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Bipolar Fuzzy Translation, Extention, and Multiplication on Bipolar Anti Fuzzy Ideals of K-algebras

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ABSTRACT :*A mapping whose real number interval [0,1] on the codomain is called a fuzzy set. Fuzzy theory is widely applied in algebraic structures. A bipolar fuzzy theory is developed from fuzzy theory which is mapping to real number interval[-1,1]. Bipolar fuzzy set is a pair of two fuzzy sets, called membership function and non-membership functions, respectively represented by positive and negative value. As well as fuzzy theory, bipolar fuzzy also applied in several algebraic structures, one of which is k-algebra. An algebraic structure which is built from group G and fulfilling several axioms is called k-algebra. Bipolar anti fuzzy is developed from the bipolar fuzzy theory. In this paper, we introduces the notion of a bipolar fuzzy translation, bipolar fuzzy extention, and bipolar fuzzy multiplication on bipolar anti fuzzy ideals of k-algebra.*

KEYWORDS *Bipolar anti fuzzy, K-algebras, ideals of K-algebra, bipolar fuzzy translation.*

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I. INTRODUCTION

A fuzzy set firstly introduced by Zadeh in 1965 was widely applied to various sciences, including algebra. In 1994 Zhang was introduced the concept of bipolar fuzzy set which is developed from fuzzy sets. A bipolar fuzzy is a pair of two fuzzy sets called membership function and non-membership functions, respectively represented by positive and negative values. Bipolar fuzzy sets are also applied to algebra, one of which is K-algebra.

K-algebra is a kind of an algebraic structure which is built by groups $(G,*,e)$ with binary operations $(\odot)$ and fulfilling the certain axioms and it is denoted by $\mathcal{K} = (G,*,\odot,e)$. This concept was discussed firstly by Akram and Dar in 2005, in their paper entitled On a K-algebra Built on a Group. The discussion was continued in 2007 where Akram and Dar wrote about homomorphismin K-algebra and fuzzyideals of K-algebra. Along with the development of fuzzy set theory, in 2008 Akram discussed the bifuzzy ideal of K-algebra. Not only fuzzy theory, but also bipolar fuzzy is applied to K-algebra. In2009 Jun et al., discussed about bipolar fuzzy translation in BCK/BCI-algebras and Akram discussed the application of bipolar fuzzy in K-algebra in his article entitled Bipolar Fuzzy K-algebras in 2010.

In this paper, we introduces the notion of bipolar fuzzy translation, bipolar fuzzy extention, and bipolar fuzzy multiplication on bipolar anti fuzzy ideals of K-algebras.

II. PRELIMINARIES

In this section we will discuss some basic theories about bipolar fuzzy translation, bipolar fuzzy extention, and bipolar fuzzy multiplication on bipolar anti fuzzy ideal of K-algebra.

Definition 2.1 (Akram and Dar, 2005)
Let $(G,*)$ is a group with an order more than 2. Define a binary operation on G as follow $\odot : G \times G \to G$

$$\odot(x,y) = x \odot y = x * y^{-1}$$

If the following axioms are hold by G:

i. $(x \odot y) \odot (x \odot z) = \left(x \odot \left((e \odot z) \odot (e \odot y)\right)\right) \odot x$
ii. $x \odot (x \odot y) = \left(x \odot (e \odot y)\right) \odot x$
iii. $x \odot x = e$
iv. $x \odot e = x$
v. $e \odot x = x^{-1}$ for every $x, y, z \in G$

then G is called K-algebra which is built by group $(G,*)$ and we denoted by $\mathcal{K} = (G,*,\odot,e)$. If $(G,*,e)$ is an abelian group, then axiom i and ii replace with
i*. $(x \odot y) \odot (x \odot z) = z \odot y$,
ii*. $x \odot (x \odot y) = y$
For every $x, y, z \in G$.

Definition 2.2 (Akram and Dar, 2007)
Let $\mathcal{K} = (G,*,\odot,e)$ is a K-algebra. A non empty set H in K-agebra $\mathcal{K}$ is called K-subalgebra if $e \in H$ and $h_1 \odot h_2 \in H$, for every $h_1, h_2 \in H$.

Definition 2.3 (Akram and Dar, 2007)
Let I is a non empty set in K-algebra $\mathcal{K} = (G,*,\odot,e)$. A set I is called ideal of $\mathcal{K}$ if the following condition satisfied for every $x, y \in G$.
i. $e \in I$
ii. $x \odot y \in I,\ y \odot (y \odot x) \in I \Rightarrow x \in I$

Definition 2.4 (Kandasamy, 2003)
Let X is a non empty set and μ_A is a mapping
$$\mu_A : X \to [0,1]$$
with $[0,1]$ is a real number interval from 0 to 1. A set A defined by
$$A = \{ (x, \mu_A(x)) | x \in X\}$$
is called fuzzy set of Ain X. $\mu_A(x)$ is called membership function for fuzzy set A.

Definition 2.5 (Akram and dar, 2007)
Let $\mathcal{K} = (G,*,\odot,e)$ is a K-algebra. A fuzzy set A of $\mathcal{K}$ is called fuzzy ideal of $\mathcal{K}$ if it satisfies:
i. $\mu_A(e) \geq \mu_A(x)$, for every $x \in G$ and
ii. $\mu_A(x) \geq \min\{\mu_A(x \odot y), \mu_A(y \odot (y \odot x))\}$, for every $x, y \in G$.

Definition 2.6 (Zhang, 1998)
Let X is a non empty set and λ_B^+ and λ_B^- is a mapping
$\lambda_B^+ : X \to [0,1]$ and $\lambda_B^- : X \to [-1,0]$
with $[0,1]$ is a real number interval from 0 to 1 dan $[-1,0]$ is a real number interval from -1 to 0. A set B defined by
$$B = \{ x, (\lambda_B^+(x), \lambda_B^-(x)) | x \in X\}$$
is called bipolar fuzzy set $B$of X. $\lambda_B^+(x)$ is called membership function for fuzzy set Adan $\lambda_B^-(x)$ is called non-membership function for fuzzy set A. Furthermore, bipolar fuzzy set written as $B = (\mu^+, \mu^-)$.

Definition 2.7 (Akram et al, 2010)
Let $\mathcal{K} = (G,*,\odot,e)$ is a K-algebra. Bipolar fuzzy set $B = (\mu^+, \mu^-)$in $\mathcal{K}$ is called bipolar fuzzy subalgebra if it satisfies for every $x, y \in G$.
i. $\mu^+(x \odot y) \geq min\{\mu^+(x), \mu^+(y)\}$
ii. $\mu^-(x \odot y) \leq max\{\mu^+(x), \mu^+(y)\}$

Definition 2.8 (Ria et al, 2018)
Let $B = (\lambda^+, \lambda^-)$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$and $t' = (t^+, t^-) \in (0,1] \times [-1,0)$, for every $x \in \mathcal{K}$
i. $B(x) \geq t' \Leftrightarrow (\lambda^+(x), \lambda^-(x)) \geq (t^+, t^-) \Leftrightarrow \lambda^+(x) \geq t^+$and $\lambda^-(x) \leq t^-$
ii. $B(x) \leq t' \Leftrightarrow (\lambda^+(x), \lambda^-(x)) \leq (t^+, t^-) \Leftrightarrow \lambda^+(x) \leq t^+$and $\lambda^-(x) \geq t^-$

Definition 2.9 (Ria et al, 2018)
Let $B = (\lambda^+, \lambda^-)$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$ with
$$\lambda^+(z) = \begin{cases} t^+ & , z = x \\ 0 & , z \neq x \end{cases}$$
$$\lambda^-(z) = \begin{cases} t^- & , z = x \\ 0 & , z \neq x \end{cases}$$
Then B is called a bipolar value fuzzy point where $t' = (t^+, t^-) \in (0,1] \times [-1,0)$and support x, written as $x_{t'} = (x_t^+, x_t^-)$. $x_{t'}$ is said to belong to$B$, written as $x_{t'} \in B$ if $B(x) \geq t'$, so $\lambda^+(x) \geq t^+, \lambda^-(x) \leq t^-$. $x_{t'}$ is said not to belong to B, written as $x_{t'} \overline{\in} B$ if $B(x) \leq t'$, so $\lambda^+(x) \leq t^+, \lambda^-(x) \geq t^-$.

Definition 2.10 (Ria et al, 2018)
Let $B_1 = (\lambda^+, \lambda^-)$, $B_2 = (\mu^+, \mu^-)$ are bipolar fuzzy sets of $\mathcal{K}$,
$max\{B_1, B_2\}$ is defined as $(max\{\lambda^+, \mu^+\}, min\{\lambda^-, \mu^-\})$
$min\{B_1, B_2\}$ is defined as $(min\{\lambda^+, \mu^+\}, max\{\lambda^-, \mu^-\})$

Definition 2.11 (Ria et al, 2018)
A bipolar fuzzy set$B = (\lambda^+, \lambda^-)$ is called a bipolar fuzzy ideal of $\mathcal{K}$ if following condition hold.
i. $\lambda^+(e) \geq \lambda^+(x)and\lambda^-(e) \leq \lambda^-(x)$
ii. $\lambda^+(x) \geq min\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}and\lambda^-(x) \leq max\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$

Definition 2.12 (Ria et al, 2018)
Suppose I is a non empty subset in K-algebra $\mathcal{K}$. Bipolar fuzzy set $C_{I^c} = (C_{I^c}^+, C_{I^c}^-)$ defined by

$$C_{I^c}^+(x) = \begin{cases} 0 & , \quad x \in I \\ 1 & , \quad x \,\overline{\in}\, I \end{cases}$$

$$C_{I^c}^-(x) = \begin{cases} 0 & , \quad x \in I \\ -1 & , \quad x \,\overline{\in}\, I \end{cases}$$

is called bipolar-valued anti characteristic function.

Definition 2.13 (Ria et al, 2018)
Let $\mathcal{K} = (G, *, \odot, e)$ is a K-algebra. Bipolar fuzzy set $B = (\lambda^+, \lambda^-)$ is said a bipolar anti fuzzy ideal of $\mathcal{K}$ if the following conditions hold.
i. $x_{t'} \,\overline{\in}\, B \Rightarrow e_{t'} \,\overline{\in}\, B$
ii. $(x \odot y)_{t'} \,\overline{\in}\, B, (y \odot (y \odot x))_{r'} \,\overline{\in}\, B \Rightarrow x_{max\{t', r'\}} \,\overline{\in}\, B$
Furthermore, bipolar anti fuzzy ideal is abriviated by BAF ideal.

Theorem 2.14 (Ria et al, 2018)
If B is a bipolar fuzzy setin K-algebra $\mathcal{K}$, then axioms in Definition 3.7 are equivalent to the following axioms respectively.
a. $\lambda^+(e) \leq \lambda^+(x)$ and $\lambda^-(e) \geq \lambda^-(x)$
b. $\lambda^+(x) \leq max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$ and $\lambda^-(x) \geq min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$

III. MAIN RESULT

In this section, we discusses about the notion and the properties of bipolar fuzzy translation, bipolar fuzzy extention, and bipolar fuzzy multiplication on bipolar anti fuzzy ideals of K-algebra. For every bipolar fuzzy sets $B = (\lambda^+, \lambda^-)$ of K -algebra $\mathcal{K}$, we denote $u = 1 - sup\{\lambda^+(x) | x \in \mathcal{K}\}$ dan $i = -1 - inf\{\lambda^-(x) | x \in \mathcal{K}\}$.

Definition 3.1 (Bipolar Fuzzy Translation)
Let $B = (\lambda^+, \lambda^-)$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$and $(\gamma, \delta) \in [0, u] \times [i, 0]$. Bipolar fuzzy set $B^T_{(\gamma,\delta)} = \left(\lambda^+_{(\gamma,T)}, \lambda^-_{(\delta,T)}\right)$ is called bipolar fuzzy (γ, δ)-translation of B, where

$$\lambda^+_{(\gamma,T)}: \mathcal{K} \to [0,1], x \to \lambda^+(x) + \gamma$$

$$\lambda^-_{(\delta,T)}: \mathcal{K} \to [-1,0], x \to \lambda^-(x) + \delta$$

Theorem 3.2
If $B = (\lambda^+, \lambda^-)$ is a BAF ideal pada K-algebra $\mathcal{K}$, then bipolar fuzzy (γ, δ)-translation of $B$is a BAF ideal of $\mathcal{K}$for every$(\gamma, \delta) \in [0, u] \times [i, 0]$.
Proof:
i. B is a BAF ideal of$\mathcal{K}$, for every$x \in \mathcal{K}$obtain $\lambda^+(e) \leq \lambda^+(x)and\lambda^-(e) \geq \lambda^-(x)$.
Because of $\gamma \in [0, u]$and $u = 1 - sup\{\lambda^+(x) | x \in \mathcal{K}\}$ then

$$\lambda^+(e) + \gamma \leq \lambda^+(x) + \gamma$$

Because of $\delta \in [i, 0]$and $i = -1 - inf\{\lambda^-(x) | x \in \mathcal{K}\}$then

$$\lambda^-(e) + \delta \geq \lambda^-(x) + \delta$$

ii. $\lambda^+(x) \leq max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$

$$\lambda^+(x) + \gamma \leq max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\} + \gamma$$

$$\lambda^+(x) + \gamma \leq max\{\lambda^+(x \odot y) + \gamma, \lambda^+(y \odot (y \odot x)) + \gamma\}$$

and

$$\lambda^-(x) \geq min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$$

$$\lambda^-(x) + \delta \geq min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\} + \delta$$

$$\lambda^-(x)+\delta \geq min\{\lambda^-(x\odot y)+\delta, \lambda^-(y\odot(y\odot x))+\delta\}$$

Bipolar fuzzy (γ,δ)-translation of B is a BAF ideal of $\mathcal{K}$.

Example 3.3

Let $G=\{e,a,b,x,y,z\}$ and binary operation $\circ$ in G is defined in Table 1

Table 1: Binary Operation $\circ$ in G

$\circ$	e	a	b	x	y	z
e	e	a	b	x	y	z
a	a	b	e	z	x	y
b	b	e	a	y	z	x
x	x	y	z	e	a	b
y	y	z	x	b	e	a
z	z	x	y	a	b	e

We can prove that $(G,\circ)$ is a group and $\mathcal{K}=(G,\circ,\odot,e)$ is a K-algebra. We define a bipolar fuzzy set $B=(\lambda^+,\lambda^-)$ as follows,

$\lambda^+(x)=\begin{cases}0.03 & , \quad x=e\\ 0.4 & , \quad x\neq e\end{cases}$ and $\lambda^-(x)=\begin{cases}-0.2 & , \quad x=e\\ -0.35 & , \quad x\neq e\end{cases}$

B is a BAF ideal of K-algebra $\mathcal{K}$.

$$u=1-sup\{\lambda^+(x)|x\in\mathcal{K}\} \qquad i=-1-inf\{\lambda^-(x)|x\in\mathcal{K}\}$$
$$=1-sup\{0.03,0.4\} \qquad =-1-inf\{-0.2,-0.35\}$$
$$=1-0.4=0.6 \qquad =-1-(-0.35)=-0.65$$

If $\gamma=0.5$ and $\delta=-0.3$, then $B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$.

Theorem 3.4

Let $B=(\lambda^+,\lambda^-)$ is a bipolar fuzzy set of $\mathcal{K}$. If bipolar fuzzy (γ,δ)-translation $B^T_{(\gamma,\delta)}=\left(\lambda^+_{(\gamma,T)},\lambda^-_{(\delta,T)}\right)$ of B is a BAF ideal of $\mathcal{K}$ for every $(\gamma,\delta)\in[0,u]\times[i,0]$, then $B=(\lambda^+,\lambda^-)$ is a BAF ideal of $\mathcal{K}$.

Proof:

i. Bipolar fuzzy (γ,δ)-translation $B^T_{(\gamma,\delta)}$ of B is a BAF ideal of $\mathcal{K}$, implies $\lambda^+(e)+\gamma\leq\lambda^+(x)+\gamma$ and $\lambda^-(e)+\delta\geq\lambda^-(x)+\delta$.

Because of $\gamma\in[0,u]$ and $u=1-sup\{\lambda^+(x)|x\in\mathcal{K}\}$, then

$$\lambda^+(e)\leq\lambda^+(x)$$

Because of $\delta\in[i,0]$ and $i=-1-inf\{\lambda^-(x)|x\in\mathcal{K}\}$, then

$$\lambda^-(e)\geq\lambda^-(x)$$

ii. Bipolar fuzzy (γ,δ)-translation $B^T_{(\gamma,\delta)}$ of B is a BAF ideal of $\mathcal{K}$, it implies

$$\lambda^+(x)+\gamma\leq max\{\lambda^+(x\odot y)+\gamma,\lambda^+(y\odot(y\odot x))+\gamma\}$$
$$\lambda^+(x)+\gamma\leq max\{\lambda^+(x\odot y),\lambda^+(y\odot(y\odot x))\}+\gamma$$
$$\lambda^+(x)\leq max\{\lambda^+(x\odot y),\lambda^+(y\odot(y\odot x))\}$$

and

$$\lambda^-(x)+\delta\geq min\{\lambda^-(x\odot y)+\delta,\lambda^-(y\odot(y\odot x))+\delta\}$$
$$\lambda^-(x)+\delta\geq min\{\lambda^-(x\odot y),\lambda^-(y\odot(y\odot x))\}+\delta$$
$$\lambda^-(x)\geq min\{\lambda^-(x\odot y),\lambda^-(y\odot(y\odot x))\}$$

$B=(\lambda^+,\lambda^-)$ BAF ideal of $\mathcal{K}$.

Definition 3.5 (Bipolar Fuzzy Extention)

Let $B_1=(\lambda^+,\lambda^-)$ and $B_2=(\mu^+,\mu^-)$ is bipolar fuzzy sets of K-algebra $\mathcal{K}$. If $\lambda^+(x)\leq\mu^+(x)$ and $\lambda^-(x)\geq\mu^-(x)$ for every $x\in\mathcal{K}$, then B_2 is called bipolar fuzzy extention of B_1.

Definition 3.6 (Bipolar Anti Fuzzy Ideal Extention)

Let $B_1=(\lambda^+,\lambda^-)$ and $B_2=(\mu^+,\mu^-)$ is bipolar fuzzy sets of K-algebra $\mathcal{K}$. B_2 is called BAF ideal extention of B_1 if the following axioms are fulfilled.

i. B_2 is a bipolar fuzzy extention of B_1
ii. If B_2 is a BAF ideal of $\mathcal{K}$, then B_1 BAF ideal of $\mathcal{K}$.

Based on the definition above, $\lambda^+_{(\gamma,T)}\geq\lambda^+(x)$ and $\lambda^-_{(\delta,T)}\leq\lambda^-(x)$ for every $x\in\mathcal{K}$, we obtained several theorems as follow.

Theorem 3.7
If $B = (\lambda^+, \lambda^-)$ is a BAF ideal of K-algebra $\mathcal{K}$, then bipolar fuzzy (γ, δ)-translation $B^T_{(\gamma,\delta)} = (\lambda^+_{(\gamma,T)}, \lambda^-_{(\delta,T)})$ of $B$is a BAF ideal extention of $B$for every$(\gamma, \delta) \in [0, u] \times [i, 0]$.
Proof:
i. $B^T_{(\gamma,\delta)}$bipolar fuzzy extention of B.
ii. Suppose $B^T_{(\gamma,\delta)}$is a BAF ideal of $\mathcal{K}$, then B is not a BAF ideal of $\mathcal{K}$.

- $\lambda^+(e) + \gamma \leq \lambda^+(x) + \gamma$ and $\lambda^-(e) + \delta \geq \lambda^-(x) + \delta$

$\lambda^+(e) \leq \lambda^+(x)$ $\lambda^-(e) \geq \lambda^-(x)$

- $\lambda^+(x) + \gamma \leq max\{\lambda^+(x \odot y) + \gamma, \lambda^+(y \odot (y \odot x)) + \gamma\}$

$$\lambda^+(x) + \gamma \leq max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\} + \gamma$$
$$\lambda^+(x) \leq max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$$

and

$$\lambda^-(x) + \delta \geq min\{\lambda^-(x \odot y) + \delta, \lambda^-(y \odot (y \odot x)) + \delta\}$$
$$\lambda^-(x) + \delta \geq min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\} + \delta$$
$$\lambda^-(x) \geq min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$$

This is contrary with the statement above, it must be B BAF ideal of $\mathcal{K}$.
$B^T_{(\gamma,\delta)}$is a BAF ideal extention of B.

Example 3.8
According to example 3.3, $B^T_{(\gamma,\delta)}$is a BAF ideal of $\mathcal{K}$with $\gamma = 0.5$ and $\delta = -0.3$, then $B^T_{(\gamma,\delta)}$ BAF ideal extention of B.

Definition 3.9
Let $B = (\mu^+, \mu^-)$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$, $(\alpha, \beta) \in [-1,0] \times [0,1], \gamma \in [0, u], \delta \in [i, 0]$, then

- $\tilde{B}^{+T}_{(\beta,\gamma)} = \{x \in \mathcal{K} | \mu^+(x) \leq \beta - \gamma\}$
- $\tilde{B}^{-T}_{(\alpha,\delta)} = \{x \in \mathcal{K} | \mu^-(x) \geq \alpha - \delta\}$
- $\tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))} = \{x \in \mathcal{K} | \mu^-(x) \geq \alpha - \gamma \ dan \ \mu^+(x) \leq \beta - \delta\}$

Theorem 3.10
If $B = (\mu^+, \mu^-)$ is a BAF ideal of K-algebra$\mathcal{K}$, then$\tilde{B}^{+T}_{(\beta,\gamma)}$and $\tilde{B}^{-T}_{(\alpha,\delta)}$ ideal of $\mathcal{K}$for every$\alpha \in Im(\mu^-)$ and $\beta \in Im(\mu^+)$ with$\beta \geq \gamma$ and $\alpha \leq \delta$.
Proof:
i. Suppose $x \in \tilde{B}^{+T}_{(\beta,\gamma)} \rightarrow \mu^+(x) \leq \beta - \gamma$. Because of $\mu^+(e) \leq \mu^+(x)$ and $\mu^+(x) \leq \beta - \gamma$ then $\mu^+(e) \leq \beta - \gamma$. It is clear that $e \in \tilde{B}^{+T}_{(\beta,\gamma)}$
Suppose $x \in \tilde{B}^{-T}_{(\alpha,\delta)} \rightarrow \mu^-(x) \geq \alpha - \delta$. Because of $\mu^-(e) \geq \mu^-(x)$ and $\mu^-(x) \geq \alpha - \delta$ then $\mu^-(e) \geq \alpha - \delta$. It is clear that $e \in \tilde{B}^{-T}_{(\alpha,\delta)}$.
ii. Suppose $(x \odot y)$ and $(y \odot (y \odot x)) \in \tilde{B}^{+T}_{(\beta,\gamma)}$ implies $\mu^+(x \odot y) \leq \beta - \gamma$ and $\mu^+(y \odot (y \odot x)) \leq \beta - \gamma$.

$$max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\} \leq max\{\beta - \gamma, \beta - \gamma\}$$
$$max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\} \leq \beta - \gamma$$

Because of $\mu^+(x) \leq max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\}$ then$\mu^+(x) \leq \beta - \gamma$. It is clear that$x \in \tilde{B}^{+T}_{(\beta,\gamma)}$. In the same way for $\tilde{B}^{-T}_{(\alpha,\delta)}$, we can obtained $x \in \tilde{B}^{-T}_{(\alpha,\delta)}$. $\tilde{B}^{+T}_{(\beta,\gamma)}$ dan $\tilde{B}^{-T}_{(\alpha,\delta)}$ideal of$\mathcal{K}$.

Corollary 3. 11
If $B = (\mu^+, \mu^-)$ is a BAF ideal of K-algebra $\mathcal{K}$, then $\tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$ideal of $\mathcal{K}$for every$(\alpha, \beta) \in [-1,0] \times [0,1]$ and $(\gamma, \delta) \in [0, u] \times [i, 0]$.
Proof;
i. B is a BAF ideal of $\mathcal{K}$, implies $\mu^+(e) \leq \mu^+(x)$ and $\mu^-(e) \geq \mu^-(x)$. Suppose$x \in \tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$implies$\mu^+(x) \leq \beta - \delta$and$\mu^-(x) \geq \alpha - \gamma$.
Because of$\mu^+(e) \leq \mu^+(x)$and$\mu^-(e) \geq \mu^-(x)$then

$$\mu^+(e) \leq \mu^+(x) \leq \beta - \delta \rightarrow \mu^+(e) \leq \beta - \delta$$
$$\mu^-(e) \geq \mu^-(x) \geq \alpha - \gamma \rightarrow \mu^-(e) \geq \alpha - \gamma$$

It is clear that $e \in \tilde{B}^{T}_{((\alpha,\beta),(\gamma,\delta))}$.

ii. Suppose $(x \odot y)$ and $(y \odot (y \odot x)) \in \tilde{B}^{T}_{((\alpha,\beta),(\gamma,\delta))}$ implies
$\mu^{+}(x \odot y) \leq \beta - \delta$ and $\mu^{+}(y \odot (y \odot x)) \leq \beta - \delta$

$$max\{\mu^{+}(x \odot y), \mu^{+}(y \odot (y \odot x))\} \leq \beta - \delta$$

$\mu^{-}(x \odot y) \geq \alpha - \gamma$ and $\mu^{-}(y \odot (y \odot x)) \geq \alpha - \gamma$

$$min\{\mu^{-}(x \odot y), \mu^{-}(y \odot (y \odot x))\} \geq \alpha - \gamma$$

Because of B BAF ideal of K-algebra $\mathcal{K}$, then
$\mu^{+}(x) \leq \beta - \delta$ and $\mu^{-}(x) \geq \alpha - \gamma$
It is clear that $x \in \tilde{B}^{T}_{((\alpha,\beta),(\gamma,\delta))}$. $\tilde{B}^{T}_{((\alpha,\beta),(\gamma,\delta))}$ ideal of $\mathcal{K}$.

Theorem 3. 12
Let $B = (\mu^{+}, \mu^{-})$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$. Bipolar fuzzy (γ,δ)-translation of B is BAF ideal of $\mathcal{K}$ if and only if $\tilde{B}^{+T}_{(\beta,\gamma)}$ and $\tilde{B}^{-T}_{(\alpha,\delta)}$ ideal of $\mathcal{K}$ for every $\alpha \in Im(\mu^{-}), \beta \in Im(\mu^{+})$, and $(\gamma,\delta) \in [0,u] \times [i,0]$ with$\alpha < \delta$ and $\beta > \gamma$.
Proof:

i. Let $x \in \tilde{B}^{+T}_{(\beta,\gamma)} \rightarrow \mu^{+}(x) \leq \beta - \gamma$. Because of $B^{T}_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$, then $\mu^{+}(e) + \gamma \leq \mu^{+}(x) + \gamma$. Because$\gamma \in [0,u]$ we obtain $\mu^{+}(e) \leq \mu^{+}(x)$. So$\mu^{+}(e) \leq \mu^{+}(x) \leq \beta - \gamma$. Hence $e \in \tilde{B}^{+T}_{(\beta,\gamma)}$.

In the same way for $\tilde{B}^{-T}_{(\alpha,\delta)}$, we obtain $\mu^{-}(e) \geq \mu^{-}(x) \geq \alpha - \delta$. Hence $e \in \tilde{B}^{-T}_{(\alpha,\delta)}$.

ii. Let$(x \odot y), (y \odot (y \odot x)) \in \tilde{B}^{+T}_{(\beta,\gamma)} \rightarrow \mu^{+}(x \odot y) \leq \beta - \gamma$ and $\mu^{+}(y \odot (y \odot x)) \leq \beta - \gamma$.
$B^{T}_{(\gamma,\delta)}$BAF ideal of $\mathcal{K}$, implies

$$\mu^{+}(x) \leq max\{\mu^{+}(x \odot y), \mu^{+}(y \odot (y \odot x))\} \leq \beta - \gamma$$

Hence $x \in \tilde{B}^{+T}_{(\beta,\gamma)}$. In the same way for $\tilde{B}^{-T}_{(\alpha,\delta)}$, we obtain $x \in \tilde{B}^{-T}_{(\alpha,\delta)}$. Hence $\tilde{B}^{+T}_{(\beta,\gamma)}$ and $\tilde{B}^{-T}_{(\alpha,\delta)}$ideal of $\mathcal{K}$.

Conversely,

i. Let $x \in B^{T}_{(\gamma,\delta)} \rightarrow \mu^{+}(x) + \gamma$ and $\mu^{-}(x) + \delta$. $\tilde{B}^{+T}_{(\beta,\gamma)}$ ideal of $\mathcal{K}$, implies for every $x \in \tilde{B}^{+T}_{(\beta,\gamma)}$ there is e so $\mu^{+}(e) \leq \beta - \gamma$ and $\mu^{+}(x) \leq \beta - \gamma$, can be written as $\mu^{+}(e) + \gamma \leq \beta$ and $\mu^{+}(x) + \gamma \leq \beta$. Suppose$\mu^{+}(e) + \gamma > \mu^{+}(x) + \gamma$and$\mu^{+}(x) + \gamma = \beta$, implies$\mu^{+}(e) + \gamma > \beta$. It is contrary to the statement $\mu^{+}(e) + \gamma \leq \beta$. It must be $\mu^{+}(e) + \gamma \leq \mu^{+}(x) + \gamma$.

In the same way for $\tilde{B}^{-T}_{(\alpha,\delta)}$, we obtain $\mu^{-}(e) + \delta \geq \mu^{-}(x) + \delta$.

ii. Let$(x \odot y), (y \odot (y \odot x)) \in B^{T}_{(\gamma,\delta)}$ implies

$$\mu^{+}(x \odot y) + \gamma \qquad \mu^{-}(x \odot y) + \delta$$
$$\mu^{+}(y \odot (y \odot x)) + \gamma \qquad \mu^{-}(y \odot (y \odot x)) + \delta$$

$\tilde{B}^{+T}_{(\beta,\gamma)}$ ideal of $\mathcal{K}$, then

$$\left.\begin{matrix} \mu^{+}(x \odot y) \leq \beta - \gamma \\ \mu^{+}(y \odot (y \odot x)) \leq \beta - \gamma \end{matrix}\right\} \rightarrow \mu^{+}(x) \leq \beta - \gamma$$

or can be written as

$$\left.\begin{matrix} \mu^{+}(x \odot y) + \gamma \leq \beta \\ \mu^{+}(y \odot (y \odot x)) + \gamma \leq \beta \end{matrix}\right\} \rightarrow \mu^{+}(x) + \gamma \leq \beta$$

It can be concluded by $max\{\mu^{+}(x \odot y) + \gamma, \mu^{+}(y \odot (y \odot x)) + \gamma\} \leq \beta$.
Suppose $\mu^{+}(x) + \gamma > max\{\mu^{+}(x \odot y) + \gamma, \mu^{+}(y \odot (y \odot x)) + \gamma\}$and
$max\{\mu^{+}(x \odot y) + \gamma, \mu^{+}(y \odot (y \odot x)) + \gamma\} = \beta$, so we obtain $\mu^{+}(x) + \gamma > \beta$.
It is contrary to the statement $\mu^{+}(x) + \gamma \leq \beta$. It must be
$\mu^{+}(x) + \gamma \leq max\{\mu^{+}(x \odot y) + \gamma, \mu^{+}(y \odot (y \odot x)) + \gamma\}$.
In the same way for$\tilde{B}^{-T}_{(\alpha,\delta)}$, we obtain $\mu^{-}(x) + \delta \geq min\{\mu^{-}(x \odot y) + \delta, \mu^{-}(y \odot (y \odot x)) + \delta\}$.
$B^{T}_{(\gamma,\delta)}$BAF ideal of$\mathcal{K}$.

Corollary 3.13
Let $B = (\mu^{+}, \mu^{-})$ is a bipolar fuzzy set of K-algebrar $\mathcal{K}$. Bipolar fuzzy (γ,δ)-translation of B is BAF ideal of $\mathcal{K}$ if and only if $\tilde{B}^{T}_{((\alpha,\beta),(\gamma,\delta))}$ ideal of $\mathcal{K}$ for every $\alpha \in Im(\mu^{-}), \beta \in Im(\mu^{+})$, and $(\gamma,\delta) \in [0,u] \times [i,0]$ with $\alpha < \delta$ and $\beta > \gamma$.
Proof:

i. Let $x \in \tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))} \rightarrow \mu^+(x) \leq \beta - \delta$ and $\mu^-(x) \geq \alpha - \gamma$. $B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$, implies

$$\mu^+(e) \leq \mu^+(x) \leq \beta - \delta \text{ and } \mu^-(e) \geq \mu^-(x) \geq \alpha - \gamma.$$

$e \in \tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$.

ii. Let $(x \odot y), (y \odot (y \odot x)) \in \tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$, implies

$$\mu^+(x \odot y) \leq \beta - \delta \qquad \mu^-(x \odot y) \geq \alpha - \gamma$$
$$\mu^+(y \odot (y \odot x)) \leq \beta - \delta \qquad \mu^-(y \odot (y \odot x)) \geq \alpha - \gamma$$

$B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$, then

$$\mu^+(x) \leq \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\} \leq \beta - \delta$$
$$\mu^-(x) \geq \min\{\mu^-(x \odot y), \mu^-(y \odot (y \odot x))\} \geq \alpha - \gamma$$

$x \in \tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$.

It can be concluded by $\tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$ ideal of $\mathcal{K}$. Conversely,

i. $\tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$ ideal of $\mathcal{K}$, implies

$$\begin{matrix}\mu^+(e) \leq \beta - \delta \\ \mu^-(e) \geq \alpha - \gamma\end{matrix} \text{ and } \begin{matrix}\mu^+(x) \leq \beta - \delta \\ \mu^-(x) \geq \alpha - \gamma\end{matrix}$$

Suppose $\mu^+(e) > \mu^+(x)$ and $\mu^+(x) = \beta - \delta$ then $\mu^+(e) > \beta - \delta$. It is contrary with $\mu^+(e) \leq \beta - \delta$. It must be $\mu^+(e) \leq \mu^+(x) \leq \beta - \delta$. Hence $\mu^+(e) + \gamma \leq \mu^+(x) + \gamma \leq \beta - \delta + \gamma$. It can be conclude $\mu^+(e) + \gamma \leq \mu^+(x) + \gamma$.

In the same way we obtain $\mu^-(e) + \delta \geq \mu^-(x) + \delta$.

ii. $\tilde{B}^T_{((\alpha,\beta),(\gamma,\delta))}$ ideal of $\mathcal{K}$, implies

$$\left.\begin{matrix}\mu^+(x \odot y) \leq \beta - \delta \\ \mu^+(y \odot (y \odot x)) \leq \beta - \delta\end{matrix}\right\} \rightarrow \mu^+(x) \leq \beta - \delta$$

Suppose $\mu^+(x) > \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\}$ and $\max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\} = \beta - \delta$, we obtain $\mu^+(x) > \beta - \delta$. It is contrary with $\mu^+(x) \leq \beta - \delta$. It must be

$$\mu^+(x) \leq \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\}$$

So

$$\mu^+(x) + \gamma \leq \max\{\mu^+(x \odot y), \mu^+(y \odot (y \odot x))\} + \gamma$$
$$\mu^+(x) + \gamma \leq \max\{\mu^+(x \odot y) + \gamma, \mu^+(y \odot (y \odot x)) + \gamma\}$$

$$\left.\begin{matrix}\mu^-(x \odot y) \geq \alpha - \gamma \\ \mu^-(y \odot (y \odot x)) \geq \alpha - \gamma\end{matrix}\right\} \rightarrow \mu^-(x) \geq \alpha - \gamma$$

In the same way, we obtain

$$\mu^-(x) + \delta \geq \min\{\mu^-(x \odot y), \mu^-(y \odot (y \odot x))\} + \delta$$
$$\mu^-(x) + \delta \geq \min\{\mu^-(x \odot y) + \delta, \mu^-(y \odot (y \odot x)) + \delta\}$$

$B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$.

Theorem 3.14

Let $B = (\mu^+, \mu^-)$ is a BAF ideal of K-algebra $\mathcal{K}$, $(\gamma, \delta) \in [0, u] \times [i, 0]$ and $(\gamma', \delta') \in [0, u] \times [i, 0]$. If $(\gamma, \delta) \geq (\gamma', \delta')$, then bipolar fuzzy (γ, δ)-translation $B^T_{(\gamma,\delta)}$ of B is a BAF ideal extention of bipolar fuzzy (γ', δ')- translation $B^T_{(\gamma',\delta')}$ of B.

Proof:

i. $(\gamma, \delta) \geq (\gamma', \delta')$ implies

$$\mu^+(x) + \gamma' \leq \mu^+(x) + \gamma$$
$$\mu^-(x) + \delta' \geq \mu^-(x) + \delta$$

$B^T_{(\gamma,\delta)}$ bipolar fuzzy extention of $B^T_{(\gamma',\delta')}$.

ii. Suppose $B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$, then $B^T_{(\gamma',\delta')}$ is not BAF ideal of $\mathcal{K}$.

- Because of $(\gamma, \delta) \geq (\gamma', \delta')$, it implies

$$\left.\begin{matrix}\mu^+(e) + \gamma' \leq \mu^+(e) + \gamma \\ \mu^+(x) + \gamma' \leq \mu^+(x) + \gamma\end{matrix}\right\} \rightarrow \mu^+(e) + \gamma' \leq \mu^+(x) + \gamma'$$

and

$$\left.\begin{matrix}\mu^-(e) + \delta' \geq \mu^-(e) + \delta \\ \mu^-(x) + \delta' \geq \mu^-(x) + \delta\end{matrix}\right\} \rightarrow \mu^-(e) + \delta' \geq \mu^-(x) + \delta'$$

- $\mu^+(x) + \gamma \leq \max\{\mu^+(x \odot y) + \gamma, \mu^+(y \odot (y \odot x)) + \gamma\}$

$$\mu^+(x)+\gamma \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}+\gamma$$
$$\mu^+(x) \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}$$
$$\mu^+(x)+\gamma' \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}+\gamma'$$
$$\mu^+(x)+\gamma' \le \max\{\mu^+(x\odot y)+\gamma',\mu^+(y\odot(y\odot x))+\gamma'\}$$
and
$$\mu^-(x)+\delta \ge \min\{\mu^-(x\odot y)+\delta,\mu^-(y\odot(y\odot x))+\delta\}$$
$$\mu^-(x)+\delta \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}+\delta$$
$$\mu^-(x) \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}$$
$$\mu^-(x)+\delta' \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}+\delta'$$
$$\mu^-(x)+\delta' \ge \min\{\mu^-(x\odot y)+\delta',\mu^-(y\odot(y\odot x))+\delta'\}$$

It is contrary with the statement above, it must be $B^T_{(\gamma',\delta')}$ BAF ideal of $\mathcal{K}$.

It can be conclude by $B^T_{(\gamma,\delta)}$ BAF ideal extention of $B^T_{(\gamma',\delta')}$.

Theorem 3.15

Let $B=(\mu^+,\mu^-)$ is a BAF ideal of K-algebra $\mathcal{K}$ and $(\gamma,\delta)\in[0,u]\times[i,0]$. For every BAF ideal extention $B'=(v^+,v^-)$ of bipolar fuzzy (γ,δ)-translation $B^T_{(\gamma,\delta)}$, there is $(\gamma',\delta')\in[0,u]\times[i,0]$ so that $(\gamma,\delta)\le(\gamma',\delta')$ and B' BAF ideal extention of bipolar fuzzy (γ,δ)-translation $B^T_{(\gamma,\delta)}$.

Proof:

i. Let $(\gamma',\delta')\in[0,u]\times[i,0]$ that implies $(\gamma,\delta)\le(\gamma',\delta')$.
$$\mu^+(x)+\gamma \le \mu^+(x)+\gamma'$$
$$\mu^-(x)+\delta \ge \mu^-(x)+\delta'$$
B' bipolar fuzzy extention of $B^T_{(\gamma,\delta)}$.

ii. Suppose B' BAF ideal of $\mathcal{K}$, then $B^T_{(\gamma,\delta)}$ is not BAF ideal of $\mathcal{K}$.

- $\mu^+(e)+\gamma' \le \mu^+(x)+\gamma'$ and $\mu^-(e)+\delta' \ge \mu^-(x)+\delta'$
  $\mu^+(e) \le \mu^+(x)$ $\quad$ $\mu^-(e) \ge \mu^-(x)$
  $\mu^+(e)+\gamma \le \mu^+(x)+\gamma$ $\quad$ $\mu^-(e)+\delta \ge \mu^-(x)+\delta$
- $\mu^+(x)+\gamma' \le \max\{\mu^+(x\odot y)+\gamma',\mu^+(y\odot(y\odot x))+\gamma'\}$
  $\mu^+(x)+\gamma' \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}+\gamma'$
  $\mu^+(x) \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}$
  $\mu^+(x)+\gamma \le \max\{\mu^+(x\odot y),\mu^+(y\odot(y\odot x))\}+\gamma$
  $\mu^+(x)+\gamma \le \max\{\mu^+(x\odot y)+\gamma,\mu^+(y\odot(y\odot x))+\gamma\}$
  and
  $\mu^-(x)+\delta' \ge \min\{\mu^-(x\odot y)+\delta',\mu^-(y\odot(y\odot x))+\delta'\}$
  $\mu^-(x)+\delta' \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}+\delta'$
  $\mu^-(x) \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}$
  $\mu^-(x)+\delta \ge \min\{\mu^-(x\odot y),\mu^-(y\odot(y\odot x))\}+\delta$
  $\mu^-(x)+\delta \ge \min\{\mu^-(x\odot y)+\delta,\mu^-(y\odot(y\odot x))+\delta\}$

It is contrary with the statement above, it must be $B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$.

B' is a BAF ideal extention of $B^T_{(\gamma,\delta)}$.

Definition 3.16 (Bipolar Fuzzy Multiplication)

Let $B=(\lambda^+,\lambda^-)$ is a bipolar fuzzy set of $\mathcal{K}$ and $\rho,\sigma\in[0,1]$. Bipolar fuzzy set $B^m_{(\rho,\sigma)}=(\lambda^{+m}_\rho,\lambda^{-m}_\sigma)$ is called bipolar fuzzy (ρ,σ)- multiplication of B with
$$\lambda^{+m}_\rho:\mathcal{K}\to[0,1], x\to\lambda^+(x)\rho$$
$$\lambda^{-m}_\sigma:\mathcal{K}\to[-1,0], x\to\lambda^-(x)\sigma$$
For every BAF ideal of B, Bipolar fuzzy (0,0)- multiplication $B^m_{(0,0)}$ is a BAF ideal of $\mathcal{K}$.

Theorem 3.17

If $B=(\lambda^+,\lambda^-)$ is a BAF ideal of K-algebra $\mathcal{K}$, then bipolar fuzzy (ρ,σ)- multiplication $B^m_{(\rho,\sigma)}$ of B is BAF ideal of $\mathcal{K}$.

Proof:

i. $\rho,\sigma\in[0,1]$ implies $\lambda^+(e)\rho \le \lambda^+(x)\rho$ and $\lambda^-(e)\sigma \ge \lambda^-(x)\sigma$.

ii. B BAF ideal of $\mathcal{K}$, implies
$$\lambda^+(x) \le \max\{\lambda^+(x\odot y),\lambda^+(y\odot(y\odot x))\}$$

$$\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}\rho$$
$$\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y)\rho, \lambda^+(y \odot (y \odot x))\rho\}$$

and

$$\lambda^-(x) \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$$
$$\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}\sigma$$
$$\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y)\sigma, \lambda^-(y \odot (y \odot x))\sigma\}$$

It can be conclude $B^m_{(\rho,\sigma)}$ of B is BAF ideal of $\mathcal{K}$.

Theorem 3.18

Let $B = (\lambda^+, \lambda^-)$ is a bipolar fuzzy set of K-algebra $\mathcal{K}$. Bipolar fuzzy (ρ, σ)- multiplication $B^m_{(\rho,\sigma)}$ of B is BAF ideal of $\mathcal{K}$ if and only if B BAF ideal of $\mathcal{K}$ for every $\rho, \sigma \in [0,1]$.

Proof:

i. $B^m_{(\rho,\sigma)}$of B is BAF ideal of $\mathcal{K}$, implies $\lambda^+(e)\rho \leq \lambda^+(x)\rho$ and $\lambda^-(e)\sigma \geq \lambda^-(x)\sigma$. Because of $\rho, \sigma \in [0,1]$ we obtain $\lambda^+(e) \leq \lambda^+(x)$ and $\lambda^-(e) \geq \lambda^-(x)$.

ii. $B^m_{(\rho,\sigma)}$ BAF ideal of $\mathcal{K}$, implies

$$\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y)\rho, \lambda^+(y \odot (y \odot x))\rho\}$$
$$\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}\rho$$
$$\lambda^+(x) \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$$

and

$$\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y)\sigma, \lambda^-(y \odot (y \odot x))\sigma\}$$
$$\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}\sigma$$
$$\lambda^-(x) \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$$

B BAF ideal of $\mathcal{K}$. Conversely, according to theorem 3.15 it is clear that $B^m_{(\rho,\sigma)}$ of B BAF ideal of $\mathcal{K}$.

Example 3.19

Based on the example 3.3 $B = (\lambda^+, \lambda^-)$ defined by

$$\lambda^+(x) = \begin{cases} 0.03 & , \quad x = e \\ 0.4 & , \quad x \neq e \end{cases} \text{ and } \lambda^-(x) = \begin{cases} -0.2 & , \quad x = e \\ -0.35 & , \quad x \neq e \end{cases}$$

BAF ideal of K-algebra $\mathcal{K} = (G, \circ, \odot, e)$ with $G = \{e, a, b, x, y, z\}$.
If $\rho = \sigma = 0.5$, then $B^m_{(\rho,\sigma)}$ BAF ideal of $\mathcal{K}$.

Theorem 3.20

If $B = (\lambda^+, \lambda^-)$ is bipolar fuzzy set of K-algebra $\mathcal{K}$, $(\gamma, \delta) \in [0, u] \times [i, 0]$ and $\rho, \sigma \in [0,1]$, for every bipolar fuzzy (γ, δ)-translation $B^T_{(\gamma,\delta)}$ of B is a BAF ideal extention of bipolar fuzzy (ρ, σ)- multiplication $B^m_{(\rho,\sigma)}$ of B.

Proof:

i. $\rho, \sigma \in [0,1]$, it is clear that $\lambda^+(x)\rho \leq \lambda^+(x) + \gamma$ and $\lambda^-(x)\rho \geq \lambda^-(x) + \gamma$. $B^T_{(\gamma,\delta)}$ bipolar fuzzy extention of $B^m_{(\rho,\sigma)}$.

ii. Suppose $B^T_{(\gamma,\delta)}$ BAF ideal of $\mathcal{K}$, then $B^m_{(\rho,\sigma)}$ is not BAF ideal of $\mathcal{K}$.

- $\lambda^+(e) + \gamma \leq \lambda^+(x) + \gamma$ and $\lambda^-(e) + \delta \geq \lambda^-(x) + \delta$
  $\lambda^+(e) \leq \lambda^+(x)$ $\qquad$ $\lambda^-(e) \geq \lambda^-(x)$
  $\lambda^+(e)\rho \leq \lambda^+(x)\rho$ $\qquad$ $\lambda^-(e)\sigma \geq \lambda^-(x)\sigma$
- $\lambda^+(x) + \gamma \leq \max\{\lambda^+(x \odot y) + \gamma, \lambda^+(y \odot (y \odot x)) + \gamma\}$
  $\lambda^+(x) + \gamma \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\} + \gamma$
  $\lambda^+(x) \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}$
  $\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y), \lambda^+(y \odot (y \odot x))\}\rho$
  $\lambda^+(x)\rho \leq \max\{\lambda^+(x \odot y)\rho, \lambda^+(y \odot (y \odot x))\rho\}$
  and
  $\lambda^-(x) + \delta \geq \min\{\lambda^-(x \odot y) + \delta, \lambda^-(y \odot (y \odot x)) + \delta\}$
  $\lambda^-(x) + \delta \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\} + \delta$
  $\lambda^-(x) \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}$
  $\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y), \lambda^-(y \odot (y \odot x))\}\sigma$
  $\lambda^-(x)\sigma \geq \min\{\lambda^-(x \odot y)\sigma, \lambda^-(y \odot (y \odot x))\sigma\}$

It is contrary with the statement above, it must be $B^m_{(\rho,\sigma)}$ BAF ideal of $\mathcal{K}$. $B^T_{(\gamma,\delta)}$ BAF ideal extention of $B^m_{(\rho,\sigma)}$.

IV. CONCLUSION

In this article, we introduced the notion of bipolar fuzzy translation, bipolar fuzzy extention, and bipolar fuzzy multiplication on bipolar anti fuzzy ideal of K-algebra. We also investigate the related properties. We hope this paper can be reference for future research about fuzzy.

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