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Elastic wave propagation and applications:Cauchy, Kirchoff and Schrodinger equations

Hernán Cortez Gutiérrez,Milton Cortez Gutiérrez,Lenin Cabracancha Montesinos,Floresmilo Flores Ostos, Miguel Villafuerte Flores, Luis Garcia,Alejandro Flores Osorio, Gabriel Manes Cangana, Alex Malpica Manzanilla

Corresponding Author: Hernan Oscar Cortez Gutierrez

ABSTRACT *In this paper, we prove the local existence of elastic wave propagation for the Kirchoff equation in Sobolev spaces. The wave solution is analyzed in spaces of finite dimension. The technique used to prove our results depends on the eigenfunctions of the laplacian operator , Gronwall inequality, Caratheodory theorem and*
Approximate solutions.

KEYWORDS *elastic wave propagation, weak solution, Life sciences, fractional differential equation, center of mass.*

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I. INTRODUCTION

The conceptual terminology weak solutions and generalized solution is known as a really solution in life sciences. Cauchy ,Kirchoff and Schrodinger equations is used for a mathematical and numerical modelling of elastic waves propagations in human body.
The Cauchy problem for elastic wave propagation is the form:

$$\begin{cases} u_{tt}(x,t) - m_0 u_{xx}(x,t) = f(x,t) & (x,t) \in \quad Q =]-\infty;\infty[\times]0;\infty[\\ u(x,0) = u_0 \,, u_t(x,0) = u_1 & x \in \Omega =]-\infty;\infty[\end{cases} \quad (1)$$

In reference [0]DjairoGuedes de Figueiredoobtain a generalized solution in Sobolevspaces . A weak solution or generalized solution to partial differential equation is a function for which the derivative may not all exist but which is nonetheless deemed to satisfy the equation in some precisely defined sense. There are many different definitions of generalized solution and the most important use the notion of distributions.

The Klein-Gordon problem for elastic wave propagation is the form:

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) + m_1 u(x,t) = f(x,t) & (x,t) \in \quad Q =]-L;L[\times]0;T[\\ u(-L,t) = 0, u(L,t) = 0 & \\ u(x,0) = u_0 \,, u_t(x,0) = u_1 & x \in \Omega =]-L;L[\end{cases}$$

The Klein-Gordon problem for elastic wave propagation is the form:

$$\begin{cases} u''(t) - M(|\nabla u(t)|^2)\Delta u(t) + M_1(|u(t)|^2)u(t) = f & (x,t) \in Q \\ u(t) = 0 & \text{on } \Sigma \\ u(0) = u_0 , \quad u'(0) = u_1 & x \in \Omega \end{cases} \quad (P)$$

Thestandard notation $((\,,))$, $\|.\|, (\,,), |.|$ is used in the spaces: $H_0^1(\Omega)$, $L^2(\Omega)$and $H_0^1(\Omega)$ with the norm If $u(t) \in H_0^1(\Omega)$then $\|u(t)\| = |\nabla u(t)|$.Conditions for the functions$M$ y M_1:

I. $M, M_1 \in C^1([0;\infty[, \mathbb{R})$
II. $M(s) \geq m_0 > 0, \forall s \in [0;\infty[$
III. $M_1(s) \geq 0, \forall s \in [0;\infty[$

II. LOCAL EXISTENCE

Theorem 1. M and M_1 funciones with the conditions : I-III, $0 < T_0 < T$,If$u_0 \in H_0^1(\Omega) \cap H^2(\Omega), u_1 H_0^1(\Omega)$ and$f \in L^2\left(0, T; H_0^1(\Omega)\right)$, then $u: [0, T_0] \to L^2(\Omega)$is an elastic wave with

$$u \in L^\infty(0, T_0; H_0^1(\Omega) \cap H^2(\Omega))$$
$$u' \in L^\infty(0, T_0; H_0^1(\Omega))$$
$$u'' \in L^2(0, T_0; L^2(\Omega))$$
$$\frac{d}{dt}(u'(t), v) + M(|\nabla u(t)|^2)(\nabla u(t), \nabla v) + M_1(|u(t)|^2)(u(t), v) = (f(t), v) en D'(0, T_0) \forall v H_0^1(\Omega)$$
$$u(0) = u_0 \, y u'(0) = u_1$$

Eigenfuntions of thelaplacianoperator

$\{w_j\}_{j\in\mathbb{N}}$Basis vectors of $L^2(\Omega)$andthe approximates solutions is given by the expression:

$$u_m(t) = \sum_{j=1}^{me} g_{jm}(t) w_j(x) \in V_m$$

$g_{jm} \in C^\infty$, we obtain solutions of the form

$$(u''_m(t), v) - M(|\nabla u_m(t)|^2)(\Delta u_m(t), v) + M_1(|u_m(t)|^2)(u_m(t), v) = (f, v)$$
$$u_m(0) = u_{0m} \to u_0 in H_0^1(\Omega) \cap H^2(\Omega), \qquad u'_m(0) = u_{1m} \to u_1 in H_0^1(\Omega)$$
$$\forall v \in V_m y j = 1,2, \ldots, m.$$

$$\left(\sum_{j=1}^{m} g''_{jm}(t) w_j, w_i\right) - M(|\nabla u_m(t)|^2)\left(\Delta \sum_{j=1}^{m} g_{jm}(t) w_j, w_i\right) + M_1(|u_m(t)|^2)\left(\sum_{j=1}^{m} g_{jm}(t) w_j, w_i\right) = (f, w_i)$$

We estímate:

$$|u'_m|^2 + m_0\|u_m\|^2 + m_1|u_m|^2 \leq c + \int_0^t |u'_m|^2 ds$$

UsingGronwall :

$$|u'_m(t)| \leq c_1 \, , \|u_m(t)\| \leq c_2 \forall m, \forall t \in [0, T_m[$$

By Caratheodory theorem we can extend the solutions $u_m(t)$ to the interval $[0; T]$. Wehave

$$u_m \, boundedin L^\infty(0, T_0; H_0^1(\Omega))$$
$$u'_m \, boundedin L^\infty(0, T_0; L^2(\Omega))$$

Also:

$$\|u'_m\|^2 + m_0|\Delta u_m|^2 + m_1\|u_m\|^2 \leq c_4 + c_3 \int_0^t [\psi(s) + \psi^2(s)] ds$$

Where $\psi(s) = \|u'_m\|^2 + |\Delta u_m|^2$.
Using Gronwallinequality :

$$|\Delta u_m(t)| \leq c_3 \, , \|u'_m(t)\| \leq c_4 \forall m, \forall t \in [0, T_m[$$

We can write

$$\int_0^t |u''_m|^2 ds \leq \int_0^t M(\|u_m\|^2)|\Delta u_m| |u''_m| ds + \int_0^t M_1(|u_m|^2)|u_m| |u''_m| ds + \int_0^t |f| |u''_m| ds$$

$$|u''_m(t)| \leq c_5 \ ,$$

Then

$$u''_m \text{isboundedin} L^2(0, T_0; L^2(\Omega))$$

$$u_m \text{isboundedin} L^\infty(0, T_0; H_0^1(\Omega) \cap H^2(\Omega))$$
$$u'_m \text{isboundedin} L^\infty\left(0, T_0; H_0^1(\Omega)\right)$$
$$u''_m \text{isboundedin} L^2\left(0, T_0; L^2(\Omega)\right)$$

Observing the fact above, we put

$$u_m \rightarrow u \text{in } (0, T_0; H_0^1(\Omega) \cap H^2(\Omega))$$
$$u_m \rightarrow u \text{en} L^2(0, T_0; H_0^1(\Omega) \cap H^2(\Omega))$$
$$(\Delta u_m, w) \rightarrow (\Delta u, w), \forall w \in L^2\left(0, T_0; L^2(\Omega)\right) u'_m \rightarrow u' \text{in} L^\infty(0, T_0; H_0^1(\Omega))$$

Wehavefinally

$$u''_m \rightarrow u'' \text{en} L^2\left(0, T_0; L^2(\Omega)\right), \text{débil}$$
$$(u''_m, w) \rightarrow (u'', w), \forall w \in L^2\left(0, T_0; L^2(\Omega)\right)$$

With the same ideas we can put:$u(0) = u_0, \ u'(0) = u_1$.

III. FRACTIONAL DIFFERENTIAL EQUATION-CENTER OF MASS-CENTER OF ENERGY

In recent years, the theory of fractional differential equations has attracted interest in biophysics and developed in the monographs of Hilfer .This paper contributes to propose to solve the Cauchy ,Kirchoff and Schrodinger equations so that it can have a greater extend of studies within the physical-mathematical analysis related to fractional differential equations and center of mass. Center of energy is another terminology used in non linear biophysical problem (for a review, see ref. 2,3 and 4) and we can obtain the figure 1 in the case of the Peyrard Bishop model of DNA.

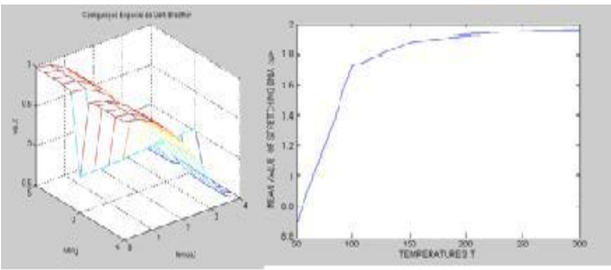

Figure 1. Breather solution of DNA modes and the amplitudes of elastic vibrational motion.

IV. CONCLUSION

We have discuss the elastic wave propagation in Cauchy, Kirchoff and Schrodinger equations.

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