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Homogenization model for the in-plane traction problem of the honeycomb core sandwich panel

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ABSTRACT: *Nowadays, honeycomb core sandwich panels are widely used in the automotive, aerospace, railway, marine industries,... Simulating and optimizing these structures is essential to achieve a lightweight and safe structure. However, numerical simulation of honeycomb structure is very tedious and time consuming. Homogenizing these structures allows us to obtain a homogeneous solid, thus making simulation and calculating much more efficient. In this paper, an analytical homogenization model for honeycomb core sandwich plate is proposed. According to this model, a 3D-Shell honeycomb core composite panel is replaced by an equivalent 3D-Solid core composite plate. A quite consistent result was obtained between the results of 3D-Shell model and homogenization 3D-Solid model, showing the accuracy and effectiveness of the proposed model for the problem of traction-compression sandwich plates.*

Keywords - *Analytic homogenization, composite sandwich, orthotropic sandwich plates, honeycomb core*

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I. INTRODUCTION

Sandwich panels have been successfully used for many years in the aviation and aerospace industries, as well as in marine, mechanical, automotive and civil engineering applications. This is due to the attendant high stiffness and high strength to weight ratios of sandwich systems [1]. Sandwich panels are often formed by adhering two high-density thin plates called face sheets or skins with a low-density core possessing less strength and stiffness. We can obtain various properties and desired performances, especially high strength to weight ratio by varying the core materials, core structures and core thickness, or material of face sheets. Many different core shapes have been applied to the construction of sandwich structures such as solid, foam, truss, web and honeycomb core. As designers in the transportation industry strive to reduce fuel consumption and improve safety, composite sandwich structures that provide improved stiffness-to-weight ratio, are becoming an attractive alternative to metals for mass transport applications. A reduction in structural weight of one large component usually triggers positive synergistic effects for other parts of the vehicle. Therefore, using composite sandwich structures not only reduces weight, thereby improving fuel economy and increasing payload capacity, but also enables the design of aerodynamic, stable vehicles with a low center of gravity [2]. Some instances of their applications in daily life are corrugated cardboard panels used for packaging, honeycomb core sandwiches used as structural floor and roof panels, metal corrugated roofs, hulks, automotive chassis and bumpers. In nature, where mechanical design required to be optimized, sandwich structures are used such as the human skull, which is made up of two layers of dense compact bone separated by a "core" of lower density material.

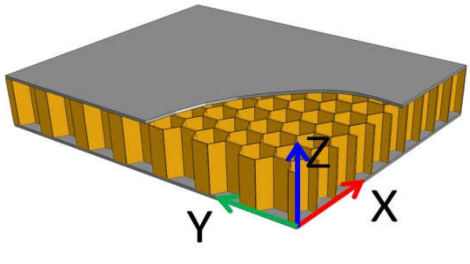

Fig. 1. The model of honeycomb core sandwich plate

The sandwich composite with honeycomb core structure is widely used in aerospace, shipbuilding, and automotive industries, as they are very light and can withstand loads much larger than weight of plate. Simulation for calculating and optimizing this type of sheet structure is important to obtain a lightweight and safe structure. However, the numerical model of the honeycomb core structure requires a lot of effort and time because of the complexion of the structure. The homogenization model of this structure allows obtaining a homogeneous core plate that is very efficient in numerical simulation (Fig. 1). Several empirical and numerical studies have been carried out to provide equivalent elastic properties. However, with the geometric properties and complex conditions of the honeycomb structure, analytical methods are always an effective homogenization method.

In this paper, we present a homogenization model to simulate the mechanical behaviors of sandwich panels with honeycomb core in traction. The homogenization is carried out by calculating analytically the in-plane properties of the honeycomb core and then this 3D-shell structure is replaced by an equivalent homogenized 3D-solid core. The simulations in case of traction load of Abaqus 3D-shell and 3D-solid model for the honeycomb core sandwich will be studied in this article. This 3D-solid homogenization model using Abaqus solid elements is very fast and has close results comparing to the 3D-shell model using Abaqus shell elements. The comparison shown many outstanding advantages of proposed model such as reduced time for modeling, time for calculation… We can use this model, of course, for other core structures, type of load or many other types of sandwich panels.

II. CACULATION OF THE IN-PLANE PROPERTIES FOR HONEYCOMB CORE SANDWICH PLATES

The homogeneous method involves replacing a heterogeneous real material with a homogeneous fictional material with equivalent macro characteristics. In order to implement the homogenization process, we must define a Representative Volume Elemental (RVE) of the material. Therefore, the results of homogenization on this RVE will represent the behavior of the whole plate. A RVE must satisfy a number of conditions: It must be large enough for the size of heterogeneity to represent the material and must be the same between regions; It must be small enough for the size of the structure that we can considered as a uniform stress or deformation state. In the case of a honeycomb structure, a RVE can be selected as shown in Fig. 2.

For honeycomb core sandwich plates, elastic modules E_x and E_y are calculated by applying displacements in the x and y directions. In the classical homogenization theory [4], the traction properties are determined on a single honeycomb cell without the effect of the skins and the properties depend only on the bending behavior of the honeycomb walls. In the present study, the skins are assumed to be very hard compared to the walls of the honeycomb core, so that the deformation of the honeycomb walls is determined by the skins. Therefore, the effect of traction or compression of thin walls dominates their bending effect. As a result, for a regular hexagonal honeycomb ($t' = 2t$ and $h = l$), the module's Young is quite proportional to (t/l) (tensile wall) instead of $(t/l)^3$ (bending wall, in [4]). If the height of the honeycomb core is very small or if we cut the honeycomb core close to the hard skins, then the honeycomb core will deform like the skins (same ν) and it will only behave traction and thus bending, as well as the effect of bending can be ignored, because:

$$\kappa \frac{t}{l} \gg \kappa' \left(\frac{t}{l} \right)^3 \quad (1)$$

Indeed, if $t = 0.19$ mm, $l = 4.62$ mm, this ratio is 600 times.

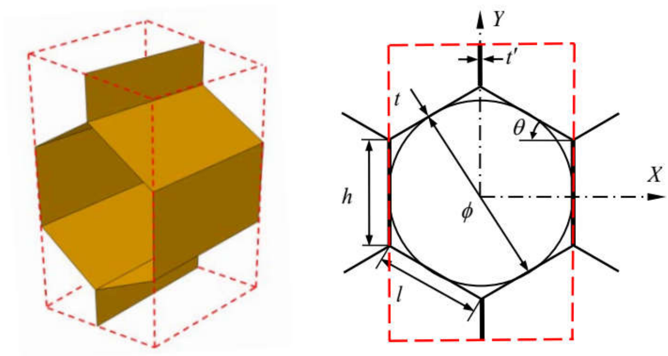

Fig. 2. Representative volume elemental (RVE) for honeycomb core

1. Traction along direction x

Considering the model constructing for slices located far away from the two skins, we establish the equation of internal force balance on five walls of EA, AC, CB, CD, and DF. Perform a displacement at the center of the core from the h/2 to the end position for points A, C and D, we have an equivalent structure as shown (Fig. 3). The problem becomes as follows: A and B are fixed on the skin, what force must be applied at D to get the displacement $u_0 = 1$? We have:

$$\varepsilon_x = \frac{u_0}{2l\cos\theta} \quad ; \quad \varepsilon_y = \frac{v_0}{h + l\sin\theta} = -\nu\varepsilon_x = -\nu\frac{u_0}{2l\cos\theta} \tag{2}$$

$$\Rightarrow v_0 = -\nu u_0 \frac{h + l\sin\theta}{2l\cos\theta} \tag{3}$$

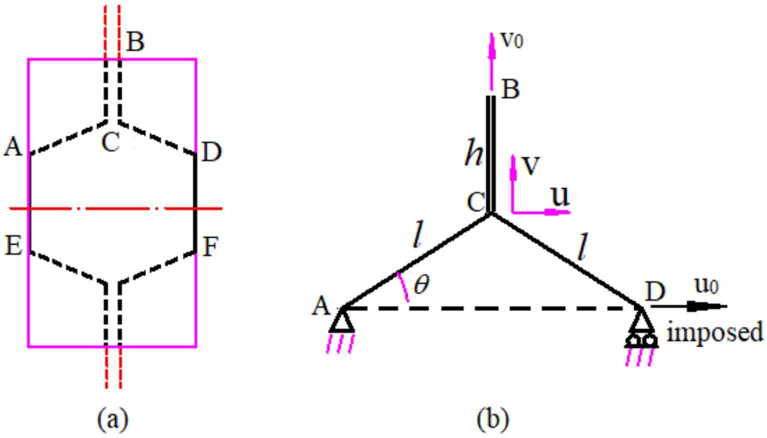

Fig. 3. Model for calculating elastic modulus E_x for a REV of honeycomb core

$$\varepsilon_l = \frac{1}{l}\left(u\cos\theta + v\sin\theta\right) \tag{4}$$

$$N_l = \sigma_l.b.t = \frac{Ebt}{l}\left(u\cos\theta + v\sin\theta\right) \tag{5}$$

$$N_h = E\frac{v_0 - v}{h}.2.bt \tag{6}$$

Balancing of node at C, we have:

$$N_h = 2N_l\sin\theta \tag{7}$$

From (5), (6) and (7), we have:

$$\frac{2\,Ebt}{h}\left(v_0 - v\right) = 2\sin\theta\frac{Ebt}{l}\left(u\cos\theta + v\sin\theta\right) \tag{8}$$

$$\Rightarrow l\left(v_0 - v\right) = h\sin\theta\left(u\cos\theta + v\sin\theta\right) \tag{9}$$

On the other hand, we have:

$$u = \frac{1}{2}u_0 \quad ; \quad v_0 = -\nu u_0\frac{h + l\sin\theta}{2l\cos\theta} \tag{10}$$

$$\Rightarrow v = \frac{v_0 - \frac{h}{4l}\sin^2\theta.u_0}{1 + \frac{h}{l}\sin^2\theta} \tag{11}$$

We have:

$$P_x = N_l\cos\theta = \frac{Ebt}{l}\left(u\cos\theta + v\sin\theta\right)\cos\theta \tag{12}$$

$$E_x^* = \frac{\sigma_x^*}{\varepsilon_x^*} = \frac{P_x}{\left(h + l\sin\theta\right)b}.\frac{2l\cos\theta}{u_0} \tag{13}$$

From Eq. (12) and (13), we have:

$$E_x^* = \frac{2E.b.t\left(\mathrm{u}\cos\theta + \mathrm{v}\sin\theta\right).\cos^2\theta}{\left(h + l\sin\theta\right)b.u} \tag{14}$$

2. Traction along direction y

For traction along y direction, we impose a displacement $v_0 = 1$, we have:

$$\varepsilon_y = \frac{\mathrm{v}_0}{h + l\sin\theta} \quad ; \quad \varepsilon_x = \frac{\mathrm{u}_0}{2l\cos\theta} = -\nu\varepsilon_y = -\nu\frac{\mathrm{v}_0}{h + l\sin\theta} \tag{15}$$

$$\Rightarrow \mathrm{u}_0 = -\nu.\mathrm{v}_0\frac{2l\cos\theta}{h + l\sin\theta} \tag{16}$$

$$\varepsilon_l = \frac{1}{l}\left(\mathrm{u}\cos\theta + \mathrm{v}\sin\theta\right) \tag{17}$$

From Eq. (8), we have:

$$\mathrm{v} = \frac{\mathrm{v}_0 - \frac{h}{2l}\sin\theta\cos\theta.\mathrm{u}_0}{1 + \frac{h}{l}\sin^2\theta} \quad ; \quad \mathrm{u} = \frac{1}{2}\mathrm{u}_0 \tag{18}$$

On the other hand, we have:

$$P_y = N_h = \frac{2\,\mathrm{Ebt}}{h}\left(\mathrm{v}_0 - \mathrm{v}\right) \tag{19}$$

$$E_y^* = \frac{\sigma_y^*}{\varepsilon_y^*} = \frac{P_y}{\left(2l\cos\theta\right)b}.\frac{h + l\sin\theta}{\mathrm{v}_0} \tag{20}$$

From Eq. (19) and (20), we have:

$$E_y^* = \frac{\sigma_y^*}{\varepsilon_y^*} = \frac{E.2tb\left(\mathrm{v}_0 - \mathrm{v}\right)}{\left(h.2l\cos\theta\right)b}.\frac{h + l\sin\theta}{\mathrm{v}_0} \tag{21}$$

For sandwich plates with very small core heights, we will use the Poisson's ratio similar to the Poisson's ratio of two skins. However, if the plate has a large core height, we can use Gibson's formula to calculate the Poisson's ratio [4]:

$$\nu_{yx} = \frac{\left(\frac{h}{l} + \sin\theta\right)\sin\theta}{\cos^2\theta} \tag{22}$$

$$\nu_{xy} = \frac{\cos^2\theta}{\left(\frac{h}{l} + \sin\theta\right)\sin\theta} \tag{23}$$

III. NUMERICAL VALIDATION OF HOMOGENIZATION MODEL

To validate the proposed homogenization model (3D-solid), we used a honeycomb core sandwich plate of length L = 160 mm, width B = 194 mm. The two skins of the sandwich are made from unidirectional non-woven linen and combined with Acrodur® resin forming a multi-layer plate. A skin of the sandwich consists of three layers oriented 0°, 90° and 0°. The mechanical properties of a skin are presented in Table 1. The total thickness of the skin is 0.6 mm. The mechanical properties of the core are presented in Table 2. For the numerical simulation of honeycomb sandwich plates, we first mesh the skins by 1248 quadrilateral elements S4R with 1320 nodes and mesh the core of the plate by 143633 quadrilateral elements S4R with 142632 nodes in Abaqus to achieve the 3D-Shell Model. For equivalent plates, we still use two real skins, but the solid core of the homogenization model will be meshed by 3861 solid elements C3D8R with 5440 nodes in Abaqus to achieve the 3D-Solid Model. The comparison of results allows assessing the effectiveness and accuracy of the proposed homogenization model.

Table 1. Parameters of the layers forming the skins of sandwich plate

E_1 (MPa)	E_2 (MPa)	ν_{12}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	Thickness (mm)
18000	2000	0.4	8500	10	10	0.2

Table 2. Parameters of the paper forming the honeycomb core of sandwich plate

E_1 (MPa)	E_2 (MPa)	ν_{12}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	Thickness (mm)
3292	1594	0.42	788	10	10	0.19

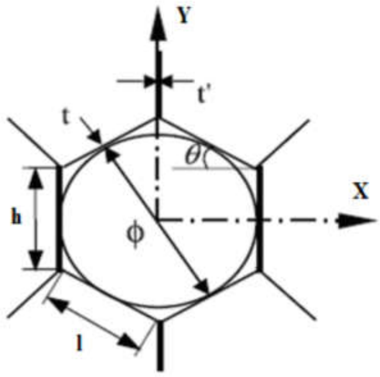

Fig. 4. Geometric shape of a honeycomb cell

Table 3. Parameters of the paper forming the honeycomb core of sandwich plate

ϕ (mm)	*θ (°)*	*l=h (mm)*	*t (mm)*	*t' (mm)*	*Height (mm)*
8	30	4.62	0.19	0.38	17

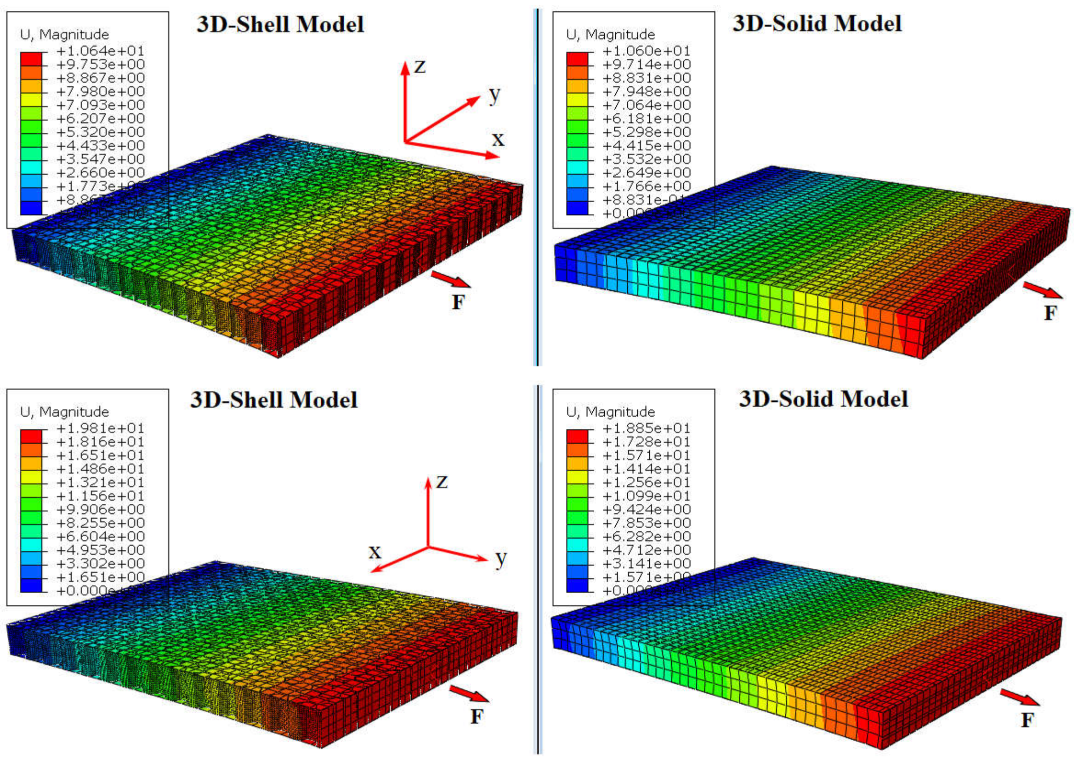

Fig. 5. Displacement and deformation of the honeycomb core sandwich under traction

From the shape of a honeycomb cell (Figure 4 and Table 3) and the mechanical properties of the core (Table 2), a homogenization model is used to calculate the properties of an equivalent homogeneous solid. In two types of simulation (3D-shell and 3D-solid Model), a rigid plate is attached to one side of the honeycomb sandwich plate to better apply force. Calculations by the 3D-solid model are very fast, while the 3D-shell calculations take a lot of time. The comparison of the results obtained by the two models as well as the percentage error of these results is presented in Table 4. For traction along x and y, we find that 3D-shell simulation uses more than 18 times of the CPU time comparing to the 3D-solid model. The numerical results given by the two models are nearly identical. The comparison of the results allows demonstrating the effectiveness and accuracy of the proposed homogenization model.

Table 4. Comparison between 3D-Shell and 3D-Solid Model for traction along x and y

F=200 kN	Traction along x		Traction along y	
	Displacement U_1 (mm)	CPU Time (s)	Displacement U_2 (mm)	CPU Time (s)
3D-Shell Model	10.467	276	19.50	270
3D-Solid Model	10.424	15.3	18.85	17.0
Error (%)	-0.41	18 times	+3.33	16 times

The deformation shape and displacement values of the honeycomb sandwich plate obtained by the 3D-Shell simulations and the 3D-Solid homogenization model are shown in Fig. 5. We see that the 3D-Shell model give the results are very close to the 3D-Solid homogenization model. The comparison shows that the 3D-Solid homogenization model proposed for honeycomb core composite panels is quite accurate and effective.

IV. CONCLUSION

In this paper, we have proposed an analytic homogenization model for the traction problems (along x and y) of a sandwich panel with the honeycomb core. The comparison of the results obtained by the 3D-Shell and 3D-Solid simulations have proved the validation of the present homogenization model for traction problems. The present homogenization model allows us to largely reduce not only the time for the geometry creation and FEM calculation, but also the computational hardware requirements for the large sandwich panels. From this model, it is possible to implement homogenous models for other loading cases such as bending, shearing in plane, transverse shear, torsion... with consideration of the influence of the skins and effect of honeycomb core height. This homogenization model can be used not only for honeycomb core sandwich plates, but also for naval and aeronautic composite structures.

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