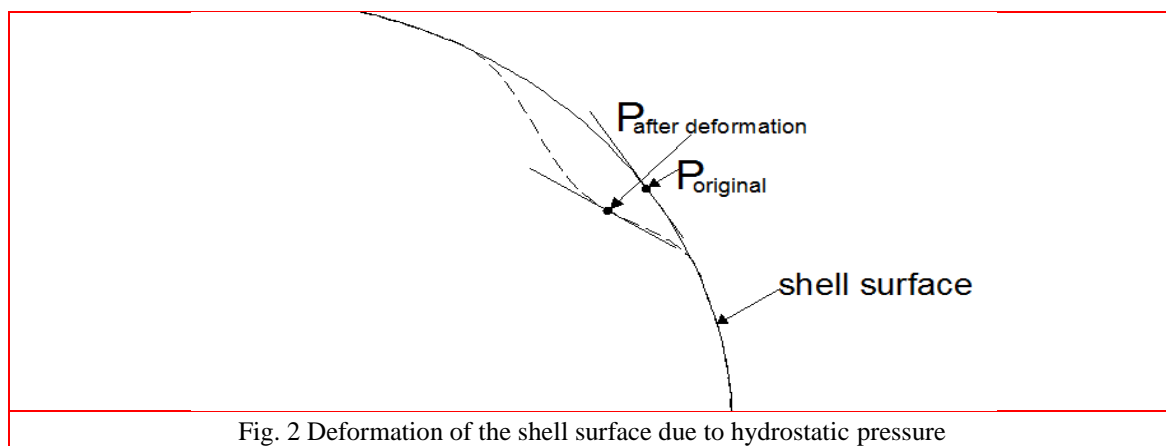


Fig. 2 Deformation of the shell surface due to hydrostatic pressure

The thickness of the composite subsea shells is lower than that of the conventional isotropic shells. As the thickness is lower, the deformation due to external hydrostatic pressure will be higher. Due to the hydrostatic pressure which acts normal to the shell surface, the shell deforms as shown in Fig.2. The hydrostatic pressure follows the deformed shell surface. This change in direction and subsequently magnitude of the pressure force is termed as the follower force effect.

### Formulation of linear elastic stiffness matrix

The displacement fields are based on first order shear deformation theory and are given by Reddy (1997). With these displacement fields, a linear strain – displacement relations and constitutive relations between stress resultants and strains are obtained. With suitable element shape functions and geometry of the element, the linear elastic stiffness matrix of the element is obtained as

$$k = \int_A [B]^T [D][B] dA \quad (1)$$

Where [B] is the strain- displacement matrix and [E] is the matrix relating stress resultants and the strains

### Formulation of geometric stiffness matrix

The geometric stiffness matrix is formulated based on the nonlinear strain-displacement relations given by Teng and Hong (1998). The work done by prebuckling stresses on nonlinear buckling strains is given by (Zienkiewicz and Taylor, 2000)

$$K_g d\{u\} = \int \{\sigma\} d[B_{nl}]^T dV \quad (2)$$

Where [B<sub>nl</sub>] is the incremental nonlinear strain displacement matrix. Rewriting the equation (2), the geometric stiffness matrix is given by

$$K_g = \int_A [G]^T [S][G]dA \tag{3}$$

Where [G] is the matrix of first order differential terms of shape functions while [S] is the matrix of stress resultants of the linear analysis

3.3 Formulation of pressure stiffness matrix

Pressure stiffness matrix is calculated from the potential energy change due to the action of hydrostatic pressure. The expression for change in potential energy is given by [Loganathan et.al, 1979]

$$\hat{\Pi} = -\frac{1}{2} \int \lambda p (w_0 (\beta_1 + \beta_2) - u_0 \phi_\theta - v_0 \phi_\varphi) dS \tag{4}$$

The work done is considered negative since the hydrostatic pressure acts inward. The equation (4) can be written as

$$\hat{\Pi} = \{u\}^T [k_p] \{u\} \tag{5}$$

The square matrix [kp] is called the pressure stiffness matrix.

$$[k_p] = p \int_A [N]^T [L] dS \tag{6}$$

where [N] is the shape function matrix and the [L] is the first order differentiation of [N].

**IV. AVAILABLE ELEMENTS IN COMMERCIAL FINITE ELEMENT SOFTWARE**

A large number of finite elements have been developed to model shells and has been used by finite element packages such as ABAQUS and ANSYS. Laminated composite shells are modeled using specialized elements which take into account the effect of transverse shear deformations.

Various shell elements are present in ABAQUS which can be used for modeling and analyzing composite shells. S4 and S4R are four noded quadrilateral elements, the first one with full integration and the second with reduced integration. Similarly three noded triangular elements (S3/S3R) and eight noded quadrilateral elements (S8R) are also present in the package. All these elements account for transverse shear deformations (ABAQUS Documentation, 2006).

ANSYS uses two shell elements for the analysis of laminated composite shells. SHELL181 is a four noded shell element with six degrees of freedom per node. It is well suited for linear, large rotation nonlinear applications. SHELL281 is an eight noded element with similar properties as the four noded elements. Both the elements are based on first order shear deformation theory (ANSYS Release Notes, 2009).

These elements are plate elements with 6 degrees of freedom per node. This shows that all these are flat shell elements and a refined mesh will be required to model and analyse the subsea shells discussed earlier. These elements are not based on shell theories and so the accurate results are difficult to obtain. Numerical investigations presented below depict this fact. The results of the linear elastic analysis are presented in Table 2

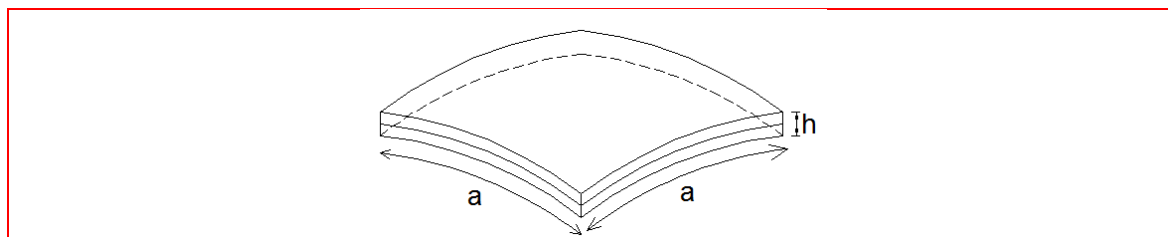


Fig.3. Geometry of the shallow shell panel

Table 2 Static response of anisotropic shallow shell

	ANSYS (SHELL281)	ABAQUS (S8R)	Somasekhar et.al, (1987)	Noor and Mathers (1975)
$\bar{w} = \frac{wE_2 h^3}{pa^4}$	5.2x10-4 (324 elements)	13.5 x 10-4 (225 elements)	5.27x10-4 (196 elements)	5.17x10-4
$\bar{N}_{\varphi\varphi} = \frac{N_{\varphi\varphi}}{pa}$	11.4 (62500 elements)	-	6.0 (196 elements)	6.0
$\bar{M}_{\varphi\varphi} = \frac{M_{\varphi\varphi}}{pa^2}$	7.35x10-3 (62500 elements)	-	4.38 x10-3 (196 elements)	4x10-3

The results from Table 1 indicate that ANSYS predicts acceptable values of deflection with tolerable number of elements, but the ABAQUS calculates a higher value of deflection. ABAQUS output does not contain the stress resultants and ANSYS forecasts intolerable values of the same even using large number of elements.

#### Linear Buckling Analysis of Orthotropic Cylindrical Shell

An orthotropic cylindrical shell reported by Hur et.al (2008) has been considered here for the linear buckling analysis. They have conducted both the experimental and finite element analysis of the shell. The characteristic features of the shell are as follows

Diameter of the cylinder = 316mm, length = 600mm. One end of the cylinder has been fixed and the other end has been assumed to be clamped. There have been 12 layers of (0/90)<sub>s</sub> each layer having a thickness of 0.105mm. E<sub>1</sub>=162 GPa, E<sub>2</sub>= 9.6 GPa, G<sub>12</sub>= 6.1 GPa, G<sub>13</sub>= 6.1 GPa, G<sub>23</sub>= 3.5 GPa,  $\nu_{12}$ =0.298.

The critical buckling pressure calculated using ABAQUS has been found to be 0.659 MPa using 680 elements. The same result using ANSYS has been obtained as 0.612 using 720 elements. The experimental results presented by Hur et.al (2008) have been 0.55 MPa, while the finite element result by them has been 0.641 MPa.

#### Appropriate finite element for the analysis of subsea shells

The stresses required for the design of composite subsea shells are the three membrane stresses, the transverse shear and interlaminar stress which are obtained from the linear elastic analysis. The output obtained by the commercial software include the abovementioned quantities. However, it may be noted that the abovementioned output has been realized for a fine mesh. The demand for a fine mesh can be overcome by suitably selecting the shape functions and efficiently describing the displacement field in including the transverse shear deformations. Such higher order elements when employed with isoparametric features map themselves into curved geometry and turnout to be well suited for doubly curved shell surfaces. Based on the abovementioned features a 9 noded, triangular, doubly curved, finite element based on Novozhilov shell theory with 5degrees of freedom per node viz., the three translations and the two rotations has been proposed with nonlinear variations of transverse shear strains across the thickness incorporated.

### V. CONCLUSION

Applications of composite subsea shells have been discussed. Finite element analysis of two typical components of composite subsea shell structures have been carried out using the commercial finite element packages ANSYS and ABAQUS. The results indicate the necessity of a new shell finite element to be used for the analysis of underwater composite shells. Based on the output, a new shell finite element has been proposed.

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