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Structural Analysis of Sandwich Submarine Structures

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Abstract: - The concept behind sandwich construction and its application in naval and commercial fields is highlighted in this paper with special reference to underwater shell forms. This paper also examines the various shell finite elements generated by researchers for the analysis of sandwich shells. A comprehensive overview of finite elements available for analysis of sandwich structures in commercial software packages is presented. The accuracy of the axisymmetric finite element sandwich shell is evaluated by comparing the results with the numerical results available by analysing the shell using ANSYS. The results of reasonable accuracy have been realized at the cost of heavy computation using the software. The need for a sandwich shell finite element based on sandwich shell theory has been justified.

Keywords: - Axisymmetric *shell finite element, sandwich, submarine;*

I.   INTRODUCTION

Submersible vehicles with pressure hulls are finding a growing range of applications in both naval and commercial fields. Naval uses include deployment and mine countermeasures and various elements of submarine and antisubmarine warfare. Commercial uses include oceanographic surveying, inspection and repair of cables, pipelines and other underwater installations for exploration and exploitation of ocean resources. It is being observed that more than 70% of underwater reaches lying at a depth of 3000m to 6000 m are yet to be explored. Hence a definite need is felt to develop and design deep submergence hulls which are light weight and capable of withstanding extreme underwater conditions.

Subsea shells in the form of circular cylinders are most commonly preferred because of its beneficial hydrodynamic form. Circular cross-section is the most efficient structural shape to withstand the high pressures on the hull at a given depth with the lowest stresses and optimum space utility. A number of wall architectures are available for cylindrical pressure hull such as pure monocoque or corrugated construction, ring stiffened and circumferentially tube stiffened configurations.

Structures that combine high stiffness, strength, and mechanical energy absorption with low weight are needed for naval applications. Sandwich structures are the most impressive and viable option because of its weight saving potential. Sandwich construction can be described as hybrid construction consisting of two outer dense facings with a light weight core. The facings usually provide the load carrying capability for sandwich construction. The primary requirement of submersible pressure hulls is to resist hydrostatic pressure exerted by water. The hydrostatic pressure induces the direct force resultants, but because of axisymmetric loading and geometry, the inplane shear force resultants and moment resultants vanish. Under a hydrostatic load, the main function of the core is to stabilize the facings and provide enough thickness to maintain shell buckling strength. For a long sandwich under uniformly distributed external pressure, the stresses and the displacements do not depend on the axial coordinate. The axisymmetric distribution of external forces produces stresses identical at all crosssections and dependent only on the radial coordinate. When hydrostatic pressure exceeds a critical value, shell buckling may occur to the pressure hull due to inplane compressive stress resultants induced in the shells. The sandwich shells are thick by generic and are immune to buckling failure which is usually associated

with shell membranes carrying the compressive stress resultants. Unlike ordinary shells, the sandwich shells are basically three layered structure in which transverse shear has significant contribution to the shell deformation.

A sandwich construction is characterized by very high flexural strength, low lateral deformations, higher buckling resistance and higher natural frequencies. The high stiffness to weight ratio makes this kind of structural element a very attractive design option in weight critical structures. The sandwich configurations have attracted significant interest particularly in marine application for its good out-of-plane stiffness and strength, high inplane stretching resistance along with excellent capacity to mitigate the underwater propagated shocks. The thickness of a structure can be increased by introducing a low modulus core in between the laminates even without incurring the weight penalty from adding extra laminate layers or stiffeners. In effect the light weight core acts as a separator between the load-bearing facings [1].

The low weight of vessels can enable increased payload with reduction in space requirements, fuel consumption and environmental emissions. Sandwich structures being generally very stiff require very less to almost no stiffeners to be dispensed with, giving smooth surfaces and a compact structure. In sandwich underwater shell the elastic stability is derived from the core separated facings and not from ring stiffeners and bulkheads as in its metal counterparts. The shell wall is very well capable of maintaining its shape under external load [2].

A typical composite sandwich structures construction reduces the weight by 30% to 70%, compared to traditional stiffened steel structures and thus allowing to achieve the operational capabilities that is almost impossible in case of traditional steel construction. Thus the pressure hull of subsea shell or habitat with fiber reinforced sandwich construction can greatly minimize the total weight of underwater vehicle and can be considered as an innovative and suitable replacement over traditional metallic ring stiffened pressure hulls [3].

Sandwich construction with composite facings is usually noncorroding giving low maintenance and low running costs. The thermal conductivity through the thickness for fiber reinforced multilayer sandwich construction is very low and hence increases the strength at elevated temperatures. Signature management is an important consideration for reducing the vulnerability, detection and attack of underwater vehicles. Use of such construction in naval vehicles, reduces the scope of acoustic signatures through incorporation of damping properties, as well as reduction of magnetic and electrical signatures [4]. Carbon reinforced plastics can have good absorption properties with regard to electromagnetic waves, giving good stealth properties

Composite sandwich constructions are less sensitive to fatigue and have high underwater shock resistance than with timber, which was previously used for mine counter-measure vessels. A longer time of sea exploitation is possible for sandwich construction as compared to stiffened metal structures which are weakened by the appearance of stress-concentrations due to welds sharp edges and cutouts in the structure. The physical and mechanical properties of the FRP faces can be tailored by proper selection of laminate materials, their stacking sequence and orientation. Suitable selection of fiber orientation and stacking sequence and core configuration can result in substantial improvements of the buckling strength [5].

Graphite/Epoxy Boron/Epoxy and Glass/Epoxy are the best choices available for composite facings material. For composite sandwich facings, the main reinforcement fibre materials are glass (E-glass, R-glass, S-glass), aramids (Kevlar, Twaron), carbon, polyester, High Performance Polyethylene and various hybrids/combinations. The foam cores for underwater applications are developed which are far lighter and can be tailored to optimize dynamic performance. The main foams in use are PVC (various types), polymethacrylimide (PMI), polyetherimide (PEI) and phenolic [6].

Regarding the structural analysis of underwater sandwich shells, two sensitive areas can be identified. The former is regarding the determination of stresses for facing and core. The later being the incorporation of strength properties of the component.

II.   FINITE ELEMENT ANALYSIS OF SANDWICH SHELLS

The earlier class of sandwich elements have been based on Reissner and Mindlin assumptions. The popularity of the Reissner-Mindlin element type arises partly from the simplicity in the formulation which requires only Co continuity, and partly from its general applicability to predict accurate results in global structural response. Three dimensional or layer-wise finite element analysis must be used to evaluate stresses for multilayered sandwich structures. The computational cost of 3D finite element analysis is very high because a

large number of elements are required in the thickness direction, while for the layerwise finite element analysis, the number of degrees of freedom increases with the increase in layer numbers [7].

In order to analyse thick sandwich structures with reduced computational effort two options have been suggested. The first option involves generation of elements based on higher order shell theories and second option includes elements in which inplane displacements is represented by zigzag theories. Many researchers developed higher-order theories in which the displacements of the middle surface are expanded as cubic functions of the thickness co-ordinate and the transverse displacement is assumed to be constant through the thickness. This displacement field leads to the parabolic distribution of the transverse shear stresses. Higher order elements allow for nonlinear warping of the cross-section by using extra degrees of freedom. This warping function is selected to achieve a distribution of transverse shear stresses vanishing at the exterior surfaces. The use of extra degrees of freedom in higher order elements involves higher order resultant moments and shears which have little physical meaning [8,9].

The zig-zag and layerwise theories within the concept of a sublaminate finite element formulation can also be used for analysis of sandwich shells. The variations of in-plane and transverse displacement components are chosen so as to satisfy the continuity of interlaminar stresses. A higher-order layerwise theory based on a Taylor's series expansion for in-plane displacements and constant variation of the transverse displacement component was proposed by Polit and Touratier [10]. They introduced additional unknowns into the in-plane displacement expressions that were later eliminated by imposing the continuity of interlaminar shear stresses, thus reducing the overall unknowns to displacements and slopes. A C1 continuous element is utilized in the finite element implementation. However, these formulations require an excessive number of degrees of freedom to be defined within each element if the sandwich panel contains many layers.

In the sandwich shells where transverse shear stress effects are of great importance, many high order shell theories exist, but few numerical tools have been developed based on this. These numerical tools are not pure structural models and suffer of the classical shear and membrane locking. The need is felt to generate a new finite element based on a sandwich shell theory which is simple to use, free from classical numerical pathologies. This paper addresses the development of a finite element involving physically meaningful displacements, rotations and shear strains as nodal parameters and which is efficient in computing both displacements and stresses for multilayered composite sandwich shells.

III.   FORMULATION FOR AXISYMMETRIC SANDWICH SHELLS

For an axisymmetric thin shell, the geometry of the shell surface is governed by the meridian alone and thus it is sufficient to define the shape of the meridian curve to idealize the shell form. A typical subsea shell generally consists of a large diameter cylindrical pressure hull with spherical, torispherical or hemi ellipsoidal dome ends. The differing diameter cylinders in a submarine are usually joined by conical sections. So all most all the sections of a subsea shell are axisymmetric and this considerably simplifies structural analysis of such shell forms by removing the curse of dimensionality. The three dimensional shell is reduced to one dimensional for the sake of analysis with both the shell and loading being axisymmetric in submarines provided the thickness of the shell is constant at a node and not the function of. Highly favourable stiffness to weight ratio of a sandwich construction has promoted its use in weight critical structures. A comprehensive effort has been made by Smith et al (1991), Liang et al (2002) and Lee *et al* (2013) [11] to examine the use of fibre-reinforced polymer composites in the pressure hulls of submersible vehicles.

Several authors have developed suitable singly and doubly curved axisymmetric finite elements for the analysis of homogenous shells of revolution. Numerous investigators have associated themselves with generation of different types of two and three dimensional sandwich shell finite elements, hardly few have explored the advantage of axisymmetry in the analysis of sandwich shells especially for subsea applications. The choice of an axisymmetric shell finite element is evident but very few finite elements are dedicated to this kind of analysis with all the above capabilities without numerical problems In fact, it is very difficult to achieve an efficient element with these capabilities for sandwich structure analysis.

Khojasteh-Bhakt [12] employed a doubly curved element for which the positions, slopes and curvatures of the shell meridian match at nodal circles .This element was modified by Abel [13] for sandwich application which did not impose any restrictions on relative thickness and material properties of core and

facing. A cubic polynomial was used to approximate transverse displacement and linear polynomial for inplane displacements and transverse shear strains for core and facings. The use of separate order polynomial for transverse displacements and meridional displacement induced inconsistencies in the inplane stresses at junctures where segments with different radii of curvature meet. To circumvent this inadequacy Sharifi (1973), modified the Abel's Linear Shear Strain element by using cubic polynomial for inplane and out of plane displacement and quadratic expansion for transverse shear strains of facing and core. However for Sharifi element, only the position and slope of the curve defining the shell geometry was matched at nodal circles [14]. The nodal compatibility curvature of shell geometry which has been ensured in Abel LSSE element was missing in the Slope compatible Quadratic Shear Strain element (SCQSSE) proposed by Sharifi.

The relation between inplane stresses and strains for axisymmetric deformation are represented by a state of generalized plane stress. The stresses acting in the axisymmetric shell for each layer includes the meridional stress, the hoop stress and transverse shear strain. As the shell is symmetric with respect to z axis, the inplane shear stress and the transverse shear strain $\tau_{\theta\zeta}$ are omitted from the stress-strain relationships. The stiffness matrix is derived by application of variational principle to the total potential of the structure. The total potential involves the strain energy stored in the structure and external work done by the load vector corresponding to the displacements.

IV. FINITE ELEMENTS AVAILABLE IN COMMERCIAL SOFTWARE PACKAGES

Most commonly available finite element softwares provide sandwich elements based on equivalent single layer model. The finite element package ANSYS provides SHELL91 used for layered applications of a structural shell model or for modelling thick sandwich structures. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

SHELL281 in ANSYS is suitable for analyzing thin to moderately-thick layered shell structures using first-order shear-deformation theory. The element has eight nodes with six degrees of freedom at each node: three translations and three rotations about the x, y, and z-axes. It is well suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness and follower effects of distributed pressures is accounted in nonlinear analyses.

SOLID46 is a layered version of the 8-node structural solid element in ANSYS designed to model layered thick shells or solids. The element allows up to 250 different material layers. The element has three translational degrees of freedom at each node.[15]

3-D Laminated Sandwich General Shell Element with NKTP = 33 in NISA [16] is suited for modeling moderately thick to thin sandwich shells. The element consists of two or more isotropic or layered face sheets, which are assumed to be thin and stiff, and one or more cores, which are relatively thick and flexible. The face sheets are assumed to be in a state of plane stress and the core material sustains only transverse shear stresses. The core is identified by a material property input of EX explicitly set to zero. The element has six degrees of freedom per node: three translations and three rotations. The element can be shaped as a 4 to 12 node quadrilateral depending on the selected NORDR value.

ABAQUS [17] supports element 8-node layered doubly curved thick shell with reduced integration; having six degrees of freedom per node (three translations and three rotations) S8R for the analysis of laminated composite shells. These elements are based on first-order transverse shear flexible theory in which the transverse shear strain is assumed to be constant through the thickness of the shell. This assumption necessitates the use of shear correction factors.

In layered model through the thickness there exist at least three layers of elements. The facings are usually modeled using thin shell elements. The thin shell elements like S8R5 (with & without) composite layup options can be used depending on facing properties. The core is either modelled using thick shell element or solid brick element. The thick shell option in ABAQUS is S8R and solid element option is C3D2OR brick element. C3D20R is a 20-noded quadratic brick element with reduced integration with 8 integration points. The active degrees of freedom are the three translations for a solid element. A solid section definition is used to define the section and material properties of solid elements.

V. NUMERICAL INVESTIGATIONS

The finite element analysis of sandwich hemispherical shell is done using ANSYS. The geometry of the hemispherical shell under membrane load is shown in Fig.1. The sandwich shell is having a radius of 100 inch and subjected to pressure loads of 1psi.
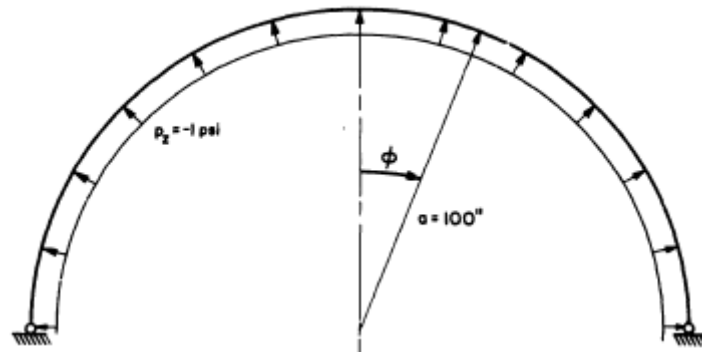

Fig.1 Hemispherical shell under membrane load

The material properties of facings and core are tabulated in table 1.The roller supports are provided at the ends to restrict meridional displacements at ends.

Table 1. Material properties of facing and core

Component	Thickness (mm)	Young's Modulus (Mpa) E	Poisson's ratio ν	Shear modulus (MPa) G
Facing	1.01	6894.75	0.3	26544.82
Core	12.7	179.26	0.3	68.94

Commonly Equivalent Single-Layer model (ESLM) is used to approximate the sandwich shell and is characterized by presence of a single element along the thickness of the shell. Core along within the facing are stacked in a sequential manner. The layup defines the material properties, thickness and orientation of each layer with reference to the coordinate system. In the ESL model, the spherical shell is meshed using SHELL281 with the facings and the core assumed to be isotropic and the loading as uniform pressure. The ANSYS finite element library doesn't have the option for an axisymmetric sandwich shell element. The symmetry boundary conditions are imposed along the meridional edge and roller support at hemispherical edge. The ANSYS finite element model is shown in Fig. 2. The present solution is obtained by generating a 2D model with 509 number of elements 1646 nodes available finite element softwares provide sandwich elements based on equivalent single layer model.
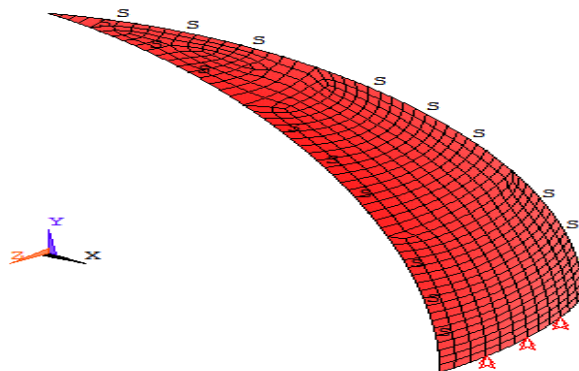

Fig.2 Finite element model for hemispherical shell

Finite Element solutions obtained using ESLM are compared with the classical solutions and tabulated in Table 2. The radial displacements obtained for ESLM of analysis are found to be faintly greater than the theoretical value.

Table 2. Comparison of results between classical solutions and ESLM

Response	Angular Position (φ) in deg	Classical Solutions	LSSE	ESLM (ANSYS)
Radial Displacement (in)	0	0.004305	0.004306	0.004321
	30	0.004305	0.004303	0.004328
	60	0.004305	0.004303	0.004321
	90	0.004305	0.004302	0.0042233
Radial Displacement (in)	0	0	0.0	0.2397×10^{-6}
	30	0	-0.46×10^{-7}	0.2386×10^{-6}
	60	0	0.69×10^{-7}	0.2394×10^{-6}
	90	0	0	0.2399×10^{-6}
Meridional extensional stress resultant (lb/in)	0	50	49.993	49.986
	30	50	49.987	49.984
	60	50	49.978	49.932
	90	50	50.023	49.937
Hoop extensional stress resultant (lb/in)	0	50	49.993	49.968
	30	50	49.981	49.938
	60	50	49.974	49.945
	90	50	49.981	49.966

Idealization for the hemisphere was done using only three LSSE elements and the results are satisfactory for both deflection and stress resultants. The meridional and radial displacements are calculated with adequate accuracy of 4 decimal places for the LSSE. The three element representation is quite enough for the idealization for the present problem with no spurious bending moments and shear forces at nodes. 509 number of SHELL 281 elements with 1646 number of nodes were required to obtain the comparable results while analyzing with ESLM using ANSYS software.

VI.   NEED FOR THE IMPROVED SANDWICH SHELL ELEMENT

In order to achieve computationally robust results for thick axisymmetric sandwich shells the existing SCQSSE must be modified by ensuring the curvature compatibility at nodes along with position and slope. The nodal compatibility of curvature is likely to improve the performance of such finite elements when the meridional curvature of shell changes in a noticeable fashion as in the case of subsea application of hemi ellipsoids or hemispherical shells. Hence a definite need is felt for such a quadratic shear strain element which possesses nodal compatibility of slope and curvature to employ it for the stress analysis of the subsea sandwich shells. Thus it is proposed to generate an axisymmetric Slope and Curvature Compatible Sandwich Shell Element (SCCQSSE) which improves the convergence of solution by using a complete displacement model to describe the field variables. This improved element would use the cubic polynomial for meridional and radial displacement thus overcoming the inaccuracies of LSSE at shell junctions of different curvatures and it uses a quadratic shear strain term which is more realistic in predicting the transverse shear even for coarser mesh. The sandwich shells used for subsea applications are generally very thick and are not prone to buckling failure unlike ordinary ring stiffened configurations. Thus the elastic response prediction which includes the deflections and stresses are essential for the design of the shell facings and core.

For sandwich structures of weak core and comparatively strong facings the shear deformation effect is much more pronounced. The transverse shearing deformation of each layer is assumed to be independent of the thickness and inplane displacements are continuous through the thickness. This element is thus applicable to structures wherein shearing deformations are significant. The advantage of use of quadratic shear model over linear shear is more evident in doubly curved shell where there is rapid variation of shear. The quadratic shear representation will produce far more satisfactory results with coarser mesh. The slope curvature compatible

QSSE considers the warping and shear strain of the facing as a nodal variable. The warping becomes significant in regions restrained sections where the variation in distribution of shear is more rapid.

The LSSE and SCQSSE generated only for isotropic facing and core. Use of composites is favored in subsea applications for its enhanced stealth properties, stiffness and strength. Thus the need is felt to modify the SCCQSSE to be used for analysis of composite sandwich shell forms that accounts for the discrete nature of each ply of the facing as well as the variation of stiffness and strength properties of the core and facing. In such situations, the constitutive law of the facings and core is modified and tailored for its application for linear elastic analysis of composite sandwich. The new constitutive relation generated depends on the material properties of the composites used and orientation of the layers of the composites in relation to the coordinate axes.

VII.　　CONCLUSION

The utility of sandwich shell form with composite facing and core has been brought here in this paper. The various types of finite elements for the analysis of sandwich shells are presented. The performance of finite elements from commercial packages and LSSE element developed by Abel has been compared with classical solutions. The need for an improved finite element has been presented.

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