

## Comparative study of Ship Response in Long and Short crested seas in Offshore West Africa

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**ABSTRACT:** The responses of a typical marine vessel are critical to marine operation and especially its crew. This response can be analyzed in the seaway medium, which could be in the form of sea energy in long and short crested wave. Hence, the comparative analysis of the vessel response in both sea state is paramount research interest. This research compares the response of the vessel in heave mode with regard to long crested and short crested waves. Strip theory and Lewis conformal mapping were used to compute the two-dimensional added mass and damping coefficient. Rhinoceros and AutoCAD software packages were used to model the vessel, which generated 43 strips and the corresponding offset table. Froude Kryloy force and hydrostatic pressure force which form the excitation force were computed simultaneously for the response in long crested and short crested waves with varying directional spread using RungeKutta 4<sup>th</sup> order equation with the aid of Mat lab. The study confirms that the linear strip approach is valid for heave response of vessel in long and short crested waves for length ratio not exceeding six. However, wide discrepancies was observed for ships in short crested wave compared to those in long crested wave where the superimposed waves of greater amplitude whose effect cancel out the one with smaller amplitude.

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### I. INTRODUCTION

The critical impact of sea going marine vessels on national economic development and security cannot be overemphasized. Ships are major critical assets in transportation, defenses, international trade and research among others. Ships are highly cost critical assets that operates in predominantly turbulent environments. The stability of marine vessel during motion is crucial to marine operations, the safety of the vessel and its crew. For these reasons, the continued research into the responses of marine vessel in various sea states continues to remain topical among researchers and engineers. Traditionally in offshore structures design, majority of the current numerical and experimental sea keeping investigation are focused on ship motion and load response in a two directional or long crested incident wave field. Realistic sea states is however, three dimensional or short crested in nature with wave component primarily and secondarily propagated from different directions towards a measurement point. Therefore, accurate prediction of ship motion and load response induced by short crested waves is of great significance for full-scale ship design.

It is important to compare the different response experienced by floating structures in long crested and short crested wave field with the same total energy in order to determine the effect of wave directionality on offshore structures.

A lot of research have been conducted on ship motion on long and short crested sea in North Sea Gulf of Mexico and elsewhere. This include the recent works of chen et al [1],[2] and [3], among others. There is however little or no studies carried out on ship motion in the sea state condition of Gulf of guinea in West Africa.

In this research, the heave motion is considered because it deals with the vertical motion of the vessel. Suppose that a ship is forced down deeper into water from its equilibrium position and suddenly release. Since its buoyant force is greater than its weight, the ship will move vertical upward, when the equilibrium position is reached the ship will continue rising because of its momentum. However, now the ship weight is greater than

the buoyant force, this will tend to show the motion of the vessel. When the velocity is zero the ship reaches its extreme position and since the weight is now greater than the buoyant force the ship will move vertically downward. The downward velocity will increase until the equilibrium position will be reached. As a result of the above phenomenon the see keeping ability of the ship is affected.

**II. MATERIALS AND METHODS**

The reponse computation of the pipe laying vessel considered for this paper would be evaluated for both long and short crested seaway as earlier explained. The mass spring damping response analogue of the mechanical syatem as tradionlly been use in modelling every ship response and this would no difference. The generalized motion response of vessel in all 6 degree can be written as

$$\sum_{j=1}^6 [(M_{ij} + A_{ij})\ddot{\eta}_j + B_{ij}\dot{\eta}_j + C_{ij}\eta_j] = \sum F_i \tag{1}$$

Where;

$M_{ij}$  =the general 6 by 6 mass matrix of the vessel

$A_{ij}$  = the global added mass 6 by 6 matrix and

$B_{ij}$  = the global hydrodynamic or potential damping 6 by 6matrix and

$C_{ij}$  = the global restoring coefficients 6 by 6 matrix.

$F_i$  =the resultant of all other forces in the ith direction 6 by 1 vector.

$\eta_i, \dot{\eta}_i, \ddot{\eta}_i$ = the displacement, velocity and acceleration vector in i nod

For this research the heave uncoupled response equation signify in 1 dof model would be considered for both long and short crested wave , the uncoupled heave equation can be written as

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 = F_3 \tag{2}$$

Where

$A_{33}$  =heave added mass due to heave response

$B_{33}$  =heave damping

$C_{33}$  = heave restoring coefficient due to heave

$F_3$  =heave excitation

$\ddot{\eta}_3, \dot{\eta}_3, \eta_3$ =heave responses (acceleration, velocity and displacement).

The hydrodynamic potentials in the form of the heave added mass and damping for this research is determined using the strip theory. This method considers a vessel to be made up of a number of two-dimensional “strips” which are rigidly connected to each other.

These strips will have a form, which closely resembles the segment of the ship which it represents. Each slice is treated hydro dynamically as if it is a segment of an infinitely long floating cylinder. This can be shown in figure 1 below.

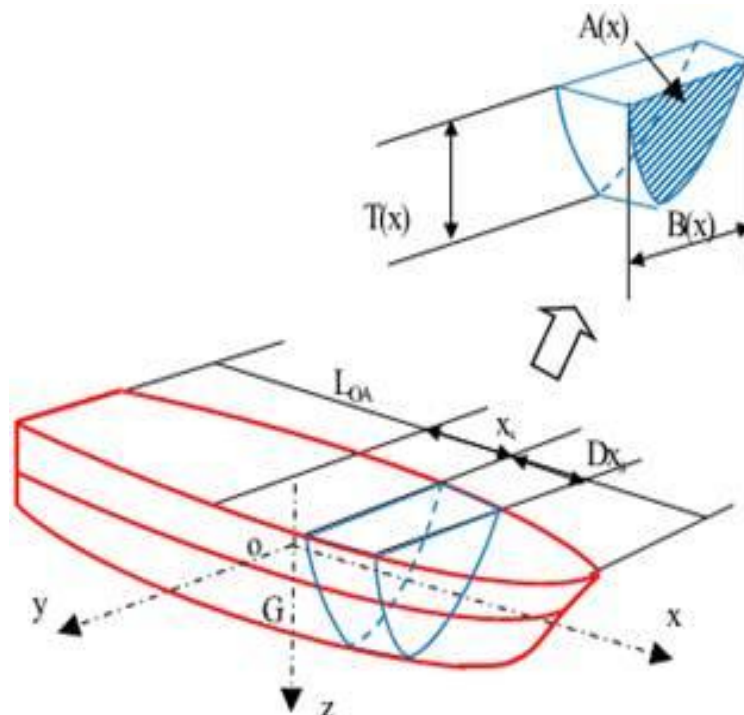


Figure 1: The vessel and a strip [4]

To determine the two dimensional added mass and damping in heave mode of the motion of ship like cross sections, these cross section are conformally mapped to the unit circle.

The general transformation formula is given by

$$Z = M \cdot \left[ \zeta \sum_{n=1}^N a_{2n-1} \zeta^{-(2n-1)} \right] \quad (3)$$

Where

$Z = x + iy$  = complex plane of the ship's cross section

$\zeta = ie^\alpha \cdot e^{-i\theta}$  = complex plane of the unit circle

M = scale factor

N = maximum number of parameters

A simple and realistic transformation of the cross sectional hull form will be obtained with N=2, the Lewis-transformation.

The Lewis-transformation is briefly described below

$$Z = M \cdot \left[ \zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} \right] \quad (4)$$

The coefficients  $a_1$  and  $a_3$  are called the Lewis-coefficients.

with:

$$x = M \cdot [e^\alpha \cdot \sin \theta + a_1 \cdot e^\alpha \cdot \sin \theta - a_3 \cdot e^{-3\alpha} \cdot \sin 3\theta] \quad (5)$$

$$y = M \cdot [e^\alpha \cdot \cos \theta + a_1 \cdot e^\alpha \cdot \cos \theta - a_3 \cdot e^{-3\alpha} \cdot \cos 3\theta] \quad (6)$$

Putting  $\alpha = 0$ , the mapped contour of the Lewis- form is expressed as follows:

$$x_0 = M \cdot [(1 + a_1) \cdot \sin \theta - a_3 \cdot \sin 3\theta] \quad (7)$$

$$y_0 = M \cdot [(1 + a_1) \cdot \cos \theta - a_3 \cdot \cos 3\theta] \quad (8)$$

With the scale factor:

$$M = \frac{B_s}{2 \cdot (1 + a_1 + a_3)} \text{ or } M = \frac{D_s}{1 - a_1 + a_3} \quad (9)$$

The rest of the conformal mapping parameters can be seen in [5],[6] and [7].

## 2.2 Excitation force

The forces and moments that arise as a result of the undisturbed pressure field of the fluid amounts to the Froude-kryov forces. These forces can be computed by integrating the pressure caused by hydrostatic pressure and by the incident wave over the wetted hull surface using the incident wave potential. Mathematically the pressure is given as

$$P^{FK} = -\rho \cdot \frac{\partial \phi(X,Y,Z,t)}{\partial t} \quad (10)$$

Where

$\rho$  = the density of the fluid.

$X, Y$  and  $Z$  are the coordinates of the transformed offsets in the global or inertial frame and  $\phi$  is the velocity potentials which is also given by

$$\phi = \zeta \cdot \frac{g}{\omega} \cdot \frac{\cosh(k(z+d))}{\cosh(kd)} \sin(kx - \omega t) \quad (11)$$

Where  $\zeta$  is the wave amplitude,  $g$  is the acceleration due to gravity,  $k$  is the wave number and  $d$  is the depth of the sea and can be given as

$$k = \frac{2\pi}{\lambda} \quad (12)$$

For a long crested irregular wave, the wave amplitude  $\zeta_{longc}$  can be computed as

$$\zeta_{longc} = \sqrt{2S(\omega)\delta\omega} \quad (13)$$

Where

$S(\omega)$  = the sea spectrum under consideration. (JONSWAP)

It is generally given mathematically as

$$S(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right) \quad (14)$$

Where

$\omega$  = wave circular frequency in radian per second. A and B are constants given in terms of the wave parameters of the significant wave heights and average time given as

$$A = 173 \frac{H_1^2}{T_1^2} \text{ and } B = \frac{691}{T_1^4}$$

Where  $T_1$  and  $H_{\frac{1}{3}}$  are the average wave period and significant wave height respectively.

For a short crested wave, the wave amplitude is given as

$$\zeta_{shortc} = \sqrt{2S(\omega, v)\delta v\delta\omega} \quad (15)$$

Where

$S(\omega, v)$  is the short wave spectrum given as

$$S(\omega, v) = \frac{2}{\pi} \cos^2(v - \mu) S(\omega) \quad (16)$$

Where;

$v$  = secondary wave direction

$\mu$  = primary wave direction

The range of the secondary wave direction can be given as

$$-v_{max} \leq v \leq v_{max} \quad (17)$$

Where  $v = \pi/2$  for practical purpose,  $v$  is considered to be  $90^\circ$  or  $\pi/2$  rad

The restoring force  $F^{restoring}$ , is the hydrostatic pressure force acting also on the wetted surface of the vessel in contact with the fluid for every time step during the motion.

It is given mathematically as

$$F^{restoring} = \rho g Z \dot{n} \quad (18)$$

Where  $\dot{n}$  is the normal unit vector in the vertical direction, it is given as

$$\dot{n} = \hat{n} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (19)$$

And

$$\hat{n} = \frac{z}{\sqrt{z^2 + y^2}} \quad (20)$$

The total heave excitation for every section can be given as

$$F_{sectional}^3 = \sum_0^{sec\_points} F^{restoring} + P^{FK} \quad (21)$$

### Numerical Solution model

The solution to equations (1) can be solved using the Runge-kutta 4<sup>th</sup> order algorithm. It is actually used for solving first order differential equation. Since the equations are second order differential equations, they have to be first reduced to two first ODE using the state space approach and then solved simultaneously.

For equations 3.3 of heave response, it can be written as

$$\eta_3 = \frac{dn}{d\omega} = y \quad (22)$$

Equation 3.13 is the velocity response of the heave motion.

Substituting these into (3.13) and (3.3)

$$(M + A_{33}) \frac{dy}{d\omega} + B_{33}y + C_{33}\eta_3 = F_3 \quad (23)$$

Making  $\frac{dy}{d\omega}$  the subject formula

$$\frac{dy}{d\omega} = \frac{F_3 - C_{33}\eta_3 - B_{33}y}{(M + A_{33})} \quad (24)$$

Now that second order ODE is reduced to first order, the algorithm can be properly applied to determine the response (displacement in this case). For every given frequency the following constants need to be derived.

$$k_1 = \omega \frac{F_3(\omega) - C_{33}\eta_3(\omega) - B_{33}y(\omega)}{M + A_{33}} \quad (25)$$

$$N_1 = \omega(y_\omega) \quad (26)$$

$$k_2 = \omega \frac{F_{3(\omega + \omega/2)} - C_{33}(\eta_{3(\omega + \frac{N_1}{2})}) - B_{33}(y_{(\omega + \frac{N_1}{2})})}{M + A_{33}} \quad (27)$$

$$N_2 = \omega(y_\omega + \frac{k_2}{2}) \quad (28)$$

$$k_3 = \omega \frac{F_{3(\omega + \omega/2)} - C_{33}(\eta_{3(\omega + \frac{N_2}{2})}) - B_{33}(y_{(\omega + \frac{N_2}{2})})}{M + A_{33}} \quad (29)$$

$$N_3 = \omega(y_\omega + \frac{k_3}{2}) \quad (30)$$

$$k_4 = \omega \frac{F_{3(\omega + \omega/2)} - C_{33}(\eta_{3(\omega + N_3)}) - B_{33}(y_{(\omega + N_3)})}{M + A_{33}} \quad (31)$$

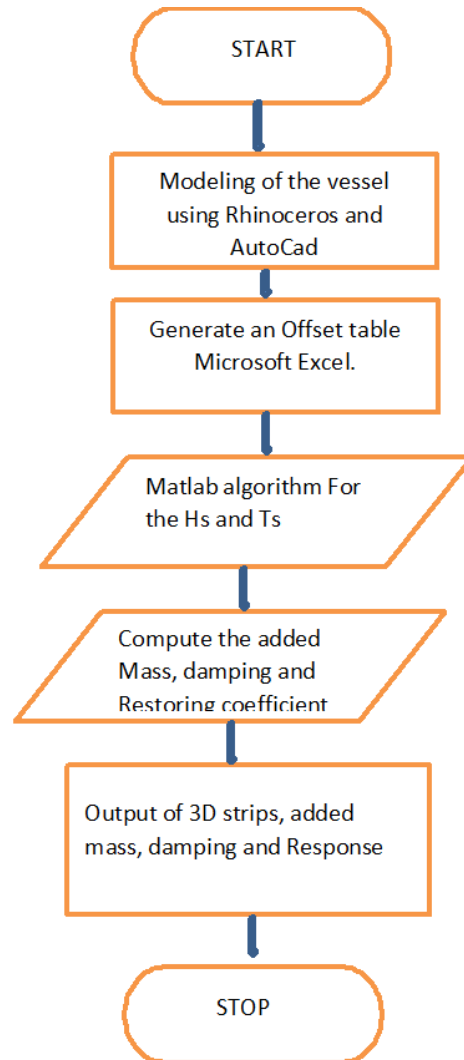
$$N_4 = \omega(y_\omega + k_4) \quad (32)$$

The responses of the heave velocity and displacement can be gotten respectively as

$$y_{\omega+1} = y_{(\omega)} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (33)$$

$$\eta_{3(\omega+1)} = \eta_{3(\omega)} + \frac{1}{6}(N_1 + 2N_2 + 2N_3 + N_4) \quad (34)$$

The heave acceleration can be obtained if necessary using the central difference scheme



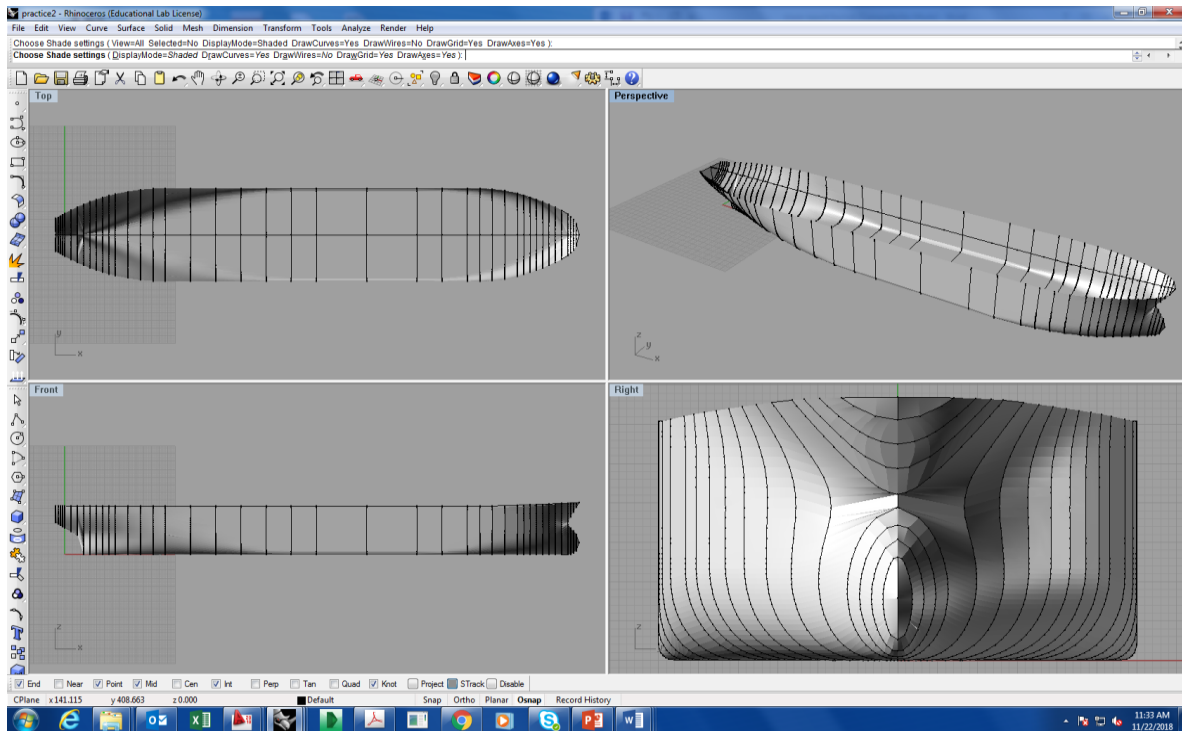
**Figure 2:** A simplified flowchart of the MATLAB source code for the simulation

### III. RESULTS AND DISCUSSIONS

The purpose of this section is to discuss and validate the results of the program developed in MATLAB for predicting the heave response of vessel in long crested and short crested waves of coast of West Africa. This program was tested against JASCON 2 vessel with its offset first imported and each sectional hydrodynamic potentials computed and summed numerically for the entire vessel length. The wave excitation considered is the Froude-Krylov Force which was also computed for each section and numerically summed. Discussions of the results is briefly outlined.

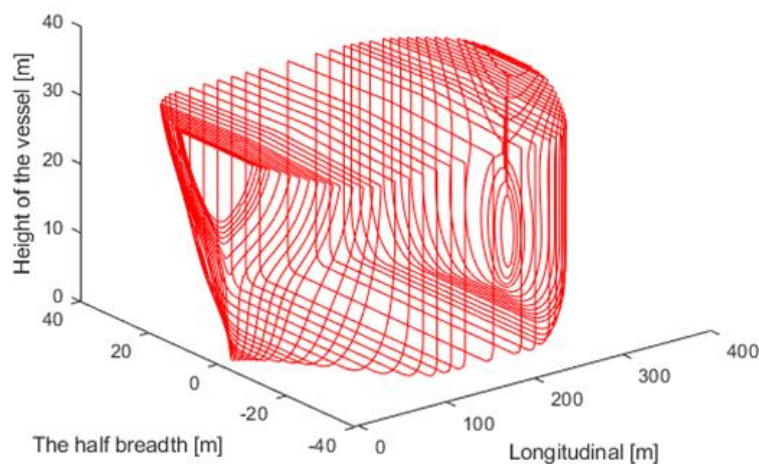
### 3.1 Validation and Numerical Results for the JASCON 17 Vessel

The modelled 3D structure of the JASCON 2 is presented in figure 4.1 below.



**Figure 3.** Modelled JASCON 2 using Rhinoceros

This program was used to validate the heave response of the JASCON 2 vessel. In line with standard practices, the body plan is first described as shown in the figure 4.1 and the vessels particulars inputted into the MATLAB program shown in appendix A. Some of the principal vessel particulars are shown in table 3.1. As earlier explained in chapter three, the strip theory is used which requires that the entire vessel be spliced into various strips. It is divided into 43 strips to improve the precision of the computations. The generated lines drawings using Rhinoceros and AutoCAD are exported to MATLAB and then plotted. The strip shape can be seen in fig 4.2.



**Figure 4:** Strip shape of the JASCON 2

The hydrodynamic potentials of the added mass and damping are computed using the strip theory algorithm of Ursell and Lewis conformal mapping approach. The results for the chosen frequency range are shown below

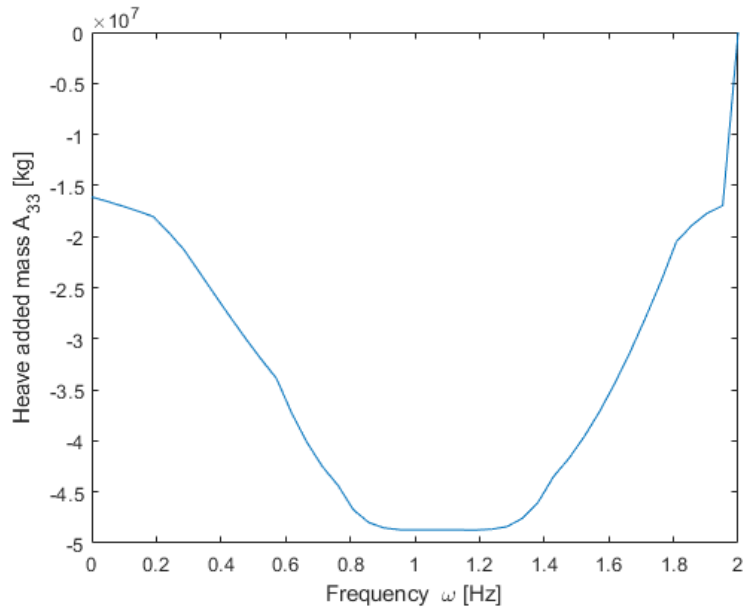


Figure 5: Added mass of the JASCON 2

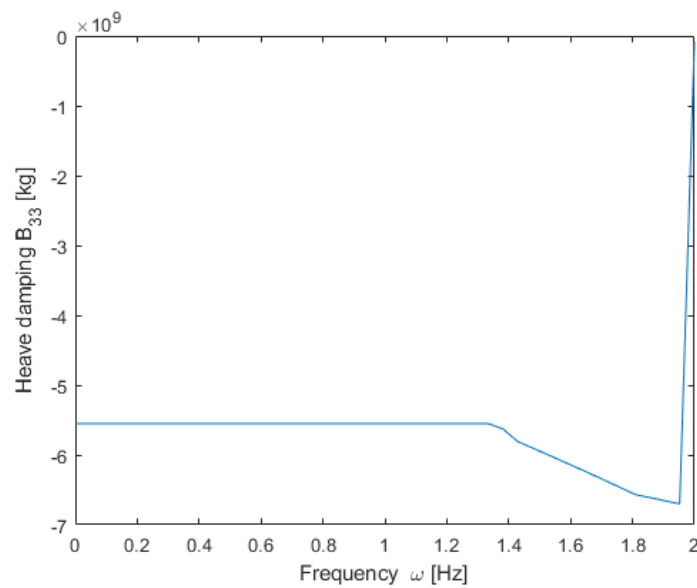


Figure 6: Hydrodynamic damping (heave) of the JASCON 2

The sea state considered for this work is the long crested regular sea with significant up crossing time of 2sec. The frequency range would be from 0 to 2Hz. The amplitude of this spectrum is computed using equation (3.70) this spectrum is given below

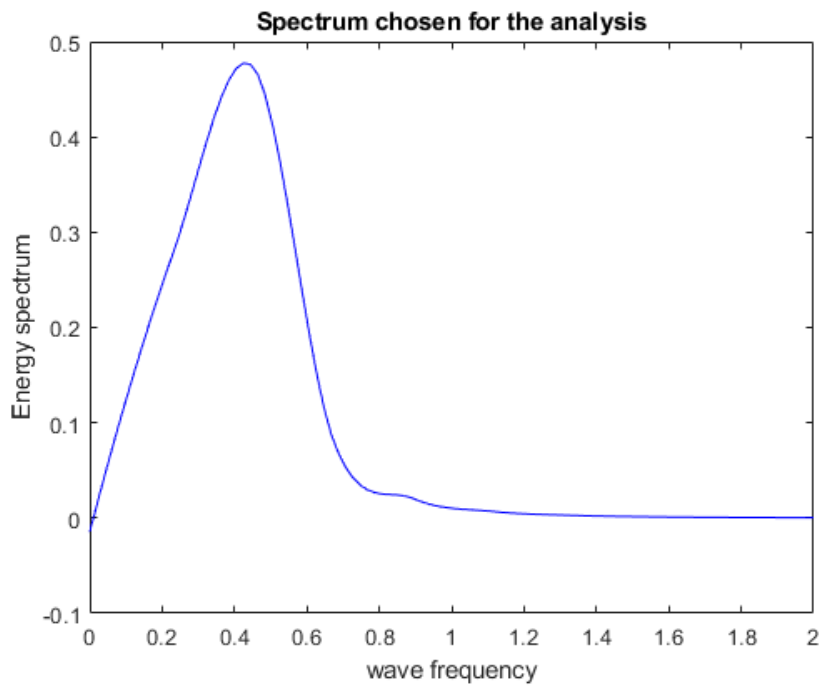


Figure 7: The JONSWAP spectrum energy

The simulation for the heave response of the vessel for long crested wave is computed and the results graphically displayed in frequency domain is shown below in figure 4.6

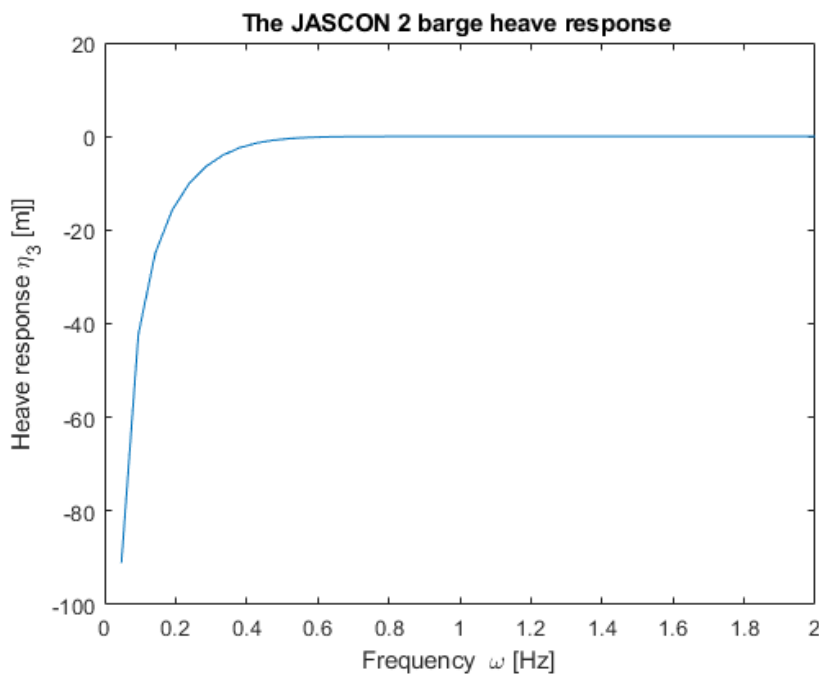
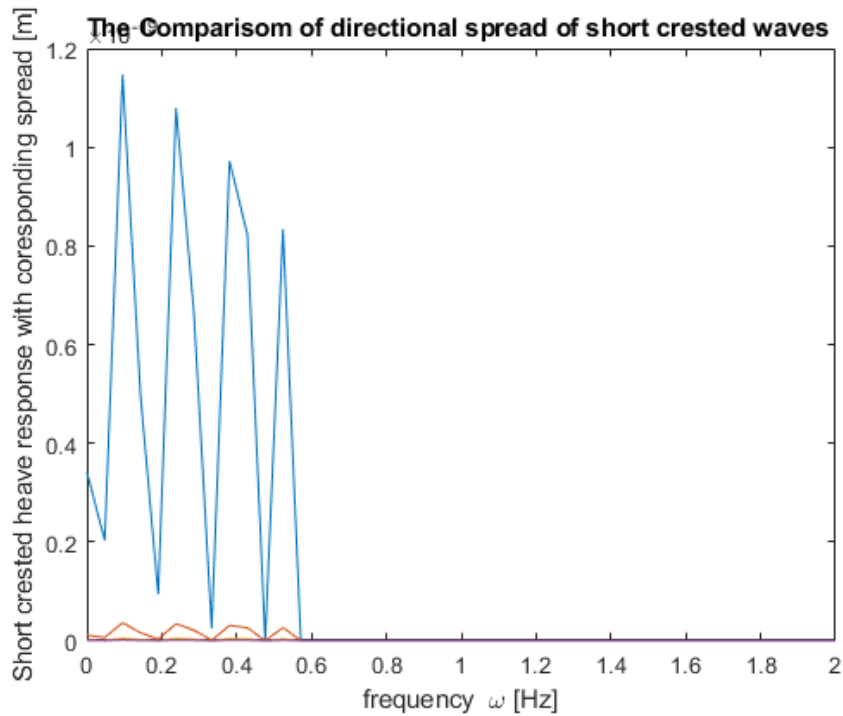


Figure 8: The JASCON 2 Barge heave response

The results indicate that the heave displacement response of the vessel with long crested wave is decaying uniformly with increasing frequency of the wave. This simulation is for quartering wave with a phase angle of  $\frac{\pi}{2}$ .

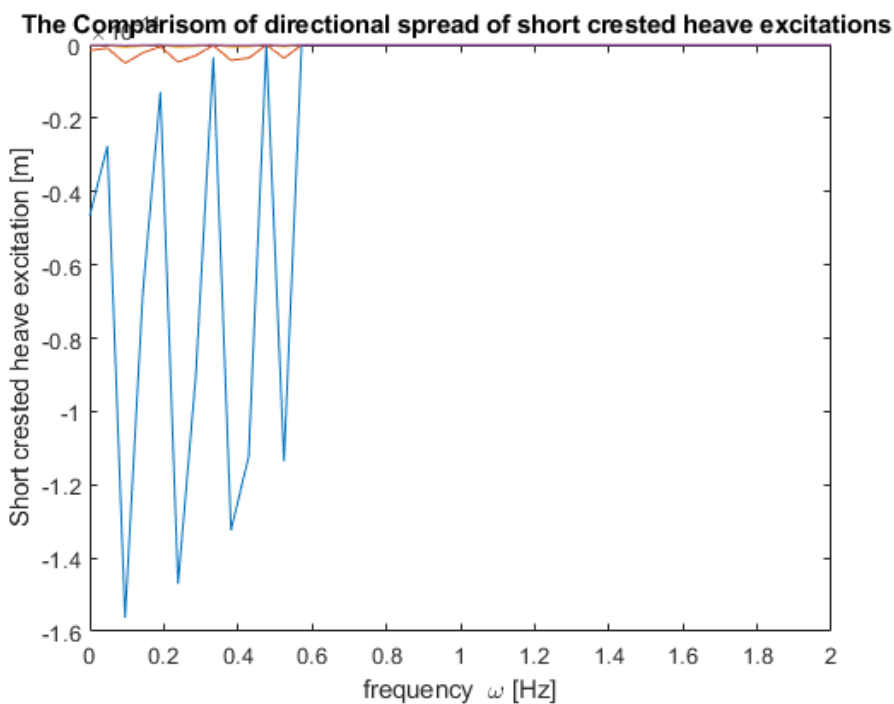
Also simulations and comparison of the heave response with long crested and short crested wave are also carried out with result shown in figure 4.7 below





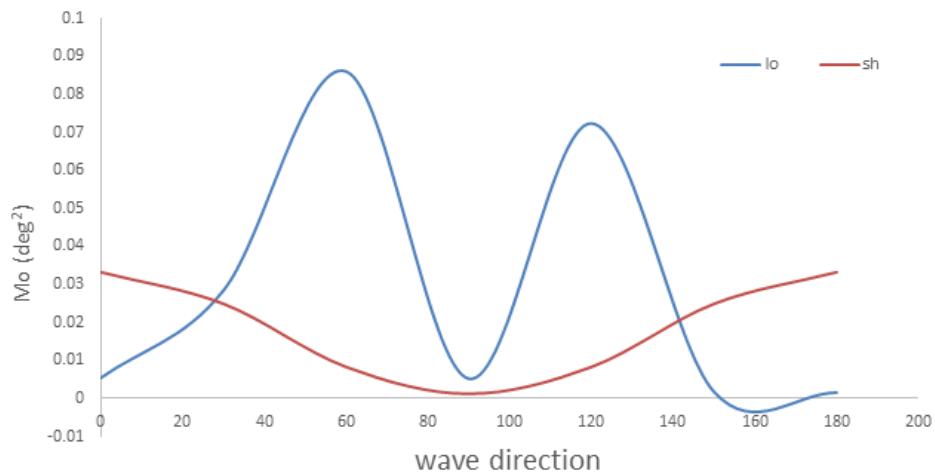
**Figure 9:** Comparison of response of long crested and short crested waves.

Simulation results shows that the short crested waves with varying spreads (0 to  $\pi$ ) had diminishing values of response. This can best be explained as the superposition of the stronger short crested waves whose amplitudes outweighs those with smaller amplitudes. As such the responses becomes infinitely minimal, hence could either excite the vessel with corresponding little or no response. It is also seen when the significant wave height is increase from 4 to 6 in figure 4.8 and 4.9.



**Figure 10** Comparison of response of long crested and heave excitation. ( $H_s$  of 4m)

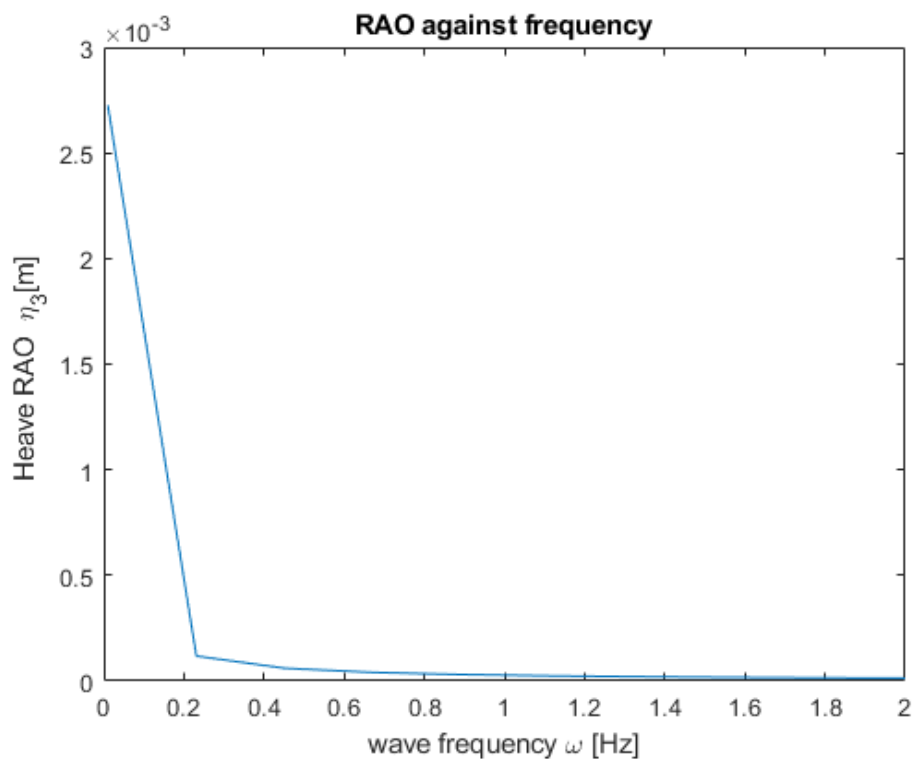
**Fig 11:** Comparison of response of long crested and short crested waves. ( $H_s$  of 6m)



**Fig 12** Variance of heaving response to short crested sea and long crested sea.

Fig 12 illustrate the variance of the heaving response of the ship. The response of the long crested sea is quite different from the short crested sea condition. At angles 45° to 150° the long crested wave sea effect on the heaving motion is higher than the short crested sea.

Thus, the extreme values can be predicted from the variance in short crested sea are smaller than those of the long crested seas. Also the response amplitude operator for the long crested seaway shows reasonable agreement with standard plots as show in figure 13 below



**Figure 13:** RAO of heave response of JASCON against the wave frequency.

#### IV. CONCLUSION

This paper work was aim at studying the response of a typical marine vessel to a long crested and corresponding short crested wave at different directional spreads in the West Africa sea state. It considered two dimensional added mass, spectral density and damping factor for computation of the heave response using the strip theory. These hydrodynamic properties were computed in frequency domain. An attempt was made in computing sectional Froude-krylov forces along and the hydrostatic pressure forces which all form the combined excitation forces.

The wave spectrum chosen for this research was carefully chosen in order to depict the idealized JONSWAP spectrum used for developed sea. The modelled equation was solved simultaneously for the responses in long and short crested waves with varying directional spread by applying the Runge-Kutta 4<sup>th</sup> order numerical method of fourth order. The algorithms were written and the source code developed in MATLAB. Based on the analysis of the result obtained the following conclusion was made.

The paper confirmed that the linear strip approach is valid for heave response of vessel in long and short crested waves for length to breadth ratios not exceeding six. However a wider discrepancy was observed for ship in short crested compare to those in long crested waves.

These discrepancies resulted basically from the short crested wave with different spread were the superimposed waves of greater amplitudes whose effects cancel out the ones with smaller amplitudes. Therefore vessels outside this limits or boundary cannot be used for this studies

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