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A Peridynamics Implements Finite Element Methodwith Truss element in Rectangle Bond (PDIFEMTRB) Captures Wood Section Fracture

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ABSTRACT
A Peridynamics theory implemented finite element method with truss element mesh in rectangle bond (PDIFEMTRB) is the method that has been used into FEA software to model wood sections fracture. To implement the FEA method with peridynamics theory, MATLAB algorithms are used to orient the truss elements in rectangle bond samples ink.file, then imported to LS DYNA user-defined subroutine (UMAT) finite element code featuring with explicit time integration. In purpose to capture crack propagation on clear wood section fracture, *MAT_003_KINEMATIC_ ELASTIC_ MATERIAL is the main UMAT using in these simulations by respecting to the bond based peridynamics theory of material point, the failure criteria are provided and when the elongation of each truss element be greater than failure strain criteria, truss elements and bonds will be automatically deleted to produce crack propagation and bond breakage on wood section fracture with well captured and validated to the realistic sample.
KEY WORDS: *Wood section fracture; PDIFEMTRB; bond based; crack propagation; elongation*

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I.   INTRODUCTION

The study of wood mechanical properties is the most competitive in wood construction technology industries. Currently, numerical simulation is the most important for verifying a material characteristic of fracture. Numerical models for wood fracture and failure are commonly based on the finite element method (FEM) [1]. The use of finite element method to model wood fracture is based on linear elastic fracture mechanics (LEFM) and derivative modeling concept that require some boundary condition such as, initial crack of predetermining size, location, and orientation which specify capable to predict the level of loading that cause the crack to propagate. Some research has focused on macrocrack, a mixed-mode I and II fracture criterion application [2]is the method used to oriented crack in both along and across the fibers in the wood section within the framework of linear elastic fracture mechanics (LEFM). The applicability is restricted to clear the wood structural dimensions, since all effect of heterogeneity are neglected, and the material is treated as a homogenous orthotropic continuum. R-curve [3]is another method used to measure the fracture-resistant curve in laminate veneer lumber of wood by using crack propagation, and fracture toughness to characterize the durability of wood and wood composites.

The new particle-based method, called Material Point Method (MPM) is another numerical method has been developed for solving problems in dynamic solid mechanics [4]. This method, the solid body is discretized into the collection of points in form as grids of pixels. The dynamics analysis proceeds by tracked on each material point and updating all required properties such as position, velocity, acceleration, stress state, etc. It has also been used to model the crack propagation geometry of the crack path in wood section fracture by modeling the realistic morphologies of earlywood and latewood [5]. However, this method is still dealing with rigidity and providing the result with large deformation and required some crack criteria. This method is still not wisely use in all structural analysis software and still needs the knowledge of language coding.

The study using FEAhas stayed meet with a lot of problems and requirements. It fails to model the interface between dissimilar materials. The failure prediction is not physically based. The failure is a dynamics

process (re-meshing), crack growth is guided, stressed and strain fields are discontinuous, and crack initiation is not resolved. The FEA with cohesive elements is required prior knowledge of the crack path. The extended finite element (XFEM) is required the external criteria for crack propagation, and the progressive in laminate is still a problem. It is also required different computational domain for coupled field analysis that it is very difficult to use with wood section [6].To deploy wood as the structural orthotropic material that can be used in the FEA software, the mechanical properties of wood must be known. LS DYNA is widely FEA software used that can be modeled composite structure including wood laminates and other fiber composite crashing and fracture. It is great for modeling crashing and other damage but it still could not be well captured on crack propagations that related to its internal meshing from software [7]. [8] use all *Mat_58 (*Mat_ Laminated_ Composite_ Fabric) and *Mat 143 (*Mat_ Wood) to model on wood in six anatomical directions and three points bending test with wood fiber oriented parallel to the beam axis by using only clear wood section. In this studying, samples are modeling in *Mat 143 Wood and *Mat 58 that all samples are used solid section with the quasi-static. However, *Mat 143 Wood and *Mat 58 still cannot use with triangular shell element mesh, it can capable only with 3D solid section, but in this studying, we use truss element meshing in 2D only.

To implements aerodynamics into FEM software with LS DYNA, *Mat_ 003_ Plastic_ Kinematics is used to instead of *Mat143 and *Mat 58 in the modeling of Wood. Even we use *Mat_ 003_ Plastic_ Kinematics, the crack propagations are still could not be well captured. In purpose to solve this problem, we have to propose a new method called truss meshing in a rectangle bond by implementing all nodes of each element meshes to get its bond with the dimension of $\delta = 3\Delta x$ each bond length base on the peridynamics bond base theory of material points. We use only bond base theory [9] with allowing one node of each element meshing to interact with other nodes of the element in its bond. When each element mesh reaches the maximum strain or failure criteria, and the energy release rate, the element will be automatically deleted to produce crack growth on the section. This method can be well captured by comparing it with the FEA samples and experiments.

II. THEORETICAL BACKGROUND

II.1 Peridynamics theory

The first peridynamics theory is called bond based was implied to the brittle material [9]. In peridynamics model, each material point interacts with its neighbors in a compact zone. The interaction between two material points is called a "bond" which is independent with each other. The pairwise bond forces are collinear with the line of a bond and have opposite directions. The bond-based peridynamics model is well developed and has been applied to simulate damage and fracture in the brittle materials [10], the reinforced concrete material [11], the composite laminate structure [12,13].

$$\rho(x)\ddot{u}(x,t) = \int_{H_x} f\left[u(x',t) - u(x,t)\right]dV_{x'} + b(x,t) \qquad (1)$$

Motion equation of peridynamics theory defines that in a reference configuration R, each material point x has a subdomain H_x with a radius of δ, which is called the material horizon, as shown in Fig.1. Point x interacts with all the points x' in its horizon through the pairwise force f, which has a unit as force per volume squared.The equation of motion at any time t for material point x can be expressed as shown in Eq. (1),where $\rho$is the density, and $u$is the displacement. The force density for point x in aerodynamics is an integration of all pairwise forces between x and x' in its horizon, which is different from the differentiation in continuum theory. b is the body force. The relative position between point$x$and x' is ξ and the relative displacement from point x and x' isη.

$$\xi = x' - x \quad \text{and} \quad \eta = u(x',t) - u(x,t) \qquad (2)$$

For each bond, the relative elongation is presented as the stretch s. The pairwise force corresponding to the linear micro elastic potential has the following form. Then the micro potential can be further expressed with the formulation containing micro modulus, stretch, and relative displacement as shown in Eq. (3-5) below.

$$s = \frac{\|\eta + \xi\| - \|\xi\|}{\|\xi\|} \qquad (3)$$

$$f(\eta, \xi) = \mu(x' - x, t)\frac{\xi + \eta}{|\xi + \eta|}cs \qquad (4)$$

$$\mu(x' - x, t) = \begin{cases} 1 & , s(x' - x, t) < s_0 \\ 0 & , s(x' - x, t) > s_0 \end{cases} \qquad (5)$$

The strain energy density in peridynamics can be calculated with integration, as shown in Eq. (6). Micro modulus $c(\xi)$ can then be calculated by equating the strain energy density in Peridynamics and that from continuum theory.

$$W = \frac{1}{2}\int \omega(\mathbf{\eta},\xi)dV = \frac{1}{2}\int_0^\delta \left(\frac{cs^2\xi}{2}\right)4\pi\xi^2 d\xi = \frac{\pi cs^2\delta^4}{4} \quad (6)$$

II.2 Peridynamics model of wood section

A Peridynamics model of wood section fracture, pairwise forces interact between material points are represent peridynamics bond which is consists of two types of different bond constant c as illustrated in Fig. 1. Matrix bonds are oriented in all directions and they represent mechanical properties of the resin material. Fiber bond which represents the mechanical properties of wood ring direction on a wood section perpendicular to the grain.
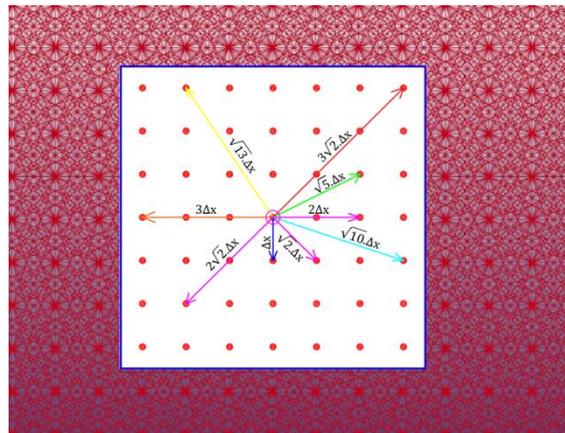
Figure 1: Truss Element in rectangle bond that a bond contains 24 truss elements with 9 different lengths

Peridynamics for wood ring bonds (c_r) and matrix bond (c_m) can be derived by equating strain energy density in continuum mechanics and peridynamics theory under expansion loading by using [15].

$$c_r = \frac{Q_{11} - Q_{22}}{\frac{1}{2}\sum_{q=1}^{Q}\xi_{qi}V_q} \quad (7)$$

$$c_m = \frac{24Q_{12}}{\pi t\delta^3} = \frac{8Q_{22}}{\pi t\delta^3} \quad (8)$$

Where

$$Q_{11} = \frac{E_{11}}{1-v_{12}v_{21}}, \quad Q_{12} = \frac{v_{12}E_{22}}{1-v_{12}v_{21}}, \quad Q_{22} = \frac{E_{22}}{1-v_{12}v_{21}}, \quad Q_{66} = G_{12}, \quad v_{12} = v_{21}\frac{E_{11}}{E_{22}}$$

In Eq. (7) and (8), ξ_{qi} stands for the initial length of the bond between material points q and i, V_q is used for defining the volume of the cubic element that includes the material points q, δ is the horizon of the material points, and t is the thickness of the plate.

II.3 Discretization method

The discretized form of the equation of motion (1) replaces the integral by a finite sum

$$\rho\ddot{u}_i^n = \sum_p f(u_p^n - u_i^n, x_p - x_i)V_p + b_i^n, \quad (9)$$

II.4. Finite element implementation

Implementation in finite element code, meshes made of truss meshing in rectangle bond elements with the appropriate stiffness properties are used to represent the aerodynamics bonds. To do this, first, observe that multiplying Eq. (9) by V_i leads to an equation of motion identical in form to that of FEA[17].

$$V_i\rho\ddot{u}_i^n = \sum_p f\left(u_p^n - u_i^n, x_p - x_i\right)V_p V_i + b_i^n V_i, \quad (10)$$

$$M\ddot{u}^n + F_T^n = F_e^n, \quad (11)$$

Where M is a lump mass matrix and, F_e^n is $b_i^n V_i$ the external force vector, and F_T^n is $\sum_p f(u_p^n - u_i^n, x_p - x_i)V_p V_i$ the internal force vector. Each diagonal term of M is ρV_i and each component of F_e^n is $b_i^n V_i$, which is the sum of all the forces from the truss elements connected to node i. Thus, the process of implementing peridynamics in an *FEA* code essentially boils down to the preprocessing task of generating a truss assembly (mesh) and stiffness properties for the truss element consistent with the peridynamics theory. In this new method, the bond changes from a circle to the rectangle with truss element mesh that each bond, there are 24 truss element meshes and 9 trusselements difference size with $\Delta x = 1.1mm$ a mesh grid. The definition of section and material properties for the peridynamics.Truss mesh in rectangle bond is again a straightforward application of the theory. The mass of the nodes is defined via lumped masses as ρV_i, and the truss elements densities are set as the properties of wood density. Because ofEq. (4) indicate that only the forces from the node in truss elements connected to other nodes are required, there is some no uniqueness in the definition of the cross-sectional area A and elastic modulus E for the truss elements. A convenient separation, which preserves the conventional units of these properties, is

$$A = (V_p V_i)^{1/3}, \qquad E = c(V_p V_i)^{2/3} \qquad (12)$$

For the truss element mesh with 9 different length elements size, then

$$A = \Delta x_j^2, E = c\Delta x_j^4, \qquad j = 1,2,3,...9 \qquad (13)$$

Also, as shown by Eq. (3), the bond stretch s is identical to the engineering strain in the truss elements, thus the fracture strain of each triangular element is

$$\varepsilon_f = S_0 \qquad (14)$$
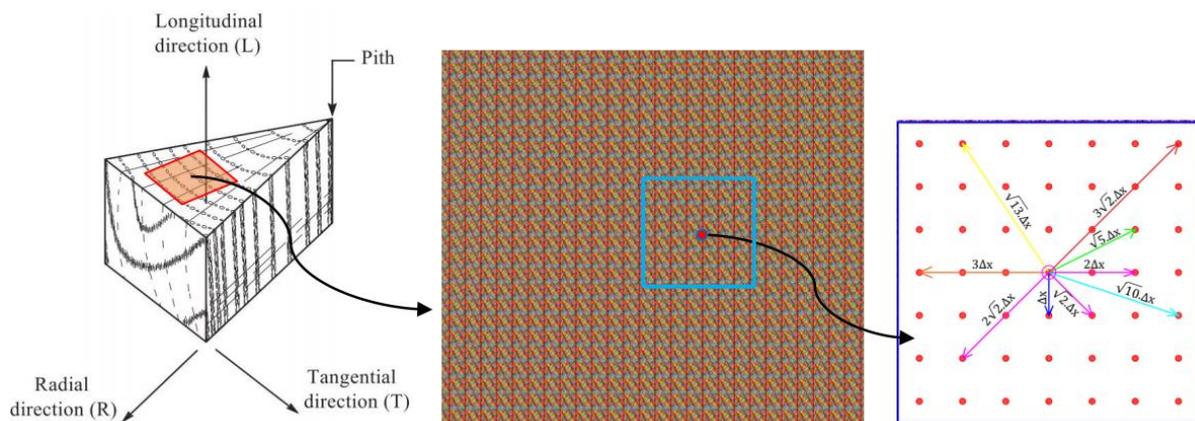
Figure 1: Truss mesh oriented in rectangle bond that a point or node interactswith all nodes in bond with 24 truss elements and 9 different elements length.

The strain S_Y at which the bonds or trusses element yield is related to Young's modulus E and the engineering ultimate stress in tension σ_{ult} by $S_Y \approx \dfrac{\sigma_{ult}}{2E}$ （15）

III. MODELING

III.1 *Problem setup*

To model wood section fracture and crack propagation between FEM and PDIFEMTRBsamples,we have to oriented with small scale perpendicular to the grain in the direction R 0-degree (perpendicular to T), 45-degree to T, and 90-degree (parallel to T direction). With the dimension side as inFig.2(a), (b), and (c) 150mm length and 150mm width with 75mm of initial crack.We apply loads to the opposite directions as in Fig.2(a) and (b) have been applied with $P = 1.5kN$ in the velocity of 2.5mm/min. MATLAB algorithms are used to orient the sample truss mesh element in rectangle bondsimport to LS DYNA*MAT_ PLASTIC_ KINEMATIC by providing the wood mechanical properties of spruce wood as in Table.1.Failure criteria for the truss element are defined. The effecting of kinematic hardening is ignored by setting $\beta = 1$. The erosion element of the failure strainparameteris determined with the failure criteria 0.009 to 0.002 for as in [14]. The verification of peridynamics theory into the FEA method,*DEFINE_ CURVE and *BOUNDARY_ PRECRIBED_ MOTION_ SET_ ID card the velocity boundary condition is applied. To plotgraphs of resultant of displacement and velocity, a point has been defined as A (x=75.48mm, y= 75.48mm).

The convergence of grid meshes and bondseffecting for these samples are chosen with $\Delta x = 1.1mm$ and $\delta = 3\Delta x$ is very suitable and reliable. The effectiveness of mesh size and bond length is very important in capturing of crack propagation and the cost of simulation [9,14,16].

Table1. Mechanical Properties and Boundary Conditions

Description	Values
Velocity Boundary condition (Vo)	2.5mm/s
Modulus Elastic in Transverse Direction (E1)	600 MPa
Modulus Elastic in Radius Direction (E2)	600 MPa
In plane poison ratio (v12)	0.0156
In-plane Shear Modulus (G12)	700 MPa
Load Boundary Condition (P)	1.5 kN
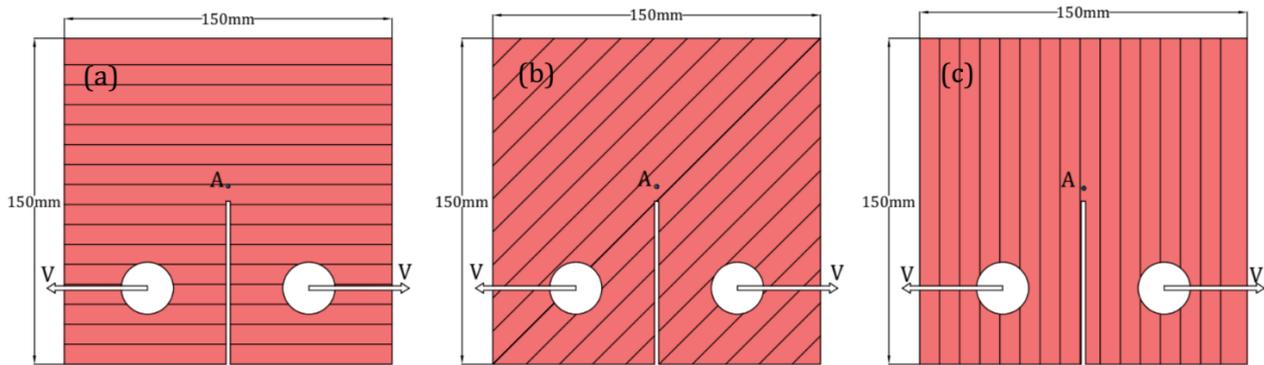
Figure 2: Wood section sample perpendicular to grain with (a) 0-degree, (b) 45-degree, and (c) 90-degree

III.2 Results
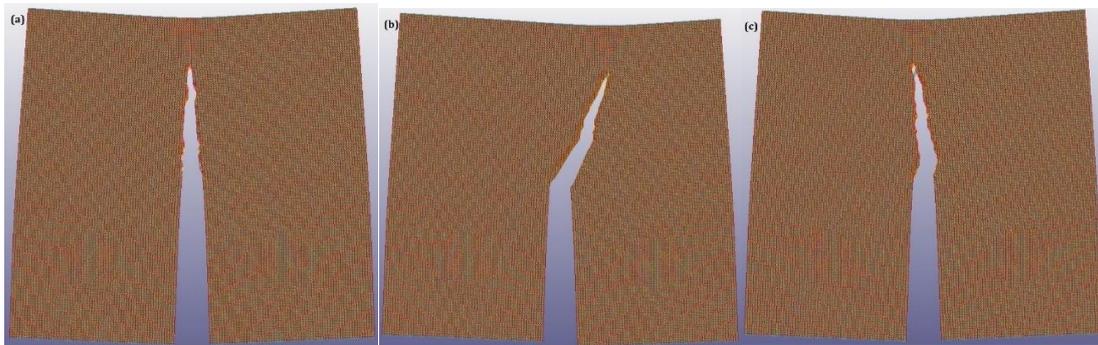
Figure 3: PDIFEMTRB Resultant Truss fracture (a) 0-degree, (b) 45-degree, and (c) 90-degree at time t = 5s
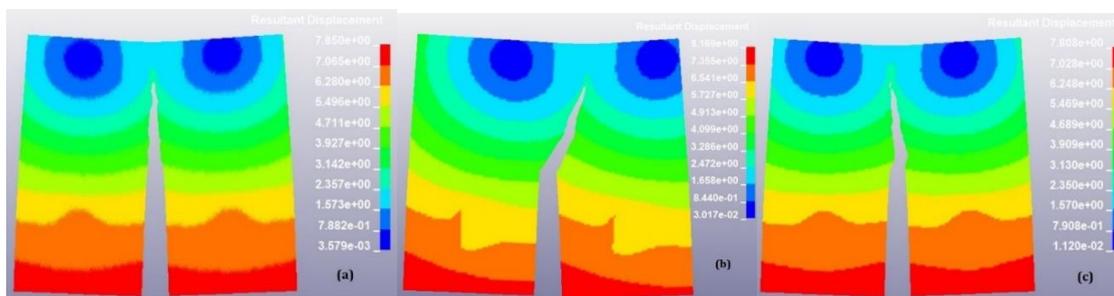
Figure 4:PDIFEMTRBResultant Displacement (a) 0-degree, (b) 45-degree, and (c) 90-degree at time t = 5s
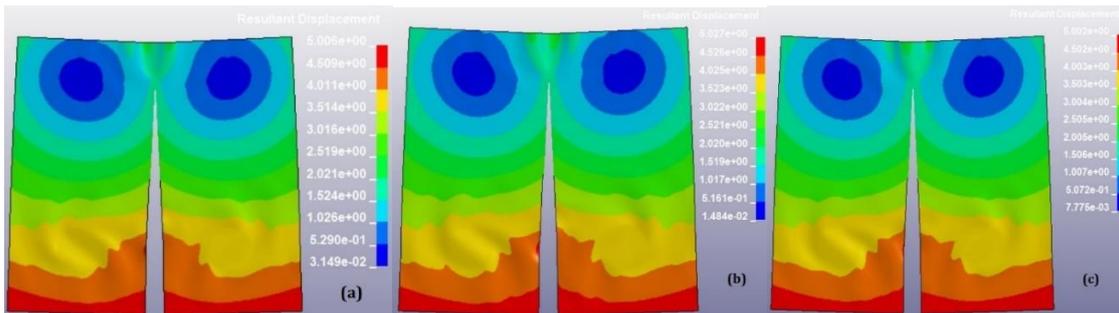
Figure 5: FEM Resultant Displacement (a) 0-degree, (b) 45-degree, and (c) 90-degree at time t = 1s
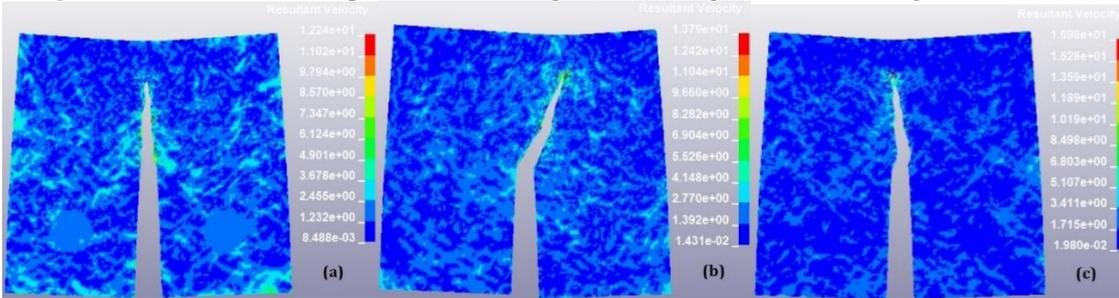
Figure 6: PDIFEMTRB Resultant Velocity (a) 0-degree, (b) 45-degree, and (c) 90-degree at time t = 5s
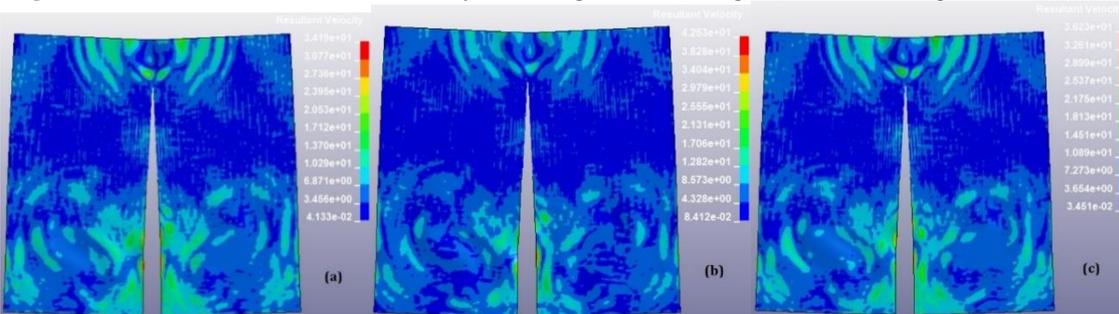
Figure 7: FEM Resultant Velocity (a) 0-degree, (b) 45-degree, and (c) 90-degree at time t = 1s
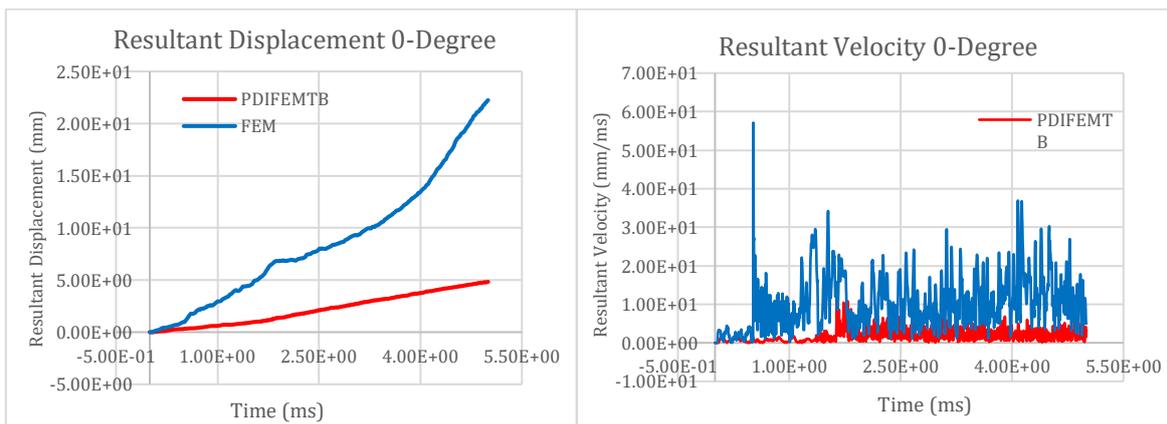
Figure 8. The Resultant Displacement and Velocity of 0-degree orientation between PDIFEMTRB and FEM
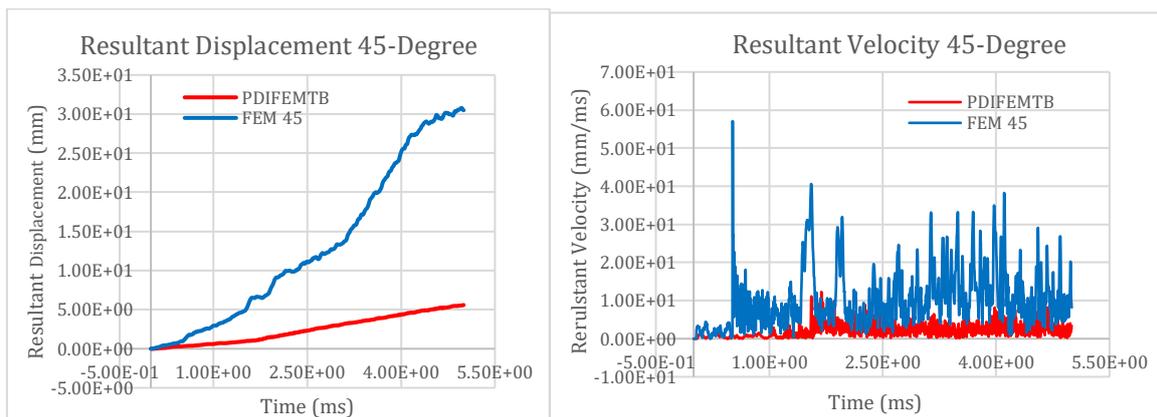
Figure 9. The Resultant Displacement and Velocity of 45-degree orientation between PDIFEMTRB and FEM
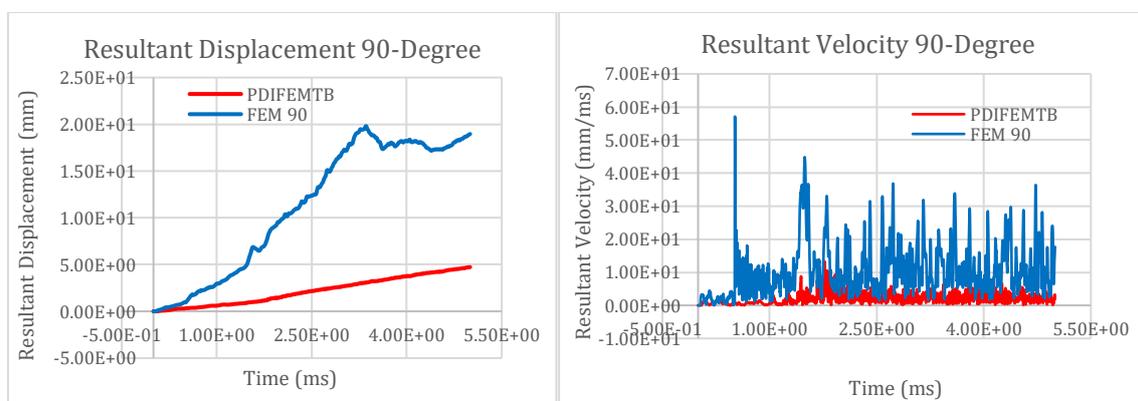
Figure 10 The resultant Displacement and Velocity of 90-degree orientation between PDIFEMTRB and FEM

III.3 *Explanation*

According to the resultillustrated inFig.4&6shown that crack propagations on wood sections arecaptured with PDIFEMTRBin resultant displacement and velocity. By using with the same wood mechanical properties, the event all sample oriented with different angles 0, 45, and 90-degree and the wood properties are the same in all directions; transverse direction and radial direction, bond breaking and crack propagations are well captured along with wood mechanical fracture and aerodynamics theory of material point [9]. Otherwise, the result from simple FEM samples as in Fig.5&7 shown t, even we use the same wood mechanical properties and failure criteria, crack propagations still can not be captured to get along with the real wood ring section fracture that why it fails to model of the wood section fracture.According to graphs inFig.8-10,all the resultant displacements and velocities have shown, PDIFEMTRB useless cost than FEM method but can well succeed illustrate the crack propagations on wood section fracture. PDIFEMTRB uses less cost because of the elongation of the interaction between two points as truss elements, but in FEM, the elongations are the interacted with four nodes or points.

IV. CONCLUSION

Peridyanamics implementsthe finite element methodwith truss mesh in rectangle bond (PDIFEMTRB) is succeed to in modelclear wood section fracture perpendicular to grain by comparing with elastic FEM method. Crack initiation and propagation perpendicular, 45-degree, and parallel to wood ring or transverse direction, arewell captured with the same as in the real wood section mechanical fracture. Samples are validated and oriented by MATLAB algorithms import into LS DYNA FEA softwareconcerningthe aerodynamics theory of material points. This paper is validated apply successfully implements peridynamics theory into LS DYNA FEA software by truss mesh bond (PDIFEMTRB) in the modeling ofclear wood section fracture.

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