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Modelling of running performances: comparisons of power-lawand logarithmic models in recreational runners.

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ABSTRACT:*A power-law modelhas been proposed for the relations between exhaustion time (t_{lim}) versus speed ($S = kt_{lim}^{g-1}$). A logarithmic model based on the decrease in the fractional use of maximal aerobic speed (MAS) was also proposed:100 S/MAS = 100 – EIln(t_{lim}/t_{MAS}) where t_{MAS} wast_{lim} corresponding to MASand slope EI an endurance index. In the present study, the relationships between speed S and t_{lim}in the power-law and logarithmic models have been compared for the values of g and EI which correspond to the same running speed at t_{lim} equal to X fold t_{MAS} (i.e. EI = (100 - 100 $(X)^{g-1}$) /ln(X)) for g equal to 0.80 (low-endurance runners), 0.90 (medium-endurance runners)and 0.95 (high-endurance runners). The shortest and largest ranges oft_{lim} corresponded to X = 2.5 and 20.For each value of X (2.5, 5, 10 and 20), the curves of the relationships corresponding to power-law and logarithmic models are superimposed for the high-endurance runners and almost superimposed for medium-endurance runners.In low-endurance runners, the difference between the curvesof power-law and logarithmic models was low (≤ 0.42%) only for the shortest range of t_{lim} (X = 2.5).Therefore, it is probably impossible to know the best model (power-law or logarithmic) when the range of performances is short (< 20 min) as in most studies on the modeling of performancesin recreational runners.A medium range between 3.5 and 35 min would be better to compare the logarithmic and power-law models.*

KEYWORDS: *Endurance, testing,model of Kennelly, model ofPeronnet-Thibault,*

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I. INTRODUCTION

1.1 Power law model (Kennelly)

In 1906, Kennelly [1] studied the relationship between running speed (S) and the time of the world records (t_{lim}) and proposed a power law:

$D_{lim} = k\, t_{lim}^{g}$ Eq.1

where k is a constant and g an exponent. This power law between distance and time corresponds to a power law between time and speed (S):

$S = D_{lim}/ t_{lim} = k\, t_{lim}^{g}/ t_{lim} = k\, t_{lim}^{g-1}$ Eq.2

Kennelly's model has been applied the best performances of elite endurance runners [2, 3, 4, 5, 6]. Exponent g of the power law model has a high interest because it can be demonstrated that it is a dimensionless index of endurance [7].Indeed, the curvature of the D_{lim}-t_{lim} equation depends on exponent g.In the elite endurance runners, the D_{lim}-t_{lim} equation is almost perfectly linear and exponent g is close to 1. In runners who are not endurance athletes, the D_{lim}-t_{lim} equationis more curved and exponent g is lower than 0.9.

In theory, parameter k should be correlated to maximal running speed [7] because k is equal to the maximal running speed corresponding to one second. Indeed, when t_{lim} is equal to 1seconde:

$S = k\, t_{lim}^{g-1} = k * 1^{g-1} = k * 1 = k$

However, parameter k is not dimensionless unlike exponent g. Indeed, if the running performances are evaluated in minutes, parameter k would be equal to the maximal speed corresponding to 1 minute.

The power laws between t_{lim} and D_{lim}can be determined by computing the regression between the natural logarithms of D_{lim} and t_{lim}:

$\ln(D_{lim}) = \alpha + \gamma \ln(t_{lim}) = \ln(k) + g \ln(t_{lim})$

$g = \gamma$ and $k = e^{\ln(k)} = e^{\alpha}$

In 1981, a similar power-law model was proposed by Riegel [8]:

$t_{lim} = aD_{lim}^{b}$
$S = D_{lim}/t_{lim} = D_{lim} / aD_{lim}^{b} = (D_{lim}^{1-b})/a$
As $D_{lim} = kt_{lim}^{g}$
$D_{lim}^{1/g} = (kt_{lim}^{g})^{1/g} = k^{1/g}t_{lim}$
$t_{lim} = D_{lim}^{1/g}/ k^{1/g} = aD_{lim}^{b}$
$a = k^{1/g}$and $b = 1/g$
In contrast with Kennelly's model, Riegel's model enables to estimate the performance (t_{lim}) of another distance. These equations of Riegel have recently been applied to a large study on 2303 recreational endurance runners [9].

1.2. Logarithmic model (Péronnet-Thibault)

In 1989, Péronnet and Thibault [10] proposed a model that took into account the contributions of aerobic and anaerobic metabolism to total energy output in function of the duration of the race. A runner is only capable of sustaining his maximal aerobic speed (MAS) for a finite period of time (t_{MAS}). Péronnet and Thibault proposed the slope of the relationship between the natural logarithm of running duration (EI) and the fractional utilization of MAS as an index of endurance capability.
$S = MAS – E \ln(t_{lim}/t_{MAS})$ Eq. 3
where E is the slope of the regression.
Then
$100 S/MAS = 100 – EI\ln(t_{lim}/t_{MAS})$ Eq. 4
where EIis an endurance index equal to 100 E/MAS

This endurance index was significantly related ($r = 0.853$) to ventilatory threshold, expressed as a percentage of MAS, in a group of marathon runners [11]. The lower the absolute value of EI, the higher the endurance capacity is assumed to be. For example, endurance indexes computed from personal best performances were equal to 8.14 for Ryun, an elite middle-distance runner and 4.07, for Derek Clayton, an elite long-distance runner. In the study byPéronnet and Thibault, the value of t_{MAS} was assumed to be equal to 7 minutes (420 s):
$100 S/MAS = 100 – EI\ln(t_{lim}/t_{420})$ Eq. 5
The value of EI can be estimated by the regression between S and the natural logarithm of $t_{lim}/420$ for the different distances:
$S = \alpha – \beta\ln (t_{lim}/420)$
When $t_{lim} = 420$, S is equal toMAS and $\ln(t_{lim}/420)$ is equalto 0.Therefore
$S = MAS = \alpha+ 0= \alpha$
$EI = 100\beta/MAS = 100\beta/\alpha$

This model does not enable the prediction of the performances of other distances. However, the estimation of the performances of other distances can be estimated from a nomogram [12]. The validity of parameter EI as an endurance index is questionable because MAS is computed assuming that the value of t_{lim} corresponding to MAS (t_{MAS}) is equal to 7 min (420s) [10],However, the value of$t_{MAS}$is probably close to 6 min in recreational athletes [13]. On the other hand, $t_{MAS}$is perhaps close to 14 min in elite endurance runners [14, 15].

1.3. Comparison of power law and logarithmic models in recreational runners.

The models of Kennelly (power law model) and Péronnet-Thibault (logarithmic model) have been compared [7, 16, 17] in elite endurance runners. In these studies, the values of exponent g and EI are highly correlated ($r > 0.99$). In a study [16], it was suggested that the estimations of the individual relationships between speed (S) and t_{lim} were similar in elite endurance runners whose values of g were between 0.90 and 0.95. But, in this study, the estimation of the relationship between speed S and t_{lim} was different when exponent g was low (0.80) and the range of t_{lim} was large (from 1 to 20 fold t_{MAS}). However, in a recent study [18] on low-endurance runners (physical education students) whose values of exponent g was lower than 0.90, the correlation between g and EI was significant ($r = 0.826$) for a small range of t_{lim} ($t_{lim}< 20$ min).

Some recreational runners are high-endurance runners but others are medium and low-endurance runners.In the present theoretical study, the speed-time curves of the power-law and logarithmic models have been compared for high-endurance runners ($g = 0.95$), medium-endurance runner ($g = 0.90$)and low-endurance runners ($g = 80$) for short and large ranges of t_{lim}in order to know which model is the best for recreational runners.

II. METHOD

The logarithmic model is normalized to t_{MAS}:
$100 S/MAS = 100 – EI\ln(t_{lim}/t_{MAS})$

The power-law model can also be normalized to t_{MAS} for t_{lim} and normalized to maximal aerobic speed (MAS) for S. For t_{lim} equal to t_{MAS}, the running speed corresponds to MAS:
$S = MAS = k\ t_{MAS}{}^{g-1}$
$k = MAS /(t_{MAS}{}^{g-1}) = MAS\ t_{MAS}{}^{1-g}$
Therefore:
$S = (MAS\ t_{MAS}{}^{1-g})\ t_{lim}{}^{g-1}$
$S/MAS = (t_{MAS}{}^{1-g})\ t_{lim}{}^{g-1} = (1/t_{MAS})^{g-1} (t_{lim})^{g-1} = (t_{lim}/t_{MAS})^{g-1}$
$100\ S/MAS = 100\ (t_{lim}/t_{MAS})^{g-1}$ Eq. 6
It is possible to compare the curves of the power-law and logarithmic model for a range of t_{lim}/t_{MAS} that corresponds to the same value of S for both models at the higher value of t_{lim}/t_{MAS}. If X corresponds to the higher value of t_{lim}/t_{MAS}:
$S/MAS = 100\ (X)^{g-1}$ for power law model
$S/MAS = 100 – EIln(X)$ for logarithmic model
Therefore, the value of EI for a given value of g is:
$100\ (X)^{g-1} = 100 – EIln(X)$
$EI = (100 - 100\ (X)^{g-1})\ /ln(X)$ Eq. 7
Similarly, the value of exponent g for a given value of EI is:
$100\ (X)^{g-1} = 100 – EIln(X)$
$(X)^{g-1} = 1 – EIln(X)/100$
$ln[(X)^{g-1}] = ln(1 – EIln(X)/100)$
$(g – 1)ln(X) = ln(1 – EIln(X)/100)$
$g = 1 + [ln(1 – EIln(X)/100)/ ln(X)]$ Eq. 8

However, the main objective of the present study is the comparison of the power-law and logarithmic models in low-endurance runners whose values of exponent g are low (≤ 0.80). Therefore, the power-law and logarithmic models were compared only with the values of EIcomputed with equation 7for the same values of exponent g (0.80, 0.90 and 0.95) for X equal to 20, 10, 5 and 2.5. Indeed, the values of g computed from EIwith equation 8increase when X decreases and exponent g becomes higher than 0.80. For example, for X = 2.5, the values of g computed with equation 7 for the values of EI equal to 4.64, 8.64 and 15 (as for X = 20) are 0.953, 0.910 and 0.839, respectively. The relationships between S/MAS and t_{lim}/t_{MAS} have been computed from equation4for the logarithmic model and equation 6 forthe power-law model.

III. RESULTS

In Figure 1A, for a large range of t_{lim} (X = 20), the values of EIcorresponding to the values of g equal to 0.95, 0.90 and 0.80 are 4.64, 8.64 and 15.0, respectively. In this case, the speed-time curves of the power-law model (blue curves) corresponding to the value of exponent g equal to 0.90 or 0.95 are superimposed with the speed-time curves of the logarithmic model (red curves). In contrast, for g = 0.80, the curves are not superimposed and the curve of the logarithmic model is higher. The maximal speed difference between the logarithmic and power law models corresponds to a 4.67 % overestimation at t_{lim}/t_{MAS} equal to 5.3. Therefore, the power-law and logarithmic models describe the performances of the low-level endurance runners differently when the range of t_{lim} is very large.

In Figure 1B, for X= 2.5, the values of EIcorresponding to the values of g equal to 0.95, 0.90 and 0.80 are 4.89, 9.56 and 18.3, respectively. As for X = 20, the speed-time curves of the power-law model are superimposed with the speed-time curves of the logarithmic model when exponent g is equal to 0.95or 0.90. However, the maximal difference between the logarithmic and power law models for exponent g = 0.80 is also very low and the maximal “overestimation” is only equal to 0.42 % for t_{lim}/t_{MAS} = 1.6.

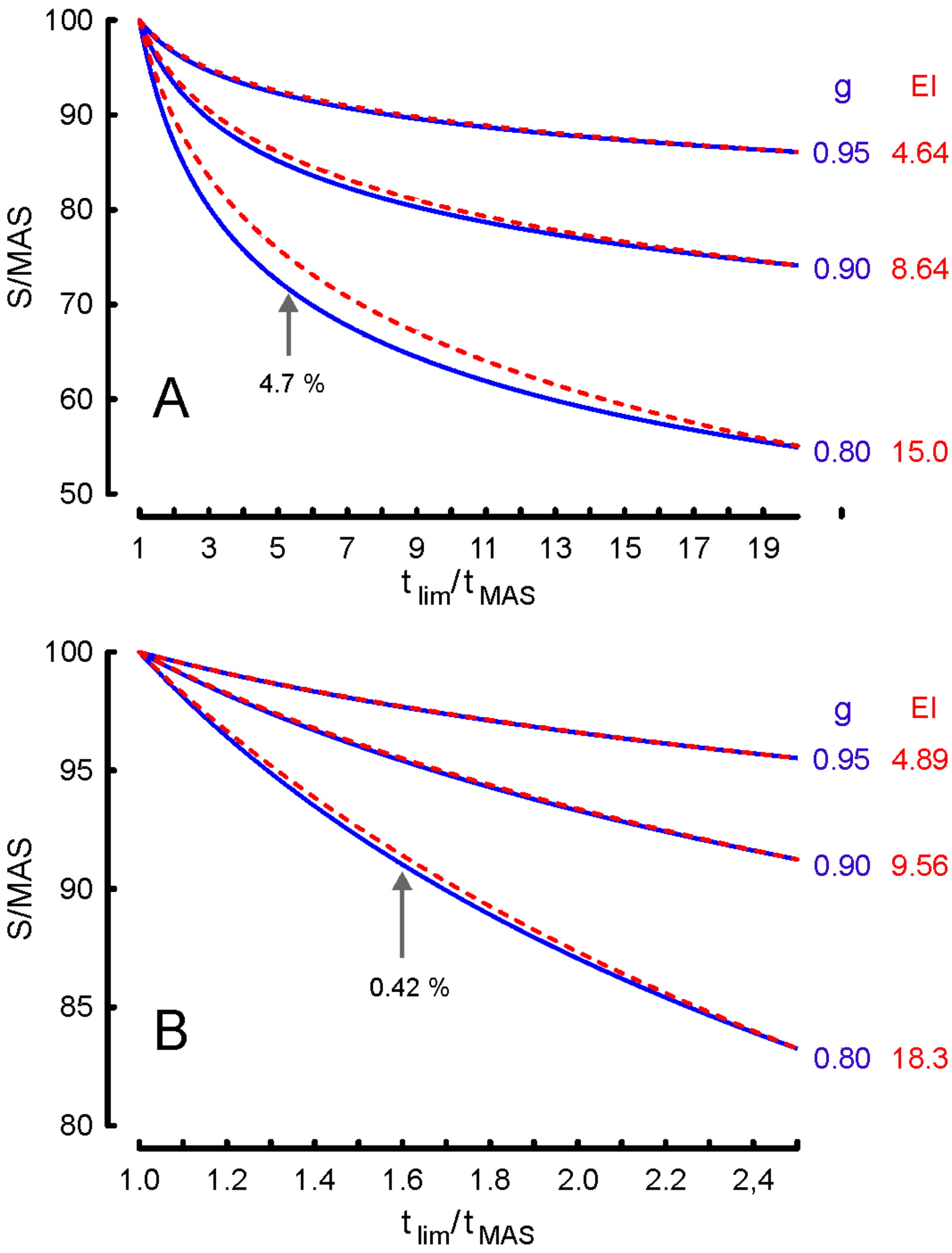

Figure 1: theoretical speed-time curves computed from the power-law model (blue solid curves) and logarithmic model (dashed red curves) with values of slope EI corresponding to the same values of S at t_{lim}/t_{MAS} equal to 20 in A and t_{lim}/t_{MAS} equal to 2.5 in B . Vertical arrows: maximal difference between power-law and logarithmic models for g = 0.80.

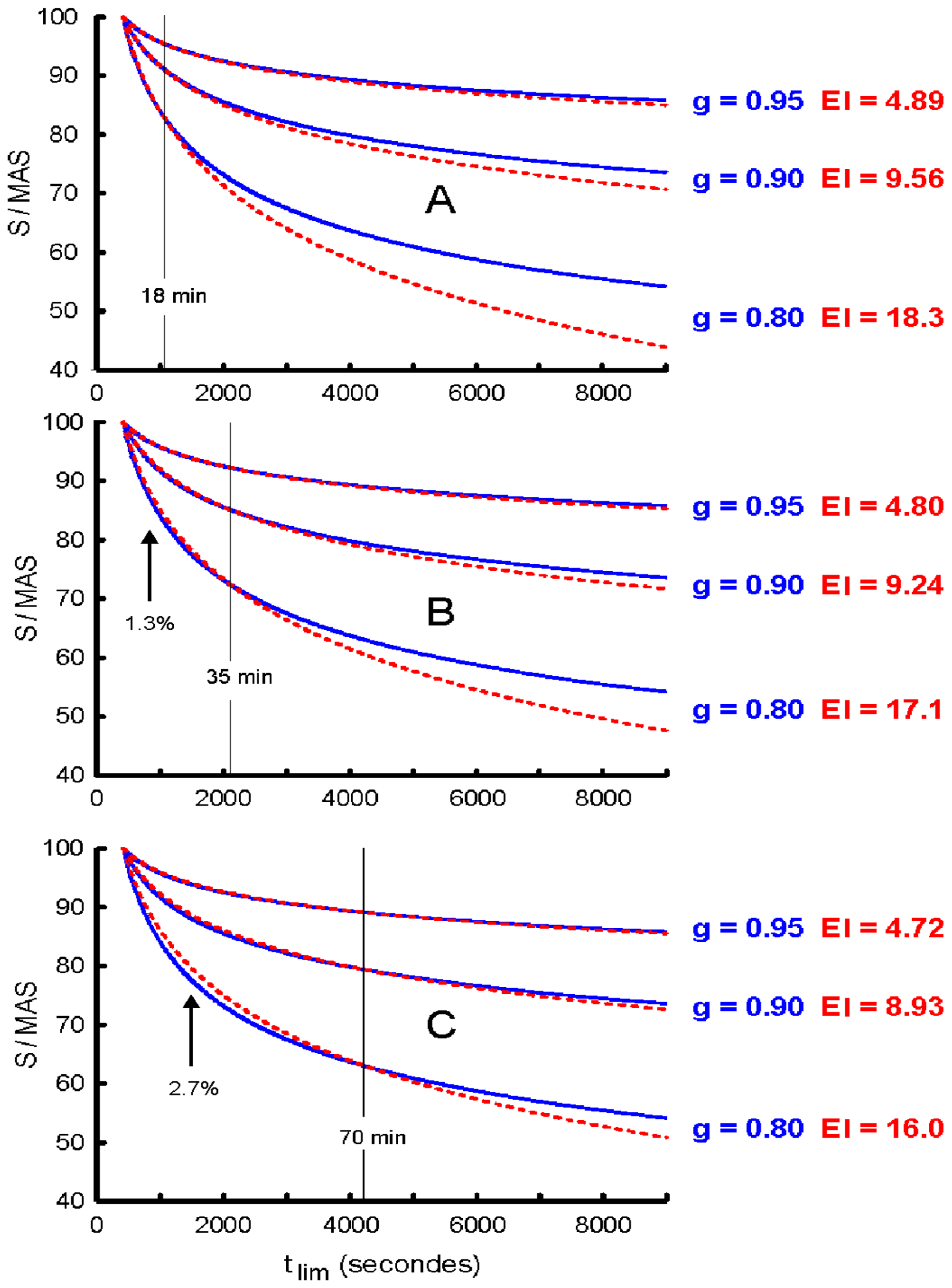

Figure 2:theoretical speed-time curves computed from the power-law model (blue solid curves) and logarithmic model (dashed red curves) andextrapolation at 9000s (t_{MAS} is assumed to be equal to 7 min) for Xequal to 2.5 (A), 5 (B) and 10 (C).Vertical arrows: maximal difference between power-law and logarithmic models for g = 0.80.

The logarithmic model was proposed for t_{lim} equal and longer than t_{MAS} ($t_{lim} \geq 7$ min). But, the other models often include shorter values of t_{lim}(about 3.5 min). For t_{lim} = 3.5 min, the curves of power-law model arehigher than those of logarithmic model and the difference are equal to 1.9 % (for X = 2.5), 2.6% (for X = 5) and 3.3 % (for X= 10).The range of t_{lim} from 3.5 to 35 min is equivalent to X = 10 (figure 2C) when t_{MAS} is equal to 3.5 min.

There are larger differences between the power-law model and the logarithmic model at times higher than 2hoursfor X equal to 2.5 (Fig 2A), 5 (Fig 2B) and 10 (Fig 2C).These differences are lower when X is higher.For X = 2.5 (fig. 2A), at t_{lim} = 9000 s, the differences are: 23.4% (g = 0.80), 4.1% (g = 0.90) and 0.9% (g = 0.95).In Figure 2C, for X = 10, at t_{lim} = 9000 s, the differences between power-law and logarithmic models are lower: 6.5 % (g = 0.80), 1.3% (g = 0.90) and 0.3% (g = 0.95).

IV. DISCUSSION

For high and medium-endurance runners (g = 0.90 and 0.95), the curves of the relationships between running speed and exhaustion time for power-law and logarithmic models are superimposed for X equal to 20 (fig. 1A), 2.5 (fig. 1B and 2A), 5 (fig. 2B) and 10 (fig. 2C). In contrast, the difference between power-law and logarithmic models is not negligible for a low-endurance runner whose exponent g is equal to 0.80 when X is high (fig 1A and fig. 2C). These results are in agreement with those of a previous study [17]. On the other hand, the curves of power-law and logarithmic models are almost superimposed for a low-endurance runner (fig 1B) when the range of t_{lim}/t_{MAS} is low (X = 2.5). Therefore, it will be difficult to study which model (power law or logarithmic) is the best for low-endurance runners when the range of t_{lim} is short, i.e. between 7 min and 20 min.The range of performances used in most of the studies on the modelling of running performances in recreational runners is about 3.5 and 15-20 min.The difference between the logarithmic and power-low models are higher at t_{lim} = 3.5 min. A medium range between 3.5 and 35 min would be better to compare the logarithmic and power-law models. A higher range (X = 10, t_{MAS} = 7 min) would be difficult in low-endurance runners becausea one-hour performance would be perhaps not reproducible if these runners do not practice long distance races.

In a recent study [9], a model adapted from Riegel’s model (equivalent to Kennelly’s model) [8] has been applied to recreational runners. In this study, the prediction of time was good for races up to half a marathon. However, theactual time of the marathon was 5% higher than the time predicted from the shorter distances with Riegel’s model [9, figure 2C]. This overestimation is similar to the difference between power-law and logarithmic models for g = 0.90whenX = 2.5 (fig. 2A) and for g = 0.80 whenX = 10 (fig. 2B).Therefore, it is possible that the prediction of the marathon performance would be better with the logarithmic model than with the modelsof Kennellyand Riegel. In figure 3, the relationships between running speed (S) and exhaustion time (t_{lim}) have be computed from the performances on 1500, 3000, 5000 and 10000 m on a track with the power-law and logarithmic models for 3 elite endurance runners who also participated and won marathon races [7].Themarathon performances were overestimated by both models but the overestimations(6.1% for Zatopek, 3.4% for Viren, and 2.1% for Gebrselassié) were slightly lower with the logarithmic models.It is likely that the overestimations of marathon performances by both models in fig. 3werepartly due to the use of lipids. Indeed, the lipids are used because glycogen stores are not sufficient for very long distances. But P/O ratio is lower for the lipids than for the carbohydrates. Therefore, the oxygen consumption is higher for the same production of ATP and the running speed corresponding to a given fraction of V_{02}max is lower. In the elite endurance runner in figure 3, the prediction of marathon performances corresponded tothe 1500 and 10000 m performances, i.e. X < 5, and the use of lipids was probably very low during these races, which could partly explain the overestimations. Moreover, the overestimations of marathon performanceswere perhaps also the the effects of age, shoes, ground (track versus road, slopes of the roads...). However, these overestimations were much lower than the overstimation by the asymptotic models generally used for modelling the endurance performances (hyperbolic model, Morton’s model and exponential model of Hopkins) [7]. In these elite runners, the logarithmic model was perhaps the best model that enables to predict marathon performance.In the study by Vickers and Vertosick on running performances in recreational athletes [9], the prediction of marathon from shorter distances included the performances of half-marathons, which corresponds to a value of X about 10, that is, races with the use of lipids.

V. CONCLUSION

The curves of the relationships between S and t_{lim} are similar for power-law and logarithmic models for the high and medium-endurance runners, whatever the range of t_{lim}. When the range of t_{lim}/t_{MAS} is short (X = 2.5), the difference between power-law and logarithmic models is also very small for a low-endurance runner whose exponent g = 0.80. Then, it will be probably difficult or impossible to know whichis the best model in low-endurance runners when both models will be applied to running performances lower than 20 min. Therefore, the results of both models should be similar in a group of recreational runners including low, medium and high-endurance runners when the range of t_{lim} is short. A medium range between 3.5 and 35 min would be better to compare the logarithmic and power-law models.

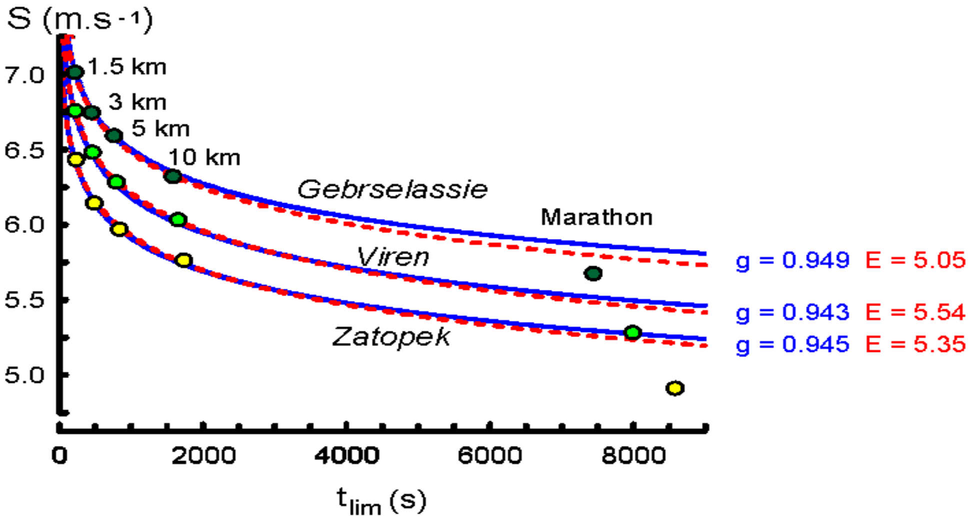

Figure 3: Relationships between running speed (S) and exhaustion time (t_{lim}) computed from the performances on 1500, 3000, 5000 and 10000 m with the power-law (blue curves) and logarithmic (dashed red curves) models for 3 elite endurance runners who also participated also to marathon (adapted from [7]).

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