American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-8, Issue-8, pp-123-131
www.ajer.org
Research Paper Open Access

A Regional-MPC for nonlinear systems with steady state multiplicity with PWA model

Anamaria De Oliveira Cardoso1 And Luís Cláudio Oliveira-Lopes2
*[*1]Science and Technology Institute, Federal University of Jequitinhonha and Mucuri Valleys, Diamantina, Minas Gerais, Brazil*
[2]Federal University of Uberlândia, Uberlândia, Minas Gerais, Brazil
Corresponding Author: Anamaria De Oliveira Cardoso

ABSTRACT : *This paper investigates the formulation of a regional model predictive controller (MPC) in the framework of hybrid systems. This MPC is designed using a collection of regional models defined with PWA (Piecewise Affine) formalism according to the region in which the output variable is in the bifurcation diagram. The results for the regional MPC are compared to those obtained with MPCs based on other model descriptions, a fully nonlinear formulation, and a locally linearized formulation. To illustrate the performance of the controller, the temperature control of a continuous stirred tank reactor is addressed. The results indicate a satisfactory performance and suggest an efficient alternative to the control of nonlinear systems of chemical processes that exhibit multiple steady states or for which several regional models are available.*

KEYWORDS Hybrid Systems, Piecewise affine, MPC, Multiple Steady States, Multimodel Control

---

Date of Submission: 05-08-2019 Date of Acceptance: 20-08-2019

I. INTRODUCTION

The hybrid system framework is used to describe systems whose behavior is described by a continuous dynamics associated with discrete events (logical) usually from the presence of on/off valves, thermostats, switches, or a level of decision involving logical if-then-else. The simulation of a hybrid system is not just combination of continuous and discrete aspects of a system. To enable the appropriate behavioral analysis of a system, it is necessary for the simulation to consider the two predominant behaviors of the system and their dynamic interaction with each other [1].

This interaction requires consistent responses in the simulation of the system behavior and appropriate control strategies for the sophisticated and complex dynamics of a hybrid system. In process control, a hybrid system can also be characterized by a digital control system (controller in discrete time), described by difference equations, which controls the continuous process described by differential equations or continuous systems with embedded digital modules [2]. In most studies, the control is based only on the continuous dynamics, neglecting the presence of these elements and their influence on the system [3].

Model Predictive Control (MPC) has peculiar features that make it as one of the most promising controllers for handling hybrid systems, such as a satisfactory adaptation to multi-variable systems, working with coupling or significant interactions between the variables, and e effective control systems for both linear and nonlinear systems that exhibit complex traits like instability. The intuitive approach to the problem of control by the MPC controller using a model to estimate the future behavior of the system is an e effective alternative for the control of hybrid systems [4].

Several studies have applied hybrid formalisms to represent the dynamic behavior of nonlinear systems. Rivotti and Pistilopoulos [5]presents applications of hybrid systems theory and MPC. The proposal shows the results of this method applied in a numerical example to the optimal control of a piecewise affine system with a linear cost function and achieves satisfactory results. Frick, Domahidi and Morari [6]use a standard branch-andbound approach with a fast interior point solver for multistage problems to solve an embedded optimization problem for mixed logical dynamical systems with a significant number of logical variables.

In this paper, PWA (Piecewise Affine) formulation is determined as the type of hybrid formalism to be used in developing the model for the system to be simulated and controlled due to emphasis given to the idea of introducing qualitative knowledge in process using logical prepositions. PWA formalism was based on the bifurcation observed in the behavior of controlled variable.

1. LOCAL LINEAR MODEL TREE AND MULTIMODEL CONTROL

Modeling is a useful tool for analysis and control of systems. Since the quality of the model typically determines an upper bounded on quality of the final problem solution, it is necessary for the modeling is developed with appropriate techniques in order to adequately describe system characteristics, such as typical behavior from nonlinearity.

Nonlinear models are the foundation for simulation, prediction and model based control in general cases. Developing such models, in most cases, is ex-pensive, time-consuming and involves many unknown parameters and heuristics [7]. Usually, chemical processes are characterized by complex nonlinear dynamics, mainly due to the chemical kinetics and reaction diffusion, biological and chemical waves [8].

Changes in some control parameters might lead to instability and might make the control task even more di cult. At first, bifurcation analysis and control theory were developed independently, although if the choice of the models chosen carefully and there is a good understanding of the dynamics of the chemical process, there can be guaranteed a more effective control system [9, 10].

Several authors have addressed the relation between controllability and stabilizability of nonlinear control systems. Abed and Fu [11] studied the local bifurcation control problem and its stabilization, defining a sufficient condition to guarantee that a nonlinear critical system can be stabilized. Intense exploration of the theory of bifurcation of nonlinear systems in engineering and science allows not only suppressing chaos but also exploiting its potential applications in control design.

Seeking a simpler approach to describe the complexity of the behavior of a nonlinear system to be used by MPC, bifurcations behavior and Local Linear Model Tree (LOLIMOT) theory and can be allies in the development of control systems for nonlinear systems for chemical process. LOLIMOT is an approximation for nonlinear systems with piecewise linear models. A tree-construction algorithm that partitions the input space by axis-orthogonal splits with an upper level loop that determines the structure with the linear local models are valid and a lower level loop for estimate parameters[12].

This approach is often used for fuzzy systems ascan be seen in the research developed for Mola et al.[13]. An identification of dynamical neurofuzzy system is proposed benefits from both LOLIMOT and the subspace identification method of N4SID to optimize the state space parameters tested on a flexible robot arm.

In Sharbafi et al. [14], LOLIMOT algorithm is used to identify the omnidirectional robotSs dynamic model. That system is subjected to a intelligent controller based on brain emotional learning algorithm. Other applications are presented in Rezaie et al. [15] and Pedram et al.[16].

The complexity of the processes in various fields results in the necessity of practical approaches to control tasks [17]. A powerful tool for that goal is Multimodel controller based in a model approach for nonlinear plant is described by combination of local linear models, each of which is valid in a particular operating region [18].

Those models could be identified around given operating points using classical method. A local controllers are tuned to each region and next the control actions of these controllers are combined formed a global controller to be implemented on the nonlinear plant [19].

Some studies have explored Multimodel controller in systems as generic air traffic control tracking [20], hydraulic turbine generating systems [19] and sensorless photovoltaic system [21]. The results show that type of control provides a good performance.

2. PIECEWISE AFFINE (PWA) HYBRID FORMALISM

The term hybrid system was primarily used by Witsenhausen [22]to describe the combination of both dynamics suggesting a linear model with states and inputs assuming discrete and continuous values. Very many formalisms have been developed to describe this type of system

A simplified definition of this type of system is used to characterize systems that have various modes of operation. Each mode of operation in the continuous dynamic process consists of diifferential equations or difference equations, while each mode switching operation occurs due to the occurrence of particular events characterizing the discrete dynamics of the system [4].

Discrete dynamics generally characterize the phase transitions of the system, in which the description is made more natural by different sets of algebraic and differential equations and the switching conditions which govern the transition from one to the other description. In the simplest cases, it is sufficient to describe the boundaries where the dynamics change, depending on the state variables of the real values of the continuous system [23].

PWA systems are defined by splitting the state and input spaces into polyhedral regions Ω_i[24]. Each region is associated with state update equations, as described in Equations 1 and 2 [25]:

$$\mathbf{x}_c(k+1) = \mathbf{A}_i\mathbf{x}_c(k) + \mathbf{B}_i\mathbf{u}_c(k) + \mathbf{g}_i, \begin{bmatrix}\mathbf{x}_c(k)\\ \mathbf{u}_c(k)\end{bmatrix} \in \Omega_i \quad (1)$$

$$\mathbf{y}_c(k) = \mathbf{C}_i\mathbf{x}_c(k) + \mathbf{D}_i\mathbf{u}_c(k) + \mathbf{h}_i \quad (2)$$

in which $\mathbf{A}_i^{n\times n}$, $\mathbf{B}_i^{n\times m}$, $\mathbf{g}_i^{n\times 1}$, and $\mathbf{h}_i^{n\times 1}$ are constants and have the appropriate dimensions. x, u and y are continuous (c) and binary (l) states, inputs and outputs, respectively, defined by Equations (3) to (5).

$$\mathbf{x}(k) = \begin{bmatrix}\mathbf{x}_c(k)\\ \mathbf{x}_l(k)\end{bmatrix}, \mathbf{x}_c(k) \in \mathbb{R}^{n_c}, \mathbf{x}_l(k) \in \{0,1\}^{n_l}, n \triangleq n_c + n_l \quad (3)$$

$$\mathbf{u}(k) = \begin{bmatrix}\mathbf{u}_c(k)\\ \mathbf{u}_l(k)\end{bmatrix}, \mathbf{u}_c(k) \in \mathbb{R}^{m_c}, \mathbf{u}_l(k) \in \{0,1\}^{m_l}, m \triangleq m_c + m_l \quad (4)$$

$$\mathbf{y}(k) = \begin{bmatrix}\mathbf{y}_c(k)\\ \mathbf{y}_l(k)\end{bmatrix}, \mathbf{y}_c(k) \in \mathbb{R}^{p_c}, \mathbf{y}_l(k) \in \{0,1\}^{p_l}, p \triangleq p_c + p_l \quad (5)$$

Such systems are linear representations used to model nonlinear processes with arbitrary occurrences over time, which makes them display the characteristics of a hybrid system. Although this formalism is a composition of linear dynamic systems invariant in time, its structural properties, such as stability, observability, and controllability, are complex, with the typical behavior of non-linear systems [23, 26].

In the last decade, there have been developed several identification methods for PWA systems from nonlinear systems with large data sets, mainly involving optimization concepts [23]. Given an MLD model, it is always possible to find an equivalent PWA model [3], enabling the transfer of useful properties and tools between classes used in analysis and control design [23]. One of the possibilities is the approximation of nonlinear dynamics with arbitrary accuracy via multiple linearization at different operating points [27].

3. Model Predictive Control (MPC) for Hybrid Systems

An optimal decision based on predictions of the behavior of the system carried out using the model, considering the current state of the process characterize a Model Predictive Control (MPC). The control policy used is presented in Bemporad e Morari [3], considering the equivalence between the MLD and PWA formalisms. The solution of optimization problem, show in Equations (6)-(10), obtain the path to the predicted behavior of the system. An observer should be introduced if the current state is not measurable.

$$\min_{u_k,\dots,u_{k+H_u-1}} J_y(k) + J_u(k) \quad (6)$$

where

$$J_y(k) = \sum_{j=H_w}^{H_p} \|\boldsymbol{y}(k+j|k) - \boldsymbol{y}_r(k+j|k)\|_{\boldsymbol{Q}_k}^{p_1} \quad (7)$$

$$J_u(k) = \sum_{j=0}^{H_u-1} \|\boldsymbol{u}(k+j|k) - \boldsymbol{u}_r(k+j|k)\|_{\boldsymbol{R}_k}^{p_2} + \|\Delta\boldsymbol{u}(k+j|k)\|_{\boldsymbol{S}_k}^{p_3} \quad (8)$$

subject to a PWA systems dynamics (Equations (1) and (2)) and

$$\Delta\boldsymbol{u}_c(k+j) = 0, j = H_u+1, \dots, H_p \quad (9)$$

$$g(\boldsymbol{x}_c(j|k), \boldsymbol{u}_c(j|k), \boldsymbol{y}_c(j|k)) \le 0, j = 1, \dots, H_p \quad (10)$$

The matrices $\boldsymbol{Q}_k$, $\boldsymbol{R}_k$ and $\boldsymbol{S}_k$ represent square weighting matrices chosen according to the control objective. In this paper, take over this is constant over the prediction horizon. H_p and H_u are the prediction and control horizons, respectively, and H_w may be used to shift the prediction. If the case addressed is a problem formed by continuous and binary variables, then the system has a hybrid description. Control problem requires the solution of a mixed integer quadratic problem to select PWA models that represent the dynamic behavior of the system in specific states [3].

4. A CONTINUOUS STIRRED TANK REACTOR

To illustrate the controller introduced in this paper, let us consider a continuous stirred tank reactor (CSTR) fed with a flow made up of reagent A, for which an exothermic irreversible reaction occurs. The states of this system are defined as the concentration of the reactant C_A, and the temperature inside the reactor, T .T_{cf} is the temperature of the coolant. A dimensionless model for this system can be represented by the Equations (11)-(13)[22]:

$$\frac{dx_1}{dt} = -\emptyset x_1 \kappa(x_2) + q(x_{1f} - x_1) \quad (11)$$

$$\frac{dx_2}{dt} = \beta\emptyset x_1 \kappa(x_2) - (q + \delta)x_2 + \delta u + q x_{2f} \quad (12)$$

$$y = x_2 \quad (13)$$

For this model, x_1 is a dimensionless variable corresponding to the concentration of A and x_2 is a dimensionless variable related to the temperature T. Sistu and Bequette defined the other dimensionless variables [28].

This system allows addressing the study and control of the significant aspects of an industrial plant description. Among these are the steady state multiplicity, as indicated in Figure 1 and its characteristic nonlinearity. In addition, one can study the existing stability problem, since the system has structural complexity featuring stable and unstable steady states and multiple output behavior in the operating region of interest.

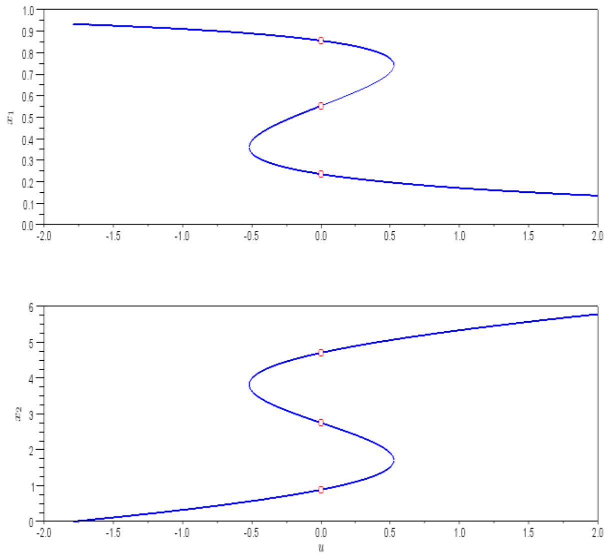

FIGURE 1:STATIC GAIN CURVE FOR THE CSTR

The nominal operating conditions are given in Table 1.

.

TABLE 1: NOMINAL OPERATIONS CONDITIONS FOR THE CSTR MODEL

Nominal conditions	
$\gamma = 20.0$	$\beta = 8.0$
$\emptyset = 0.0072$	$\delta = 0.3$
$x_{1f} = 1.0$	$q = 1.0$
$x_{2f} = 0.0$	

For these conditions, the system can achieve three different values for the steady state. One of them, $P_{ss2}: (x_1, x_2) \approx (0.5528; 2.7517)$ is unstable. In order to study the properties of the system being investigated, the system behavior around the steady state is studied with step disturbances of ± 0.5, ± 0.75, and ± 1. Figure 2-Figure **4** present process response behavior.

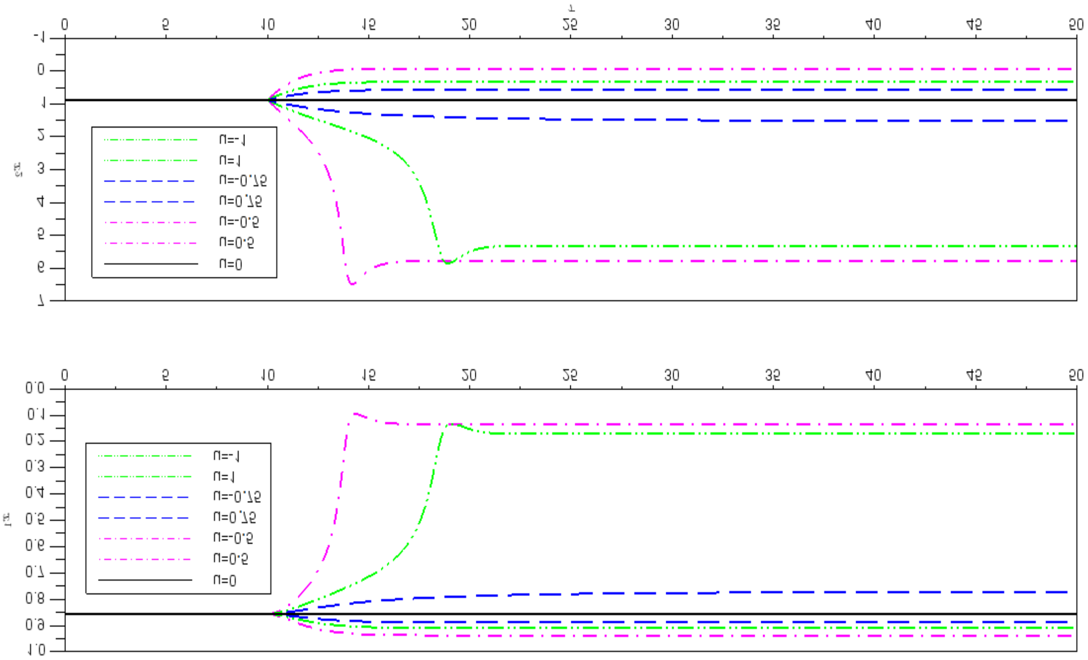

FIGURE 2: RESPONSE PROCESS BEHAVIOR IN THE NEIGHBORHOODS OF P_{ss1} BEFORE THE STEP DISTURBANCE OF $\pm 0.5, \pm 0.75$ AND ± 1 IN $\tau = 5$

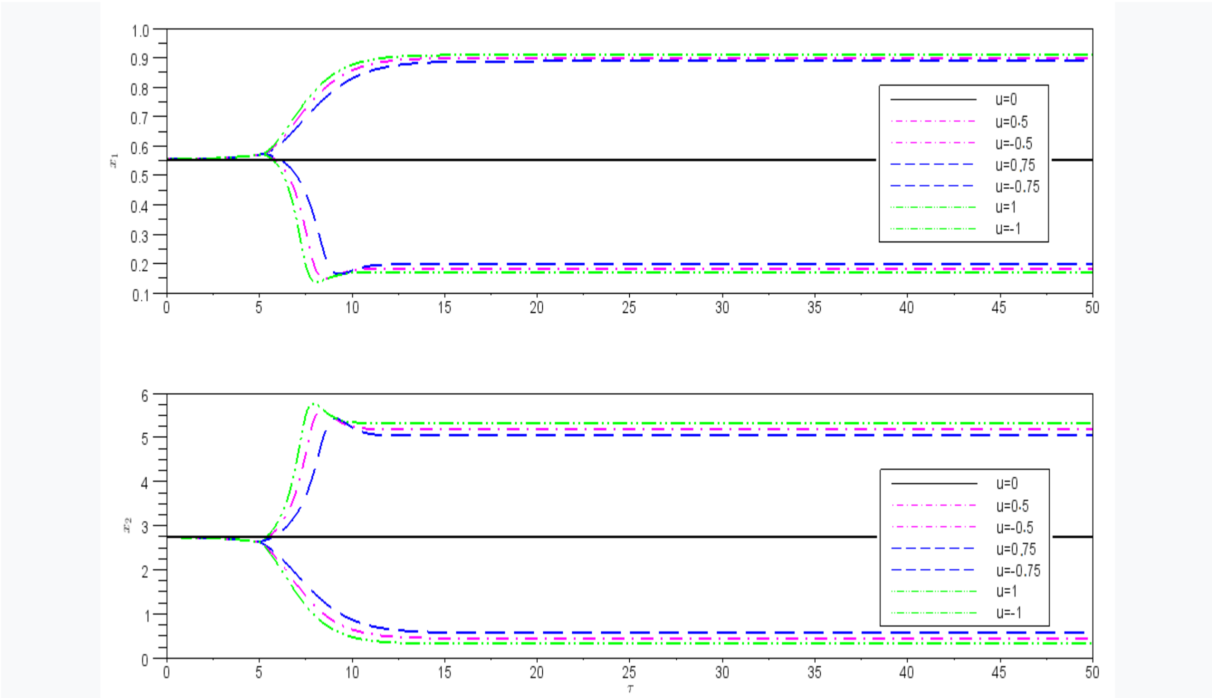

FIGURE 3: RESPONSE PROCESS BEHAVIOR IN THE NEIGHBORHOODS OF P_{ss2} BEFORE THE STEP DISTURBANCE OF $\pm 0.5, \pm 0.75$ AND ± 1 IN $\tau = 5$

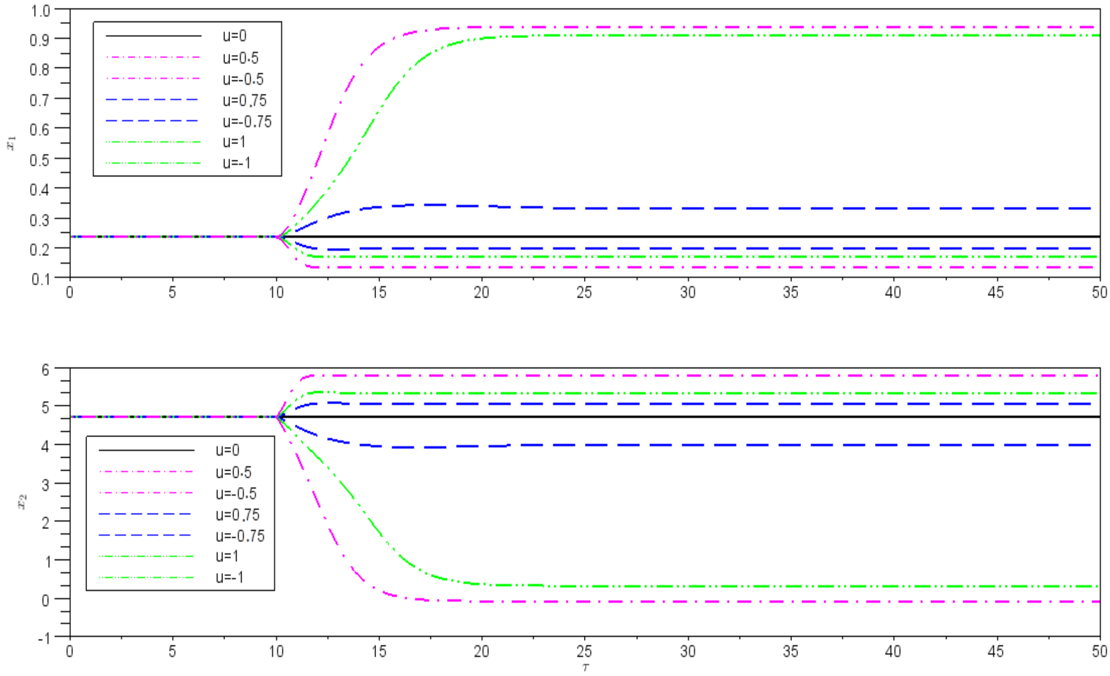

FIGURE 4: RESPONSE PROCESS BEHAVIOR IN THE NEIGHBORHOODS P_{ss3} BEFORE THE STEP DISTURBANCE OF $\pm 0.5, \pm 0.75$ AND ± 1 IN $\tau = 5$

Figures 2 and 4 presents the behavior of steady state points stable since the bounded changes in the input variable u generate bounded changes in output variable. In Figures 3, it is noticed that the system tends to reach another steady state when subjected to a limited disturbance and this new point is a stable steady state.

5. MODEL PREDICTIVE CONTROL FOR CSTR

The control technique for hybrid systems with MPC presented in Section IV is applied to the CSTR system. Three model-based predictive controllers have been investigated and compared. The first controller is based on the original nonlinear model; the hypothesis of no plant/model mismatch was assumed. The selected control policy requires the solution of a nonlinear programming problem with constraints at every sampling time.

The second controller uses a linearized model at each instant k. The linearization and discretization of the nonlinear model is carried out continuously at every sampling instant and the model found is used as a reference to predict the trajectory of the system and de ne the action of this controller. The controller requires the solution a quadratic programming problem at every sampling time.

MPC control used PWA model. The model consisted of a linear description for each region. The definition of the region for each model was a simple selection based on the bifurcation observed in the behavior of x_2. Unlike the locally linearized description, the model used by the controller is upgraded only when the region to which the states and inputs belongs changes. Binary variable in the formulation automatically performed this selection.

The controller requires the solution of a mixed integer quadratic programming problem. In this research, the problem was developed using Scilab. For all controllers, it was assumed that $H_u = H_p$ and $H_w = 0$.

In order to investigate the regional model based predictive controller, the state space of the operational region was sought, and a linearized model at each nominal state was generated such that each selected region includes a single steady state and no empty space, no intersection in the operating space was allowed. Equation (14) gives the set of discretized linear models.

$$\begin{cases} \boldsymbol{x}(k+1) = \begin{cases} \boldsymbol{A}_1\boldsymbol{x}(k) + \boldsymbol{B}_1\boldsymbol{u}(k) + b_1, & \boldsymbol{x}(k) \in \Omega_1 \\ \boldsymbol{A}_2\boldsymbol{x}(k) + \boldsymbol{B}_2\boldsymbol{u}(k) + b_2, & \boldsymbol{x}(k) \in \Omega_2 \\ \boldsymbol{A}_3\boldsymbol{x}(k) + \boldsymbol{B}_3\boldsymbol{u}(k) + b_3, & \boldsymbol{x}(k) \in \Omega_3 \end{cases} \\ \boldsymbol{y}(k) = \boldsymbol{Cx}(k) = [0\ 1]\boldsymbol{x}(k) \end{cases} \quad (14)$$

The respective regions are defined by $\Omega_1 = [0;1] \times [0;1.709[$, $\Omega_2 = [0;1] \times [1.709;3.8172[$ and $\Omega_3 = [0;1] \times [3.8172;6]$. Thus, each has its respective steady states in nominal operating conditions.
By associating a binary variable for each region defined, we have Equations (15) and (16).

$$\begin{cases} \boldsymbol{x}(k+1) = \begin{cases} \boldsymbol{A}_1\boldsymbol{x}(k) + \mathbf{B}_1\mathbf{u}(\mathrm{k}) + \mathrm{b}_1, & \text{if } \delta_1 = 1 \\ \mathbf{A}_2\mathbf{x}(\mathrm{k}) + \mathbf{B}_2\mathbf{u}(\mathrm{k}) + \mathrm{b}_2, & \text{if } \delta_2 = 1 \\ \mathbf{A}_3\mathbf{x}(\mathrm{k}) + \mathbf{B}_3\mathbf{u}(\mathrm{k}) + \mathrm{b}_3, & \text{if } \delta_3 = 1 \end{cases} \\ \mathbf{y}(\mathrm{k}) = \mathbf{C}\mathbf{x}(\mathrm{k}) = [0\ 1]\mathbf{x}(\mathrm{k}) \end{cases} \tag{15}$$

$$\begin{cases} \delta_1 = 1 \leftrightarrow x_2 < 1.709 \\ \delta_2 = 1 \leftrightarrow 1.709 \le x_2 < 3.8172 \\ \delta_3 = 1 \leftrightarrow 3.8172 \le x_2 < 6 \end{cases} \tag{16}$$

Regional-MPC has the advantage that it can be used in cases where a nonlinear model is not available, which would preclude the development of any predictive controller based on such a model as well as any controller based on the locally linearized model. Small perturbations in the plant allow the development of a regional model that can be applied as the basis for the controller, enabling a new approach for controlling a process with these characteristics.
The simulation scenario is defined by the switching in the controller setpoint between the three steady states points for input variable $u = 0$. The transition, for t_f time simulation, control objective chosen is:

- For $t < 0.8t_f$ or $t > 0.2t_f$:$x_{2ss} \approx 0.8860$
- for $0.2t_f < t < 0.4t_f$ and $0.6t_f < t < 0.8t_f x_{2ss} \approx 2.7517$
- for $0.4t_f < t < 0.6t_f$: $x_{2ss} \approx 4.7049$

The system was subjected to the action of the controllers so that the simulation scenario was kept in all three cases. The system was subjected to operating condition constraints for which the upper and lower bounds for u were - 2 and 2, respectively. Adding a control move speed constraint would not change the main aspects of the analysis.
Figure 5 presents the resulting controlled variable and input u that allow achieving the goal in the scenario selected for study.

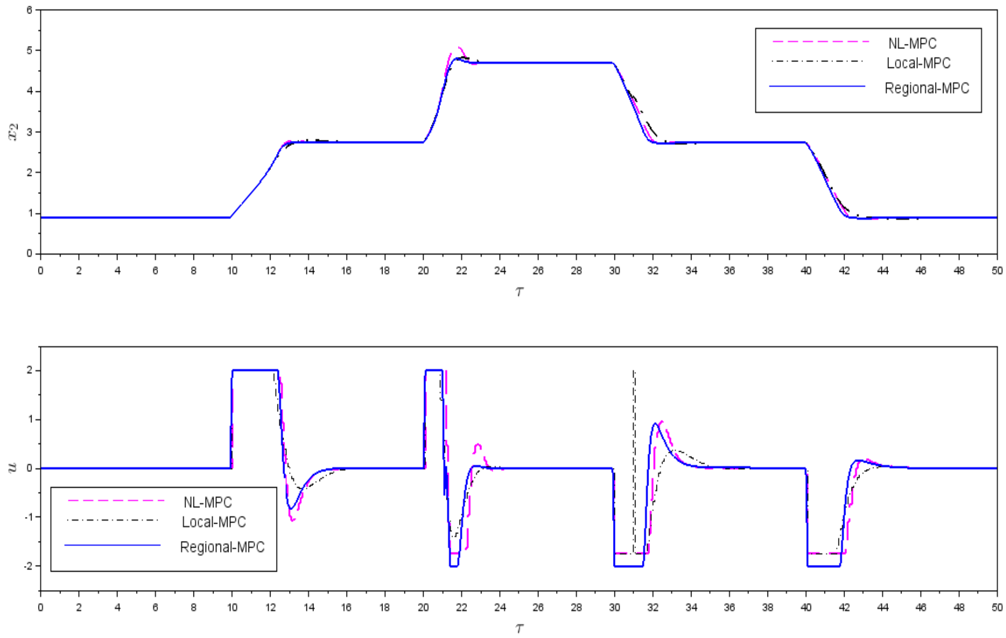

FIGURE 5:CSTR PROCESS RESPONSE UNDER THE ACTION OF THE CONSTRAINED MPC

It is possible to see that all addressed controllers managed to reach the required steady state and keep the system at this point as needed. The Regional-MPC was more sensitive when compared to others in relation to the input variable and its variation between one moment and another. However, it was able to keep the system at a particular point without great difficulty even when this point was an unstable steady state. The line used in each controller was only to indicate if the desired control was reached. Figure 6 presents the behavior of the logical variable representing the region selector of the model used.

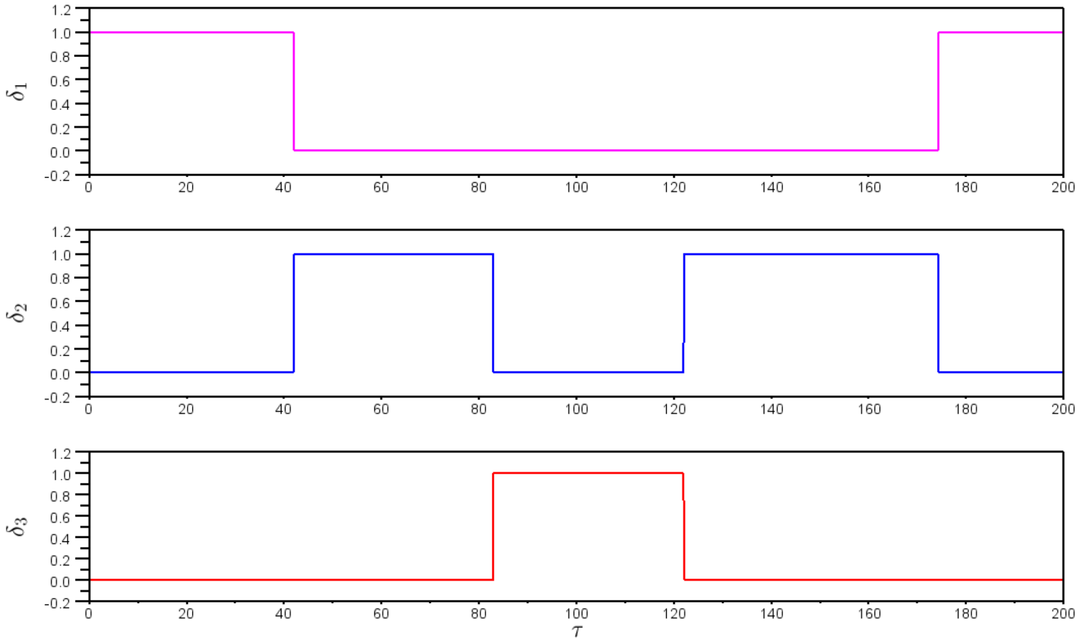

FIGURE 6: REGIONAL MODEL SWITCHES FOR THE CSTR CONTROL BY THE REGIONAL-MPC.

Local-MPC demanded more of the manipulated variable in the transition regions between the setpoints. Regional-MPC proposal presented a behavior for this variable similar to the NL-MPC although with smaller overshoot in the controlled variable. In addition, the computational effort for the resolution of the Regional-MPC is significantly smaller when compared to the controller based on the nonlinear model, reducing the simulation time by about three times when compared to NL- MPC and twice with respect to the simulation time required by Local-MPC.

The system of control can, with a satisfactorily manage regional model, the transitions required in the scenario investigated. It is seen that the activation scenario models meets the requirement of “exclusive or" for the regions of the models. So the considered plant is satisfactorily controlled in the scenario investigated.

II. CONCLUSIONS

In this paper, a model predictive control (Regional-MPC) was studied based on three different grades of models for a system that presents multiple steady states. One of the models used was based on a hybrid formalism presenting continuous and discrete dynamics. The system was satisfactorily controlled by Regional-MPC when compared with a model predictive control based on a nonlinear model.

Although Regional-MPC has been shown to be more sensitive to the tuning, the goal of the control problem was achieved and the advantage of this control is the possibility to deal with linear models in several points of interest with not much computational e ort (since it would have a linear description). However, the identification of the regions to which each continuous model is attributed is not an easy task, especially if both the model and the regions are unknown. The investigation of an appropriate region selection for the Regional-MPC would be necessary to make it a better controller alternative.

ACKNOWLEDGMENTS

This research was partially supported by FAPEMIG, Brazil

REFERENCES

[1]. T. Krilavicius, (2006) “Hybrid Techniques for Hybrid Systems,” Centre for Telematics and Information Technology (CTIT). University of Twente.

[2]. D. Mignone, (2002) "Control and Estimation of Hybrid Systems with Mathematical Optimization," Swiss Federal Institute of Technology(ETH).

[3]. A. Bemporad & M. Morari, (1999) "Control of systems integrating logic, dynamics, and constraints," Automatica, vol. 35, pp. 407–427.

[4]. M. Lazar, (2006) "Model Predictive Control of Hybrid Systems: Stability and Roboustness," Technische Universiteit Eindhoven., The Netherlands.

[5]. P. Rivotti & E. N. Pistikopoulos, (2015) "A dynamic programming based approach for explicit model predictive control of hybrid systems," Comput. Chem. Eng., vol. 72, pp. 126–144.

[6]. D. Frick, A. Domahidi, & M. Morari, (2015) "Embedded optimization for mixed logical dynamical systems," Comput. Chem. Eng., vol. 72, pp. 21–33.

[7]. O. Nelles, (1996) "Local linear model trees for on-line identification of time-variant nonlinear dynamic systems," in Artificial Neural Networks - ICANN 96, pp. 115–120.

[8]. S. H. Strogatz, (2014) Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering. Westview Press.

[9]. M. P. Vega & M. R. C. Fortunato, (2005) "Open/Closed Loop Bifurcation Analysis and Dynamic Simulation for Identification and Model Based Control of Polymerization Reactors," Comput. Aided Chem. Eng., vol. 20, pp. 1477–1482.

[10]. W. Marquardt & M. Mönnigmann, (2005) "Constructive nonlinear dynamics in process systems engineering," Comput. Chem. Eng., vol. 29, pp. 1265–1275,.

[11]. E. H. Abed & J. Fu, (1986) "Local feedback stabilization and bifurcation control, I. Hopf bifurcation," Syst. Control Lett., vol. 7, pp. 11–17.

[12]. O. Nelles, (2013) Nonlinear system identification: from classical approaches to neural networks and fuzzy models. Springer Science & Business Media.

[13]. M. Mola, M. A. Khanesar, & M. Teshnehlab, (2010)"Subspace identification of dynamical neurofuzzy system using LOLIMOT," in IEEE International Conference on Systems Man and Cybernetics (SMC), 2010, pp. 366–372.

[14]. M. Ahmad Sharbafi, C. Lucas, & A. Mohammadinejad, (2008) "Using BELBIC based optimal controller for omni-directional three wheel robots model identified by LOLIMOT," J. Comput. Robot., vol. 1, no. 1, pp. 25–30.

[15]. J. Rezaie, B. Moshiri, A. Rafati, & B. N. Araabi, (2007) "A Modified LOLIMOT Algorithm for Nonlinear Estimation Fusion," in IEEE International Conference onInformation Reuse and Integration, 2007. IRI 2007pp. 520–525.

[16]. A. Pedram, M. R. Jamali, T. Pedram, S. M. Fakhraie & C. Lucas, (2006) "Local linear model tree (LOLIMOT) reconfigurable parallel hardware," in International Symposium on Parallel Computing in Electrical Engineering, no 17, pp. 198–201.

[17]. S. Mezghani, A. Elkamel, & P. Borne, (2000) "Multimodel control of discrete systems with uncertainties," Int. J. Stud. Informatics Control.

[18]. E. Arslan, M. C. Çamurdan, A. Palazoglu, and Y. Arkun, (2004) "Multi-model control of nonlinear systems using closed-loop gap metric," in American Control Conference, 2004. Proceedings of the 2004, vol. 3, pp. 2374–2378.

[19]. I. Salhi, S. Doubabi, N. Essounbouli, & A. Hamzaoui, (2010) "Application of multi-model control with fuzzy switching to a micro hydro-electrical power plant," Renew. Energy, vol. 35, no. 9, pp. 2071–2079.

[20]. X.-R. Li & Y. Bar-Shalom, (1993) "Design of an interacting multiple model algorithm for air traffic control tracking," IEEE Trans. Control Syst. Technol., vol. 1, no. 3, pp. 186–194.

[21]. A. Rhif, Z. Kardous & N. B. Braiek, (2013) "A sliding mode multimodel control for a sensorless photovoltaic system," J. Sci. Ind. Res. (India)., vol. 71, pp. 418–424.

[22]. H. S. Witsenhausen, (1966) "A Class of Hybrid-State Continuous-Time Dynamic Systems," IEEE Trans. Automat. Contr., vol. 11, no. 2, pp. 161–167.

[23]. A. Garulli, S. Paoletti & A. Vicino, (2012) "A survey and piecewise affine system identification," in 16th IFAC symposium on system identification, pp. 344–355.

[24]. M. J. P. Garcia, (2009) "Uma Contribuição ao Controle de Sistemas Comutados," Universidade Federal de Santa Catarina, Brazil.

[25]. W. P. M. H. Heelmes, B. de Schuter & A. Bemporad, (2001)"Equivalence of Hybrid dynamical models," Automatica, vol. 37, pp. 1085–1091.

[26]. F. Borelli, (2003) Constrained Optimal Control of Linear and Hybrid Systems, vol. 290. Berlin: Springer.

[27]. F. D. Torissi & A. Bemporad, (2004) "HYSDEL- A Tool for Generating Computational Hybrid Models for Analysis and Synthesis Problems," IEEE Trans. Control Syst. Technol., vol. 12, no. 2, pp. 235–249.

[28]. P. B. Sistu and B. W. Bequette, (1991)"Nonlinear predictive control of uncertain processes: Application to a CSTR," AIChE J., vol. 37, no. 11, pp. 1711–1723.

Anamaria De Oliveira Cardoso" A Regional-MPC for nonlinear systems with steady state multiplicity with PWA model" American Journal of Engineering Research (AJER), vol. 8, no. 8, 2019, pp. 123-131