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About Electrical Resistance in Weakly Conducting Homogeneous Media

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ABSTRACT: *In the exercises of Irodov I.E. there are several tasks for finding the resistance of a system of electrodes located in a uniform weakly conducting medium. The problem of finding resistance between two metal balls is considered. It has been shown that the potentials of metal balls located in a weakly conducting homogeneous medium influence each other. This effect was taken into account by introducing image charges. Using the image method for a sphere, the dependence of the system resistance on the distance between the balls is found. Previously, only the limiting case was considered when the distance between the balls is much larger than the radius of the balls.*

KEY WORDS: *weakly conductive medium, resistance calculation, image method*

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I. INTRODUCTION

Many authors believe that to increase the standards of teaching physics in higher education institutions, more attention should be paid to solving physical problems in practical classes [1-4]. In addition to knowledge of theoretical material for solving problems, additional skills developing creativity are needed. Solving problems causes students a keen interest in physics, encourages them to analyze physical phenomena, instills a taste for original solutions, and also develops the ability to think independently and prepare answers to non-standard questions. Several popular problem books on general physics are known [5–8], among which the book by Irodov I.E. [7-8] occupies a special place. Since 1979, it has been reprinted more than 10 times and has been repeatedly translated into English [8].

The book contains about 2000 problems in all sections of the course of general physics. The variety and originality of many tasks, combined with brief theoretical information and extensive reference tables make this collection useful and convenient for students. All tasks are provided with the correct answers. The task book is recommended for university students of physical and engineering profile.

The calculation of the resistance of complex resistor compounds has always attracted the attention of physicists. In the book of problems Irodov I.E. [7, 8] there are also several tasks for finding the resistance of a system of electrodes located in a homogeneous weakly conducting medium. Our attention was drawn to some of these tasks, which allow interesting generalizations available to undergraduate students.

II. KNOWN SOLUTIONS TO SOME PROBLEMS

Here we provide solutions to some problems that seem interesting to us. Here is one of these tasks (see [7], problem 2.164):

Problem 1. Two metal balls of the same radius a are in a homogeneous weakly conducting medium with resistivity ρ. Find the resistance of the medium between the balls, provided that the distance between them is much greater than a.

We offer the following solution to this problem. Let voltage be applied to one of the balls $+\frac{U}{2}$, and to the other $-\frac{U}{2}$. Then on the first ball will be a positive charge:

$$q = 2\pi\varepsilon_0 aU \ . \qquad (1)$$

We write the Gauss theorem for electric field strength $\vec{E}$:

$$\oint_S E_n dS = \frac{q}{\varepsilon_0}. \tag{2}$$

Here S is a closed surface surrounding this charge.
Using Ohm's law in differential form:

$$\vec{j} = \frac{1}{\rho}\vec{E}, \tag{3}$$

where $\vec{j}$ – the current density, ρ – the resistivity of the medium, we rewrite (2) in the form:

$$\rho \oint_S j_n dS = \frac{q}{\varepsilon_0}. \tag{4}$$

The current flowing between the balls is:

$$I = \oint_S j_n dS, \tag{5}$$

Therefore, we rewrite (4) in the form:

$$\rho I = \frac{q}{\varepsilon_0}. \tag{6}$$

Substituting (1) in (6), we obtain

$$\rho I = 2\pi a U. \tag{7}$$

At present we find resistance

$$R = \frac{U}{I} = \frac{\rho}{2\pi a}. \tag{8}$$

Consider another problem (see [7], problem 2.162):
Problem 2. A metal ball of radius a is surrounded by a concentric thin metal shell of radius b. The space between these electrodes is filled with a homogeneous weakly conducting medium with resistivity ρ. Find the resistance of the inter-electrode gap. Consider also the case $b \to \infty$.
The resistance of a spherical layer with thickness dr:

$$dR = \rho \frac{dr}{4\pi r^2}. \tag{9}$$

Integrating this expression, we find the resistance between the electrodes:

$$R = \frac{\rho}{4\pi} \int_a^b \frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right). \tag{10}$$

In the case when $b \to \infty$, we obtain the following result:

$$R = \frac{\rho}{4\pi} \int_a^\infty \frac{dr}{r^2} = \frac{\rho}{4\pi a}. \tag{11}$$

Now, solution (8) of the first problem can be obtained using formula (11) as well. We come to the problem of two balls of radius a connected in series through a conducting medium. Moreover, we assume that the plane located in the middle between the balls has zero potential.

III. A MORE GENERAL SOLUTION TO PROBLEMS

Now let's try to answer the question: «How can the resistance depend on the distance l between the centers of the balls?» We assume that the positive charge q defined by formula (1) is in the center of the first ball and creates a potential $+\frac{U}{2}$ on it. At the center of the second ball is a charge $-q$ that creates potential $-\frac{U}{2}$ on it. We take into account the effect of this charge by introducing the charge-image $q' = q\frac{a}{l}$, which is induced on the first ball. The charge-image q' together with the negative charge $-q$ on the second ball will provide zero potential on the surface of the first ball. Thus, for the flow of electric field strength, instead of formula (2), we obtain:

$$\oint_S E_n dS = \frac{q+q'}{\varepsilon_0} = \frac{q}{\varepsilon_0}\left(1+\frac{a}{l}\right). \tag{12}$$

Given (12), instead of formula (6) we get:

$$\rho I = \frac{q}{\varepsilon_0}\left(1+\frac{a}{l}\right). \tag{13}$$

Substituting expression (1) in (13), we obtain

$$\rho I = 2\pi a U\left(1+\frac{a}{l}\right). \tag{14}$$

Given only first-order terms on $\frac{a}{l}$, we find:

$$R = \frac{U}{I} = \frac{\rho}{2\pi a\left(1+\frac{a}{l}\right)} \cong \frac{\rho}{2\pi a}\left(1-\frac{a}{l}\right). \tag{15}$$

In fact, we will have an infinite number of charges-images, and instead of the term $\left(1+\frac{a}{l}\right)$ in the denominator of expression (15), we get a geometric progression, the sum of which:

$$\left(1+\frac{a}{l}+\frac{a^2}{l^2}+\frac{a^3}{l^3}+\ldots\right) = \frac{1}{1-\frac{a}{l}}. \tag{16}$$

As a result, we conclude that formula (15) is accurate, if we neglect the small difference in the location of the charges-images, i.e. we obtain again:

$$R = \frac{\rho}{2\pi a}\left(1-\frac{a}{l}\right). \tag{17}$$

Let us make one more generalization of formula (17). Consider the case where the radius of the balls are different: the radius of one is *a*, and the other is. You can guess the resulting formula. Firstly, it should go over to formula (17) with $b = a$, and secondly, it should be symmetrical with respect to a and *b*:

$$R = \frac{\rho(a+b)}{4\pi ab}\left(1-\frac{2ab}{l\,(a+b)}\right). \tag{18}$$

IV. CONCLUSION

This work should encourage the search for problems in well-known textbooks that allow interesting generalizations available to undergraduate and graduate students. The search for more general solutions to known problems is one of the ways to improve physical education, instilling a taste for original solutions and giving impetus to the development of creativity. In the considered problem, the potentials of metal balls located in a weakly conducting homogeneous medium influence each other. This effect was taken into account by introducing image charges. Using the image method for the sphere, it was possible to find the dependence of the resistance of the system on the distance between the balls. This effect was previously neglected, considering only the limiting case when the distance between the balls is much larger than the radius of the balls $l >> a$.

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