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Quinquinomial Power Laws of Motion of Cam Mechanisms

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ABSTRACT
The quinquinomial power laws of motion make it possible to modeling laws of motion without finite and infinite spikes that result in better dynamic characteristics of high-speed, elastically deformable cam-lever systems compared to other laws of motion. Therefore, after applying a rational possibility for generating quinquinomial power laws of motion suitable for synthesizing polydyne cam mechanisms, a family of these laws has been studied.
The derived family of normalized quinquinomial power functions makes it possible to compile laws of motion without a finite and infinite spikes of cam mechanisms with better dynamic characteristics compared to trinomial and quadrinomial power laws of motion in the synthesis of high-speed, flexible cam-lever systems. This is because the parameters of the functions are derived from the condition for zeroing the first four derivatives of the normalized function at the beginning and at the end of the output move.
At low speeds, the real and the basic function of the output displacement practically coincide. At high values of speed, load, elastic deformations, and gaps of the cam-lever systems, a small part of the stroke of the executive link is lost. It is possible to preserve the type of the basic law of motion by slightly increasing the basic stroke of the output unit so as to compensate for the reduction in the travel (stroke) of the executive link.
KEYWORDS:*cam mechanisms, laws of motion, power functions*

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I. INTRODUCTION

It is generally accepted that the units of the cam mechanisms are non-gaps connected rigid bodies, whereby the mechanism generates a desired basic law of motion. In fact, the real laws of motion of the mechanisms differ as much from the basic ones as the speed of the cam, the load, the elastic deformations, and the gaps of the cam-lever systems are greater. Therefore, a number of authors [1], [2], take into account their influence on the output motion of the cam mechanisms when formulating a law of motion`s project.

The cams, designed according to polynomial laws of motion, taking into account the dynamics and deformations of the cam-operated mechanical system, are calledpolydyne cams. The synthesis of such cams is necessary for the construction of high-speed and insufficiently rigid mechanical systems, such as cam-lever distributing valves mechanisms for automotive [3], [4], and some high-speed transmission mechanisms of textile machines [2], [5].

The beginning of the development of methods for the synthesis of polydyne cams, set by Dudley, was supplemented and developed by many other authors mainly in connection with dynamic studies of cam-lever systems [6], [7], [8], [9], [10]. The main purpose of the methods is to exclude acceleration jumps, respectively of the inertial load of susceptible to flexible mechanical systems to achieve more accurate target motions with minimal vibration. If the desired law of motion needs to be observed as accurately as possible, then an additional correction is applied to the basic law of motion, which takes into account the speed of the cam, the inertial load, elasticity and gaps of the mechanical system, in order to obtain a law of motion`s project.

The basic law of motion of insufficiently rigid high-speed mechanisms is influenced most importantly by the basic second transfer function. This function, multiplied by the dynamic constant of the cam-operated mechanical system, changes the output move, as the inertial load generated by the acceleration deforms the units of the system elastically. In other words, the second derivative (the basic second transfer function) also participates in the real displacement function.

Therefore, in order to avoid jumps in the first two real transfer functions, it is necessary to avoid jumps in the next two basic transfer functions - the third and fourth. The mentioned jumps will be avoided if the transfer function and its first four derivatives are continuous functions, which means that the polynomial has at least five terms and the exponents are numbers of not less than 5. For this purpose, a family of these laws was studied, after applying a rational possibility to generate power-law quinquinomial motion laws, suitable for the synthesis of polydyne cam mechanisms.

II. DETERMINATION OF THE COEFFICIENTS OF NORMALIZED QUINQUINOMIAL POWER FUNCTIONS

In a previous article by the authors [11]a rational formula for determining the coefficients of power polynomials with any number of integer and/or non-integer exponents was derived. From this formula, at selected values j = k, m, p, q, s of the exponents, a quinquinomial power functionswithcoefficientsisformed:

$$(1)\quad \begin{cases} a_k = \dfrac{m\ p\ q\ s}{(m-k)(p-k)(q-k)(s-k)}; \\ a_m = \dfrac{k\ p\ q\ s}{(k-m)(p-m)(q-m)(s-m)}; \\ a_p = \dfrac{k\ m\ q\ s}{(k-p)(m-p)(q-p)(s-p)}; \\ a_q = \dfrac{k\ m\ p\ s}{(k-q)(m-q)(p-q)(s-q)}; \\ a_s = \dfrac{k\ m\ p\ q}{(k-s)(m-s)(p-s)(q-s)}. \end{cases}$$

These coefficients are derived from the condition of zeroing the first four derivatives at the beginning and end of the move of the output link.

Let the normalized quinquinomial power functions be determined with integer exponents from 5 to 10. The number of these laws is determined by the combinations without repetition of six elements (exponents 5, 6, 7, 8, 9, 10):

$$(2)\qquad C_6^5 = \binom{n}{p} = \frac{n\,!}{p!\,(n-p)!} = \frac{6\,!}{5!\,(6-5)!} = 6\,.$$

The values of the coefficientsa_k, a_m, a_p, a_q, a_s when k, m, p, q, s are integers from 5 to 10 are calculated from ratios (1) and recorded in Table I. Figure 1 shows the first four power polynomials $u(\xi)$ and their derivatives in the order of Table I.

Table I. *Coefficients* $a_i (i = 5, 6, 7, 8, 9, 10)$

Polynomial	a_5	a_6	a_7	a_8	a_9	a_{10}
1	126	-420	540	-315	70	0
2	112	-350	400	-175	0	14
3	94,5	-262.5	225	0	-87.5	31.5
4	72	-150	0	225	-200	54
5	42	0	-300	525	-350	84
6	0	210	-720	945	-560	126

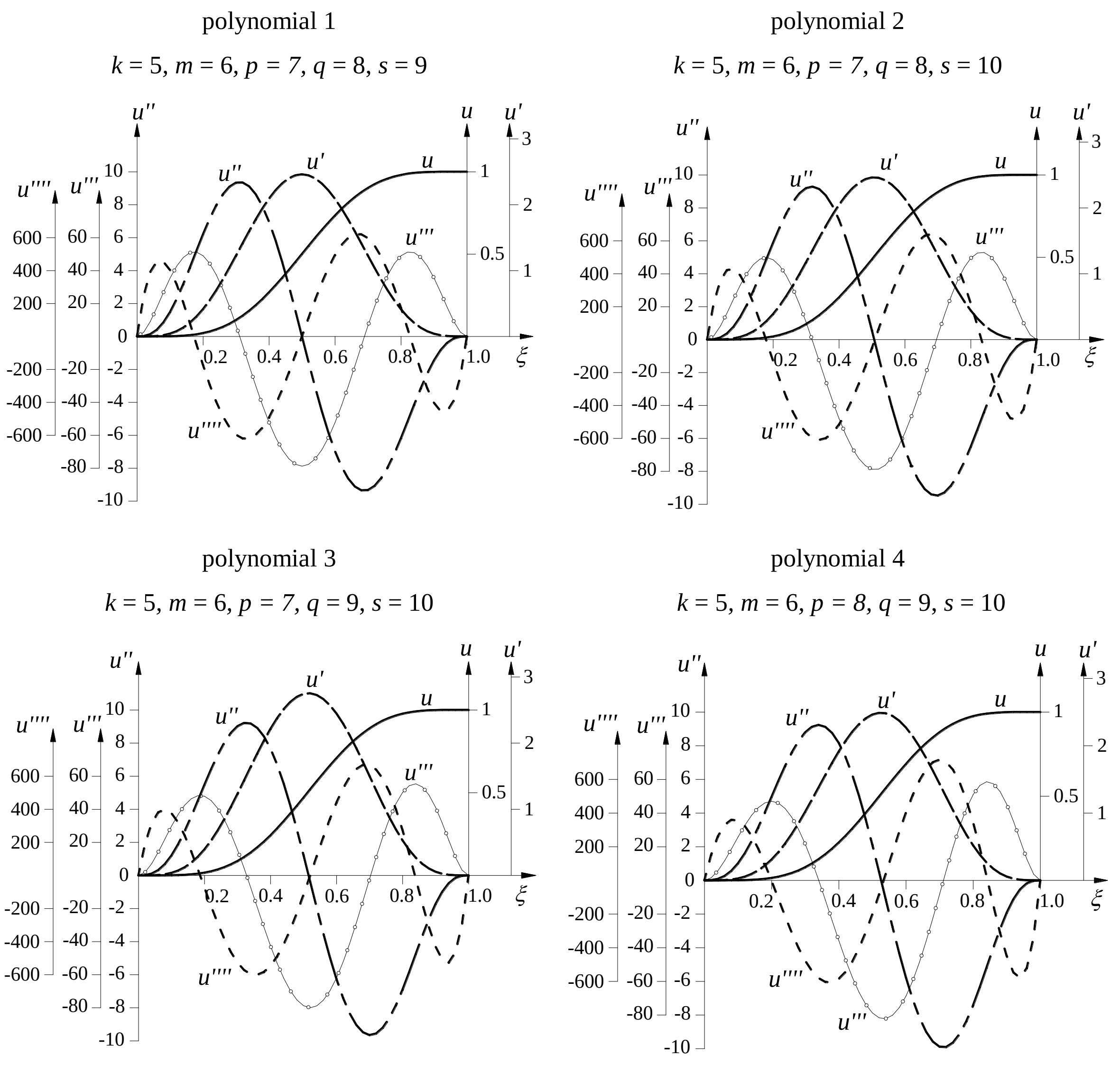

Figure 1. *Graphics of the quinquinomials of Table I*

- **polynomial 1**. For the power quinquinomial$u(\xi)$ with coefficients of row № 1 of Table I and its derivatives, is obtained:

$$(3)\qquad \begin{cases} u(\xi)=126\xi^5-420\xi^6+540\xi^7-315\xi^8+70\xi^9, \\ u'(\xi)=630(\xi^4-4\xi^5+6\xi^6-4\xi^7+\xi^8), \\ u''(\xi)=2520(\xi^3-5\xi^4+9\xi^5-7\xi^6+2\xi^7), \\ u'''(\xi)=2520(3\xi^2-20\xi^3+45\xi^4-42\xi^5+14\xi^6), \\ u''''(\xi)=15120(\xi-10\xi^2+30\xi^3-35\xi^4+14\xi^5)\,, \end{cases}$$

There is a zeroing of these functions for the interval limits $\xi\in[0,1]$.

- **polynomial 2.** For the power quinquinomial$u(\xi)$ with coefficients of row № 2 of Table I and its derivatives, is obtained:

(4)
$$\begin{cases} u(\xi) = 112\xi^5 - 350\xi^6 + 400\xi^7 - 175\xi^8 + 14\xi^{10}, \\ u'(\xi) = 140(4\xi^4 - 15\xi^5 + 20\xi^6 - 10\xi^7 + \xi^9), \\ u''(\xi) = 140(16\xi^3 - 75\xi^4 + 120\xi^5 - 70\xi^6 + 9\xi^8), \\ u'''(\xi) = 1680(4\xi^2 - 25\xi^3 + 50\xi^4 - 35\xi^5 + 6\xi^7), \\ u''''(\xi) = 1680(8\xi - 75\xi^2 + 200\xi^3 - 175\xi^4 + 42\xi^6), \end{cases}$$

There is a zeroing of these functions for the interval limits $\xi \in [0, 1]$.

- **polynomial 3.** For the quinquinomial power $u(\xi)$ with coefficients of row № 3 of Table I and its derivatives, is obtained:

(5)
$$\begin{cases} u(\xi) = 0.5(189\xi^5 - 525\xi^6 + 450\xi^7 - 175\xi^9 + 63\xi^{10}), \\ u'(\xi) = 157.5(3\xi^4 - 10\xi^5 + 10\xi^6 - 5\xi^8 + 2\xi^9), \\ u''(\xi) = 315(6\xi^3 - 25\xi^4 + 30\xi^5 - 20\xi^7 + 9\xi^8), \\ u'''(\xi) = 630(9\xi^2 - 50\xi^3 + 75\xi^4 - 70\xi^6 + 36\xi^7), \\ u''''(\xi) = 1890(6\xi - 50\xi^2 + 100\xi^3 - 140\xi^5 + 84\xi^6) \end{cases}$$

There is a zeroing of these functions for the interval limits $\xi \in [0, 1]$.

- **polynomial 4.** For the power quinquinomial$u(\xi)$ with coefficients of row № 4 of Table I and its derivatives, is obtained:

(6)
$$\begin{cases} u(\xi) = 72\xi^5 - 150\xi^6 + 225\xi^8 - 200\xi^9 + 54\xi^{10}, \\ u'(\xi) = 180(2\xi^4 - 5\xi^5 + 10\xi^7 - 10\xi^8 + 3\xi^9), \\ u''(\xi) = 180(8\xi^3 - 25\xi^4 + 70\xi^6 - 80\xi^7 + 27\xi^8), \\ u'''(\xi) = 720(6\xi^2 - 25\xi^3 + 105\xi^5 - 140\xi^6 + 54\xi^7), \\ u''''(\xi) = 2160(4\xi - 25\xi^2 + 175\xi^4 - 280\xi^5 + 126\xi^6) \end{cases}$$

There is a zeroing of these functions for the interval limits $\xi \in [0, 1]$.

The extremum values of the quinquinomial power laws of Figure 1 are recorded in Table II to assist engineers in selecting a basic law of motion suitable for the design of polydyne cams.

***Table II.** Comparison table of the extremums of functions u ', u '' and u ''' of Figure 1*

Functions $u(\xi)$	$u'_{\max}$	$u''_{\max}$	$u''_{\min}$	$u'''_{\max}$	$u'''_{\min}$
$u = 126\xi^5 - 420\xi^6 + 540\xi^7 - 315\xi^8 + 70\xi^9$	2.461	9.372	-9.372	51.428	-78.75
$u = 112\xi^5 - 350\xi^6 + 400\xi^7 - 175\xi^8 + 14\xi^{10}$	2.463	9.288	-9.484	53.116	-78.992
$u = 0.5(189\xi^5 - 525\xi^6 + 450\xi^7 - 175\xi^9 + 63\xi^{10})$	2.753	9.228	-9.659	55.428	-79.963
$u = 72\xi^5 - 150\xi^6 + 225\xi^8 - 200\xi^9 + 54\xi^{10}$	2.456	9.232	-9.932	58.651	-82.224
$u = 42\xi^5 - 300\xi^7 + 525\xi^8 - 350\xi^9 + 84\xi^{10}$	2.526	9.388	-10.363	63.261	-86.74
$u = 210\xi^6 - 720\xi^7 + 945\xi^8 - 560\xi^9 + 126\xi^{10}$	2.602	9.893	-11.058	70.104	-95.289

From the graphs of Fig. 1 and Table 2 it is seen that as the values of the exponents increase in not very large limits, the extremums of the functions u', u'' and u''' increase, being zeroed for the limits of the argument $\xi = \varphi / \Phi_1 \in [0;\ 1]$.

A detailed solution to the question of the laws of motion and synthesis of cam mechanisms was made by Galabov, Roussev, and Paleva-Kadiyska in [12].

III. CONCLUSION

The derived family of normalized quinquinomial power functions makes it possible to compile laws of motion without a finite and infinite spikes of cam mechanisms with better dynamic characteristics compared to trinomial and quadrinomial power laws of motion in the synthesis of high-speed, flexible cam-lever systems. This is because the parameters of the functions are derived from the condition for zeroing the first four derivatives of the normalized function at the beginning and at the end of the output move.

At low speeds, the real and the basic function of the output displacement practically coincide. At high values of speed, load, elastic deformations, and gaps of the cam-lever systems, a small part of the stroke of the executive link is lost. It is possible to preserve the type of the basic law of motion by slightly increasing the basic stroke of the output unit so as to compensate for the reduction in the travel (stroke) of the executive link.

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