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Penalty Method for a Non Linear Coupled System

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ABSTRACT
In this paper we prove the existence of the solution of a nonlinear coupled system in Sobolev space under certain conditions. we are concerned with the method penalized for that system. The study for a coupled system using the method penalized was handled by Lions, J.L. [5]. *Further results can be found in Christian Clason, Karl Kunisch, Armin Rund*[4]
KEYWORDS: *Penalty method, coupled system.*

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I. INTRODUCTION

The purpose of this paper is to give a relatively simple proof to the existence of a nonlinear coupled system in Sobolev space using a penalty method. It is worthwile to notice that the coupled systems have received a great deal of interest from the mathematicians in the past years or so, due in particular to their applications in optimal control problems which involve the minimization of an objective function subject to constraints on the state variables and control inputs. In general, this will lead to a non-linear constrained optimization problem.

II. MATERIALS AND METHODS

We introduce the Sobolev space and stablish some properties which are essential tools used in subsequent sections.
Notation and preliminaries

$H^1(\Omega)$ is a Hilbert space with the inner product

$((u,v)) = (u,v) + \sum_{i=1}^{3}\left(\frac{\partial u}{\partial x_i},\frac{\partial v}{\partial x_i}\right)$, where $(,)$ denote the inner product in

$L^2(\Omega)$ which is as well a Hilbert space of measurable functions $u:\Omega \to \mathbb{R}$ such that

$$\int_\Omega |u(x)|^2\,dx < \infty$$

$L^2(\Omega)$ is equipped with the norm

We will consider the non linear coupled system in Sobolev space.

$$Ay - y^3 = u \quad , \ \text{in}\Omega$$
$$A^*y - 3y^2p = (y - y_d)^5 \quad , \ y_d \in L^6(\Omega) \quad (1.1)$$
$$y = p = 0 \qquad \text{on} \quad \partial\Omega$$

In addition

$$(p + Nu, v - u)_{L^2(\Omega)} \geq 0 \;, v \in U \subset L^2(\Omega)\;, \quad N \in \mathcal{L}\big(L^2(\Omega); L^2(\Omega)\big) \quad (1.2)$$

Where N is Hermitian and positive definite and

$$Ay = -\sum_{i,j}^{3} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial y}{\partial x_j}\right) + a_0(x)y$$

is a continuous linear mapping of $H_0^1(\Omega)$ to $H^{-1}(\Omega)$.We assume that U is a closed convex set of $L^2(\Omega)$ such that for every $v \in U$ we have $Z(v) \neq \emptyset$
Here

$$Z(v) = \{y \in L^6(\Omega);\ Ay - y^3 = v \;,\ \text{in}\Omega\,, y = 0 \text{ on } \partial\Omega\}$$

Furthermore $a_0(x) \geq 0$ for all x in $\bar{\Omega}$ and $a_{ij}(x)$are functions in $L^\infty(\Omega)$ satisfying the following condition
$\sum_{i,j}^3 a_{ij}(x)\xi_i\,\xi_j \geq \alpha\|\xi\|^2$, for all x lie in $\bar{\Omega}$ and $\xi \in \mathbb{R}^3$
Let A , N and U be as above , suppose that one of the conditions holds
i) $0 \in U$ and $\|z_d\|_{L^6(\Omega)} \leq C$for some constant $C > 0$
ii) There exists a closed convex set $K \subset L^2(\Omega)$ so as well a non-empty open set $V \subset \Omega$ such that$U = K + L^2(\mathrm{V})$
Then, there exists $(u, y, p) \in U \times L^6(\Omega) \times L^2(\Omega)$ satisfying the following conditions

$$Ay - y^3 = u \;,\ \text{in}\Omega \quad (1.22)$$

$$A^* y - 3y^2 p = (y - y_d)^5 \quad (1.23)$$

$$y = p = 0 \quad \text{on} \quad \partial\Omega \quad (1.24)$$

$$(p + Nu, v - u)_{L^2(\Omega)} \geq 0 \;, \text{for every } v \in U \quad (1.25)$$

And (u, y) is a solution of the problem (1.3

III. CONCLUSIONS

Throughout this work, it has been used Lax Milgram's Theorem for showing the existence of solution of the nonlinear elliptic equation. So that, we have used as well the unique prolongation for elliptic equation (see Scott, A.- Silvestre, L.[8]). Whereas for the non linear coupled system in Sobolev space, it wassuggested adding the penalty term to the functional $J(v, z)$ in such a way that one obtained certain convergences through the minimizing sequence. On the other hand we used the estimate technics and method of the functional analysis so as some embedding theorems in Sobolev space.

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