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Control System Design of Linear Quadratic Proportional-Integral-Plus (LQ-PIP) Controller for MIMO Systems

Ubong Ukommi[1], Mbetobong Charles Enoh[1], Ibiangake Etie Akpan[1]
[1](Department of Elect/Elect Engineering, Akwa Ibom State University, Nigeria)
Corresponding Author: Ubong Ukommi

ABSTRACT : *MIMO systems are systems with more than one control input and outputs variables. This work is based on the True Digital control design implementation for MIMO systems. In our previous paper; "optimal decoupling control design for multivariable processes: the quadruple tank application", the SISO control technique was implemented. The PIP formation allows for the implementation of an SVF control action with complete decoupling or by optimal LQ-PIP control design. In this paper, we derived the SVF control law and a* Non-minimal State Space *(NMSS) equation for a MIMO system. Their Transfer Function Matrix (TFM) contains n X m transfer function (TF), and each TF has a relationship between the input and output. The Two input, Two-output, DT TFM model represented in terms of the left matrix fraction description (LMFD) is considered. The optimal LQ-PIP FB gain matrix is designed to minimise the LQ cost function. Also, we designed the multivariable decoupling control with the ability to dynamically decouple control loop interactions.*

KEYWORDS Multivariable Systems, continuous-time systems, multivariable discrete-time system, gain matrix.

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I. INTRODUCTION

According to Wang et al [1], the conventional framework of model predictive control, designed using a state-space model, consists of an observer and a state feedback controller. Subsequently, an on-line optimization scheme is applied to calculate the state feedback control law subject to plant operational constraints [2]. In the context of discrete-time MPC, the possibility of using a non-minimal state-space (NMSS) representation of the controlled system can help avoid the need for an observer as proven in [3]. The article [3] proposed a model predictive control scheme based on a non-minimal state-space (NMSS) structure. This combination was able to yield a continuous-time state-space model predictive control system that permits hard constraints to be imposed on both plant input and output variables, whilst using NMSS output-feedback with no observer needed. In addition, a comparison between the NMSS and observer-based approaches using Monte Carlo uncertainty analysis was conducted. The results showed that the former design is considerably less sensitive to plant-model mismatch than the latter. Furthermore, by simulation studies, the article also investigated the role of the implementation filter in noise attenuation, disturbance rejection and robustness of the closed-loop predictive control system. The results showed that the filter poles became a subset of the closed-loop poles and this provided a straightforward method of tuning the closed-loop performance to achieve a reasonable balance between speed of response, disturbance rejection, measurement noise attenuation and robustness [3].

II. SYSTEMMODEL AND REPRESENTATION

The system model of the multivariable process using the concept of a nonlinear Quadruple tank application in our previous work [2] is adopted in paper as shown in figure 1. In brief description, the TITO Quadruple Tank Process consists of four interconnected identical water tanks, two pumps and two valves that allow the inflow of water into the upper and lower tanks. The tanks are piled orderly in a vertical manner with one tank over another.

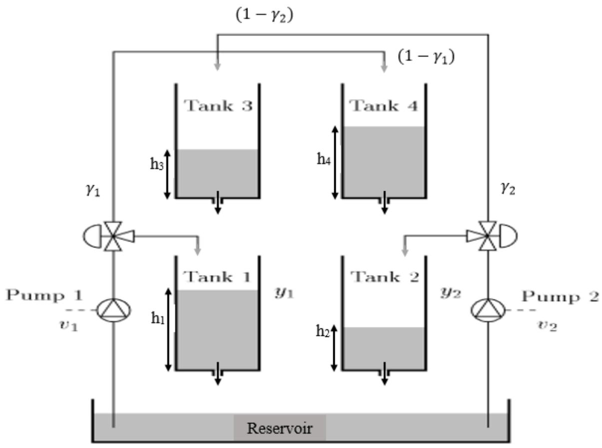

Fig. 1. Schematic diagram of the quadruple tank process

Non-minimal State Space (NMSS) Model Representation

A multivariable discrete-time system presented in [2] will be represented in a transfer function matrix form as shown.

$$y(k) = \left(A(z^{-1})\right)^{-1} B(z^{-1})u(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 1$$

where y(k) and u(k) are vectors of r input and p output variables respectively. G(z^{-1})isa matrix of TF model similar to the general DT-TF model for a SISO system [1].

$$y(k) = [y_1(k)y_2(k) \ \dots \ y_p(k)\ ]^T \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 2$$

$$u(k) = [u_1(k)u_2(k) \ \dots \ u_r(k)\ ]^T \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 3$$

where $A(z^{-1})$ and $B(z^{-1})$are polynomials express as

$$B(z^{-1}) = B_1 z^{-1} + B_2 z^{-2} + \cdots + B_m z^{-m} (B_m \neq 0) \quad - \quad - \quad - \quad - \quad 4$$

$$A(z^{-1}) = 1 + A_1 z^{-1} + A_2 z^{-2} + \cdots + A_n z^{-n} (A_n \neq 0) \quad - \quad - \quad - \quad 5$$

In a multi-variable system, $A_i(i = 1,2, \dots, n)$are P x $P$and $B_i(i = 1,2, \dots, m)$ are P x r matrices, while i is a p x p identity matrix. The NMSS state vector $x(k)$ comprises vectors of past and present system output and past inputs, and defined as;

$$\mathrm{x}(k) = [y(k)^T \dots y(k-n+1)^T \quad u(k-1)^T \dots u(k-m+1)^T \ \ z(k)^T\ ]^T \quad - \quad - \quad 6$$

The command input vector $y_d(k)$ and $z(k)\ in$ [1], for a MIMO system will now be in the form;

$$y_d(k) = [y_{d1}(k)y_{d2}(k) \quad \dots \ y_{dp}(k)\ ]^T \quad - \quad - \quad - \quad - \quad - \quad - \quad 7$$

$$z(k) = [z_1(k)z_2(k) \quad \dots \ z_p(k)\ ]^T \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 8$$

the NMSS representation can be derived as;

$$x(k0 = Fx(k-1) + Gu(k-1) + D_{yd}(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad 9$$

and the associated output equation:

$$y(k) = Hx(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 10$$

where the state transition matrix F, input vector G, command input vector D and output vector H are defined as follows:

$$F = \begin{pmatrix} -A_1 & -A_2 & \vdots & -A_{n-1} & -A_n & B_2 & B_3 & \vdots & B_{m-1} & B_m & 0 \\ I_p & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & I_p & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & I_p & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & I_r & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & I_r & \vdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 \\ A_1 & A_2 & \vdots & A_{n-1} & A_n & -B_2 & -B_3 & \vdots & -B_{m-1} & -B_m & I_p \end{pmatrix} \quad - \quad - \quad - \quad 11$$

$$G = [B_1\ 0\ \ 0 \ldots\ 0\ \ I_r\ 0\ \ 0\ \ \ldots\ \ \ 0\ \ -B_1\ ]^T \quad - \quad - \quad - \quad - \quad - \quad 12$$

$$h = [I\ \ 0\ \ \ldots 0\ 0\ \ 0\ \ \ 0 \ldots\ \ 0\ \ \ 0\ \ \ 0] \quad - \quad - \quad - \quad - \quad - \quad - \quad 13$$

$$D = [0\ 0\ \ 0 \ldots\ 0\ \ 0\ 0\ 0\ \ \ldots\ \ \ 0\ \ I_p\ ]^T \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 14$$

Here, I_p and I_r denote p x p and p x r identity matrices, respectively. The PIP state variable control law associated with the multivariable NMSS model shown in [1] takes the form

$$u(k) = -Kx(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 15$$

where K is the controller gain matrix in the multivariable system.

Linear Quadratic LQ-PIP controller for MIMO Control

The SVF control law in equation 15 takes the same form and the NMSS can also be formulated for the LQ PIP controllers. The FB gain K is designed to minimise the quadratic function J_m as presented in [1][7] as;

$$J_m = \frac{1}{2}\sum_{k=0}^{\infty} x(k)^T Qx(k) + u(k)^T Ru(k) \quad - \quad - \quad - \quad - \quad - \quad 16$$

where $Q = q^T q$ is a positive semi-definite symmetric state weighting matrix and $R = r^T r$ is a positive definite symmetric input weighting matrix. q and r are the associated choleski factors. Equation 16 is the infinite time optimal LQ cost function for multi-variable system similar to SISO cost function in [2]. The PIP-LQ controllers ensures a better closed-loop performance, with little or no cross-coupling.

Implementation of Multivariable LQ-PIP control law

The multivariable weighting matrices Q and R are represented as;

$$Q = diag(\bar{y}_1\ \ldots\ \ \bar{y}_n \bar{u}_1\ \ \ldots\ \ \bar{u}_{m-1}\bar{z}) \quad - \quad - \quad - \quad - \quad - \quad 17$$

$$R = \left[\frac{{u_1}^w}{m}\ \ \ldots\ \ \frac{{u_r}^w}{m}\right] \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 18$$

where Q defines the measured input, measured output and the integral of error states

$$\bar{y}_i(i = 1\ \ \ldots\ \ n) = \left[\frac{{y_1}^w}{n}\ \ \ldots\ \ \frac{{y_p}^w}{n}\right] \quad - \quad - \quad - \quad - \quad - \quad - \quad 19$$

$$\bar{u}_i(i = 1\ \ \ldots\ \ m-1) = \left[\frac{{u_1}^w}{m}\ \ \ldots\ \ \frac{{u_r}^w}{m}\right] \quad - \quad - \quad - \quad - \quad - \quad 20$$

$$\bar{z} = \left[{z_1}^w\ \ \ldots\ \ {z_p}^w\right] \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 21$$

${y_1}^w$, … ${y_p}^w$, ${u_1}^w$ … ${u_r}^w$and${z_1}^w$ … ${z_p}^w$are the weighting parameters associated with integral of error state, all present and past inputs and outputs variables respectively carefully selected in the design process. The control gain matrix K solved from the NMSS state transition matrix F, the state input vector matrix G, the state weighting matrix Q and the input weighting matrix R for a MIMO system is represented as

$$K = (R + G^T PG)^{-1} G^T PF \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 22$$

$$= [L_0 L_1 \ \ldots \ L_{n-1} M_1 \ \ldots \ M_{m-1} - K_1] \quad - \quad - \quad - \quad - \quad - \quad - \quad 23$$

where the matrix P is the steady state solution of the discrete time matrix Riccati equation as shown below:

$$P - F^T PF + F^T PG(R + G^T PG)^{-1} G^T PF - F = 0 \quad - \quad - \quad - \quad - \quad - \quad 24$$

The gain matrix K is usually in form of the output and input feedback matrices $L(z^{-1})$and $M(z^{-1})$given as

$$L(z^{-1}) \ = L_0 + L_1 z^{-1} + \cdots + L_{n-1} z^{-n+1} \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 25$$

$$M(z^{-1}) \ = M_1 z^{-1} + M_2 z^{-2} \ldots + M_{m-1} z^{-m+1} \quad - \quad - \quad - \quad - \quad - \quad - \quad 26$$

substituting equation 12 into 15, the optimal state variable control law that minimises multivariable system LQ cost function becomes

$$u(k) = -(R + G^T PG)^{-1} G^T PFx(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 27$$

PIP Decoupling Control by Combined Algebraic Pole Assignment

Different decoupling techniques for a multivariable system have being presented in various literature. Morgan Jr in [3] researched on the design and synthesis of non-interacting control systems, In [4], the authors carried out the technique of decoupling multivariable systems by SVF. Lees et al. in [5] investigated the nonminimal state feedback approach to multivariable control of glasshouse climate where the decoupling techniques were also implemented. The PIP decoupling control technique tuned by combined algebraic pole assignment, with closed loop responses shaped by the desired pole positions will be implemented here. This model-based multivariable controller has the ability to dynamically decouple the control channels and reduce or completely remove the interactions in the control model. This is an advantage over the multiple-loop SISO controllers [1][12][13]. The control law is then modified by introducing an additional control gain matrix M_0into the nominal PIP gain matrix K and expressed as;

$$u(k) = -Kx(k) - M_0 u(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 28$$

substituting the gain matrix K and the state vector x(k), the modified control law becomes

$$u(k) = -\left[[L(z^{-1})]y(k) - M_0 u(k) - M_0(z^{-1})u(k) + K_1 z(k)\right] \quad - \quad - \quad - \quad - \quad 29$$

$$u(k) = -[L(z^{-1})y(k) - M^*(z^{-1})u(k) + K_1 z(k)] \quad - \quad - \quad - \quad - \quad - \quad 30$$

where $M^*(z^{-1})$ is given as

$$M^*(z^{-1}) = M_0 + M_1 z^{-1} + M_2 z^{-2} + \cdots + M_{m-1} z^{-m+1} \quad - \quad - \quad - \quad - \quad - \quad 31$$

The control gain matrix K solved from the control law with a modified control gain matrix

$$K = [Lz^{-1} + M^* z^{-1} - K_1] \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 32$$

The closed loop TF Matrix can be determined from the relationship between y(k) and yd(k) as;

$$\bar{A}(z^{-1})y(k) = \bar{B}(z^{-1})y_d(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 33$$

Where

$$\bar{A}(z^{-1}) = (1 - z^{-1})[A(z^{-1}) + B(z^{-1})\{I + M*(z^{-1})\}^{-1} L(z^{-1})] + B(z^{-1})\{I + M*(z^{-1})\}^{-1} K_1 \quad - \quad 34$$

$\bar{B}(z^{-1}) = \{I + M * (z^{-1})\}^{-1}K_1$ - - - - - - - - 35

the combined decoupling and pole assignment algorithm can be obtained if matrices $L(z^{-1})$, $M * (z^{-1})$ and K_1 are chosen such that;

$\bar{B}(z^{-1})$ is diagonal and non-singular
$\bar{A}(z^{-1})$ is diagonal and its zero is placed at its desired location in the complex z-plane.

SISO Control System Design for QTP

Here, we focus on the design of a multivariable system where an independent SISO controller is implemented as shown in fig.2. Decentralised control remains popular in the industry, despite the increasing developments of advanced controller used in multivariable processes. In designing a decentralized control system, the following steps were carried out;
(i) Selection of the best inputs-outputs pairing based on RGA Approach [7][8][9].
(ii) Design of a decentralized SISO controller for each control loop. The controller ensures that the set-point objectives were met by generating appropriate control action u, where V_i^0 is the control input to the nonlinear Quadruple Tank Process. For control purposes, the controllers are designed based on the linearised model in [2] using an operating point of V_i^0 and h_i^0, while controlling the nonlinear system. To control the water level in the lower two tanks (1 and 2) in fig.1 despite the presence of coupling (load disturbance).

The decentralized control has been researched in different literatures. Decentralized control is concerned mainly with stability and uses of the time domain. Some practical approaches to the design of decentralized controllers have evolved in the independent design procedure as shown in [6]. The issue of interaction between the loops is considered first using the RGA, and the SISO controllers are then designed independently.

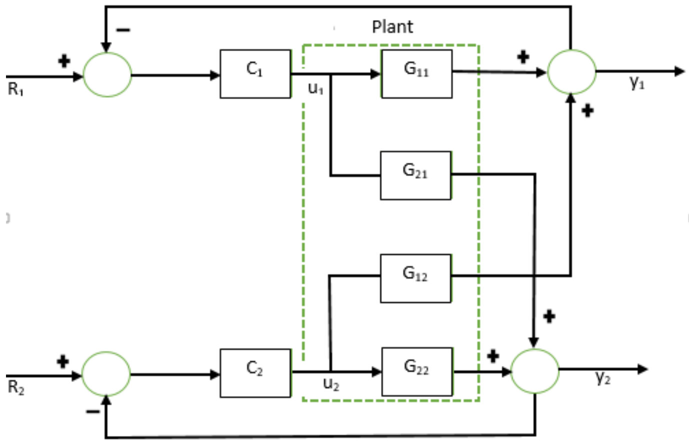

Fig.2. Decentralized SISO control structure with two controllers

Input-Output Paring based on RGA Approach

For the selection of best input-output paring in the design of SISO decentralized controllers, the RGA tool was used such that interaction is minimum. As presented in theory the element of each row and column of the RGA matrix sum up to one [10][11][16].

Decentralized PI Controllers

In [2], the design analysis was applied to the Quadruple Tank Process. The controller parameters that define the PI controller was obtained by closed loop model and tuned by pole placement assignment. The controller structure and parameters adapted optimally to the nonlinear Quadruple Tank Process model. The parameters of the PI controller are designed on the basis of the derived PI control law.

From RGA analysis, the multivariable discrete-time SISO system transfer function is

$y(k) = \frac{B(z^{-1})}{A(z^{-1})}u(k)$ - - - - - - - - - 36

The controller polynomials are derived from the control law. The CLTF obtained by block diagram reduction method is given as

$$G_{PI} = \frac{QB}{PA+QB} \qquad 37$$

The aim is to designate closed loop poles to an explicit position by equating the characteristics equation to a user specified design polynomial as presented in [12]. The dynamics of the closed loop system are governed by their denominators. From our paper in [2], if the characteristic equation equals the desired closed loop characteristic polynomial D(z), then;

$$D(z) = 1 + d_1(z^{-1}) + d_2(z^{-2}) \qquad 38$$

where d_1 and d_2 are the coefficients of the desired closed loop characteristic equation obtained by deriving a quadratic expression based on the desired poles selected. The different closed-loop poles were varied and [0.9 0.9] selected as the solutions to the quadratic equation to determine the coefficients of the closed-loop characteristic equation. Therefore,

$$(z^{-1} - 0.9)(z^{-1} - 0.9) = 1 + d_1(z^{-1}) + d_2(z^{-2}) \qquad 39$$

$$z^{-2} - 1.8z^{-1} + 0.81 = 1 + d_1(z^{-1}) + d_2(z^{-2}) \qquad 40$$

The controller polynomials $Q(z^{-1})$ and $P(z^{-1})$are related to the model polynomial $A(z^{-1})$ and $B(z^{-1})$via the diophantine equation.

Decentralized LQ-PIP Controllers

PIP design has numerous advantages. In particular, its structure exploits the power of SVF methods, where the vagaries of manual tuning are replaced by either pole assignment of LQ design. The optimal LQ-PIP control is implemented here. For the SISO control system a FB gain k is designed to minimise the LQ cost function. Based on the defined TF

$$G(z^{-1}) = \begin{bmatrix} \frac{0.0416z^{-1}}{1-0.984z^{-1}} & 0 \\ 0 & \frac{0.03094z^{-1}}{1-0.984z^{-1}} \end{bmatrix} \qquad 41$$

Considering the TF G_{11} for controller C_1 in figure 1,

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})}u(k) \qquad 42$$

Polynomials $A(z^{-1})$ and $B(z^{-1})$ are expressed as,

$$B(z^{-1}) = 0.0416z^{-1} \qquad 43$$

$$A(z^{-1}) = 1 - 0.984z^{-1} \qquad 44$$

Substituting these polynomials in equation 42 gives,

$$y(k) = \frac{0.0416z^{-1}}{1-0.984z^{-1}}u(k) \qquad 45$$

Considering the TF G_{22} for controller C_2 in figure 1, and $y(k)$ in equation - - - 42

Polynomials $A(z^{-1})$ and $B(z^{-1})$ are expressed as,

$$B(z^{-1}) = 0.03094z^{-1} \qquad 46$$

$$A(z^{-1}) = 1 - 0.989z^{-1} \qquad 47$$

Substituting these polynomials in equation 42 gives,

$$y(k) = \frac{0.03094}{1-0.989z^{-1}}u(k) \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 48$$

The LQ-PIP controller implemented in an incremental form is used to control the nonlinear QTP. This is due to the fact that Integral of error state is unknown apriori and incremental control law algorithm however has no knowledge of Integral of error state. Thus, problems associating with potential anti-wind up phenomenon are avoided

III. RESULTS AND DISCUSSION

A controller is a dynamical system. A change in the parameters of a dynamical system will naturally result in changes of its output. Although results of the optimal LQ-PIP designs demonstrate that good control performance with minimal cross-coupling terms ca be obtained, full dynamic decoupling can yet be obtained by SVF decoupling design. Table 1 shows the simulation results Decentralised SISO LQ-PIP Controller. On the average, smaller set-point to output error (IAE) is obtained from the decoupled control, however at the expense of a little more controller effort (IAC) as shown in fig. 3,4 and 5.

Table 1: Performance Evaluation of a Decentralised SISO LQ-PIP Controller

γ_1	γ_2	IAE_1	IAE_2	IAC_1	IAC_2
0.7	0.6	0.0207	0.0192	1.4223	2.4664
0.9	0.6	0.0233	0.0191	0.8630	3.029
0.9	0.9	0.0188	0.0186	1.9856	1.8942

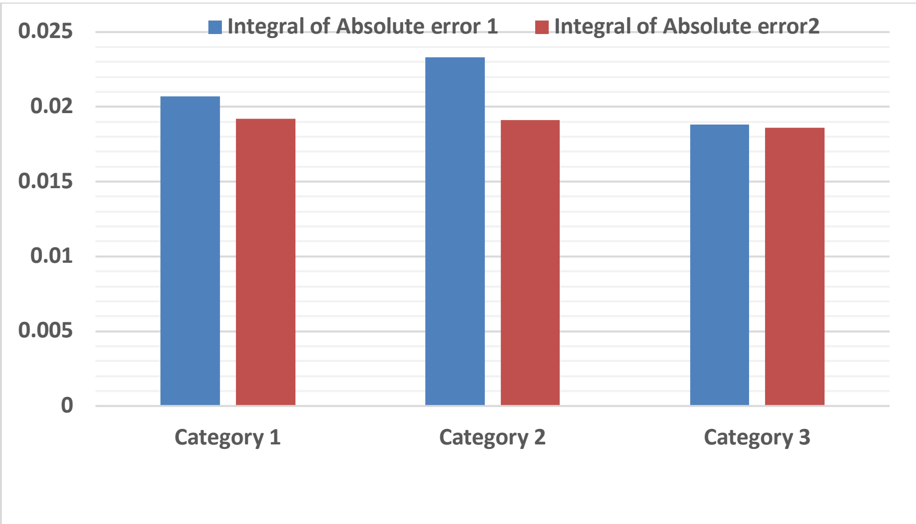

Fig. 3. Representation of IAE for decentralised LQ-PIPSISO controller

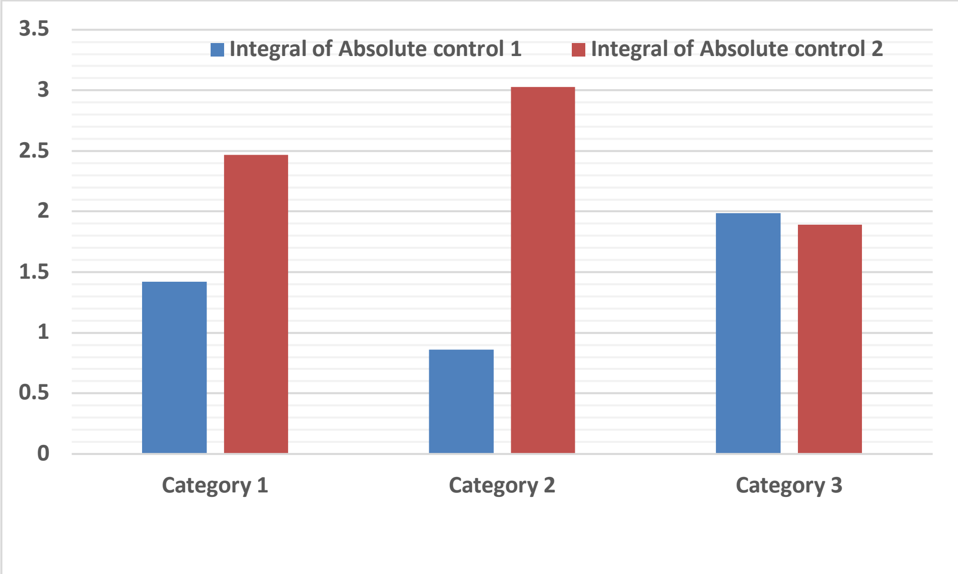

Fig.4. Representation of IAC for decentralised LQ-PIPSISO controller

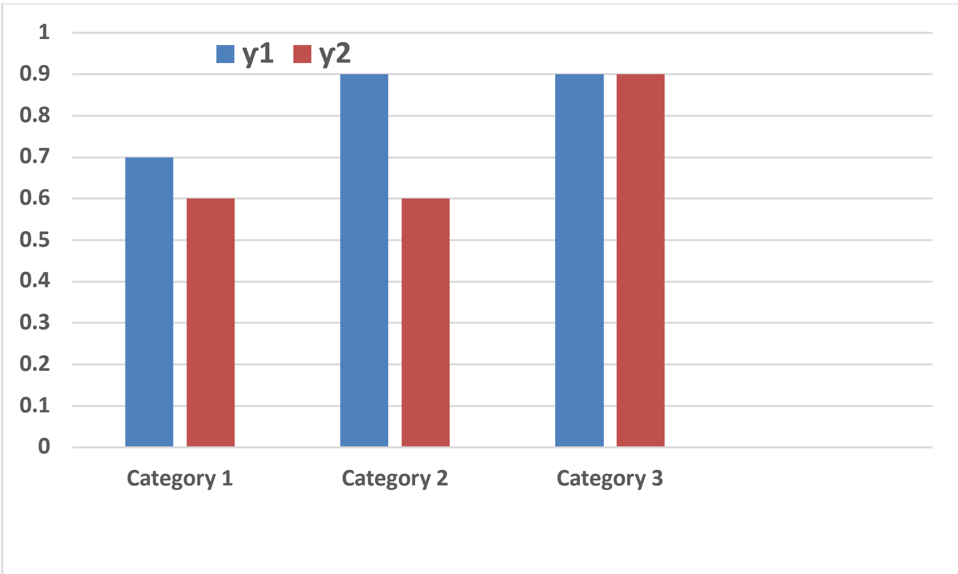

Fig. 5: Representation of yˊfor decentralised LQ-PIP SISO controller

IV. CONCLUSION

This paper gives an intensive summary of the obtained results using the non-linear model with minimal phase characteristics. The motivation of this paper was to illustrate the various advance control techniques in a multivariable process with application to a QTP. For excellent knowledge of the implementation preformed, the mathematical analysis for individual controllers and their respective control laws, were theoretically derived for clarity and completeness. The controllers were simulated, and results obtained.

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